– Generalized Fermi Acceleration –
application to relativistic outflows

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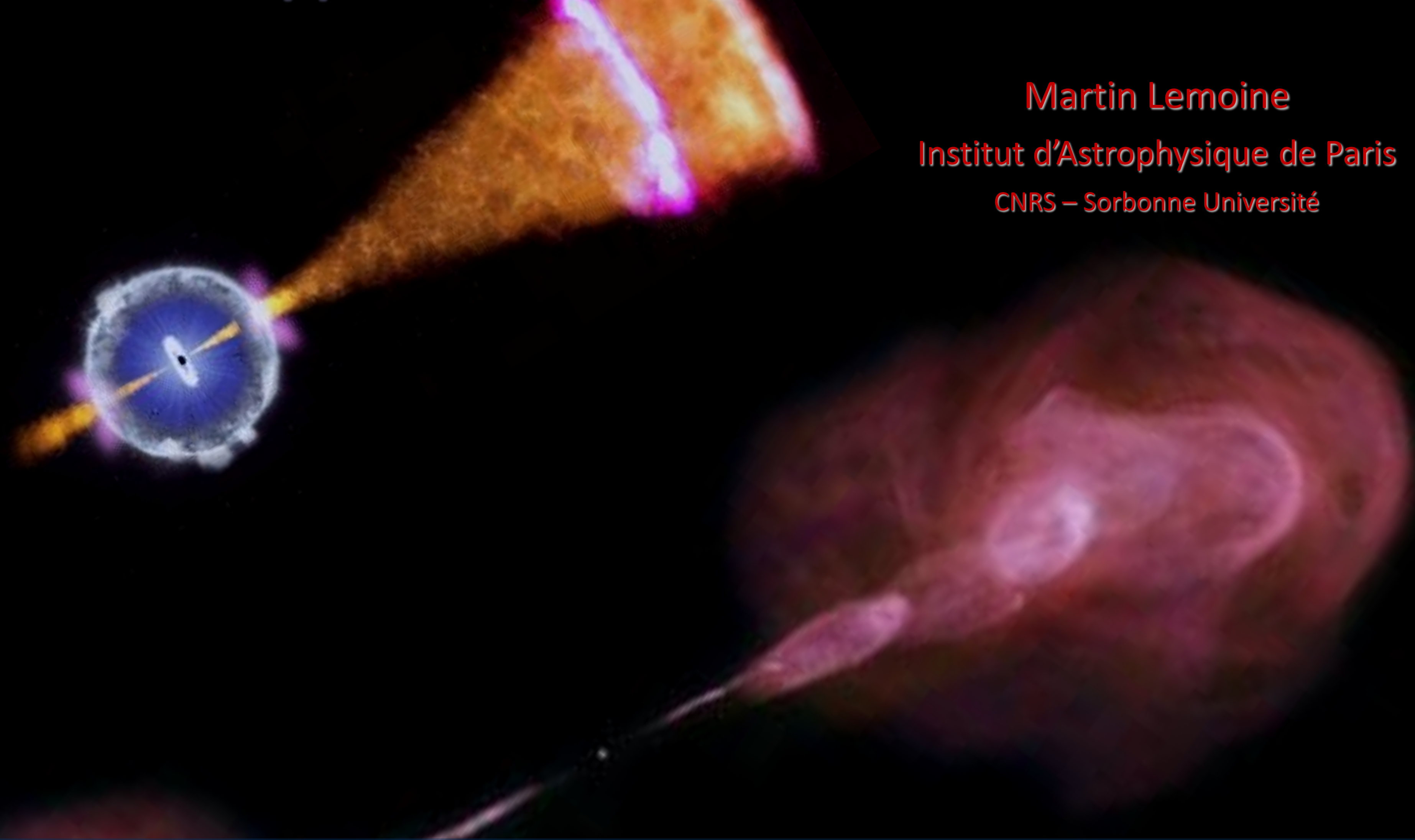
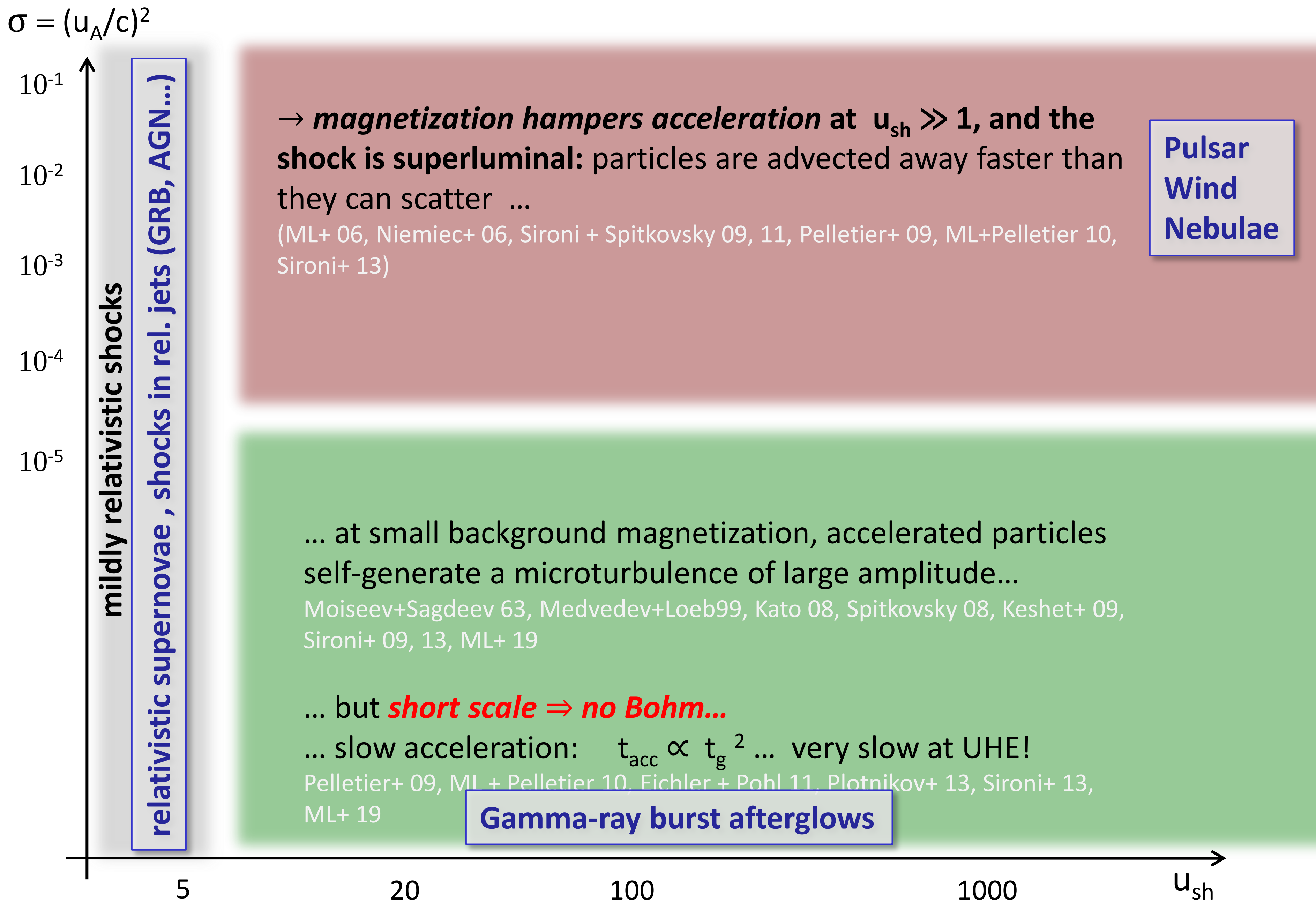


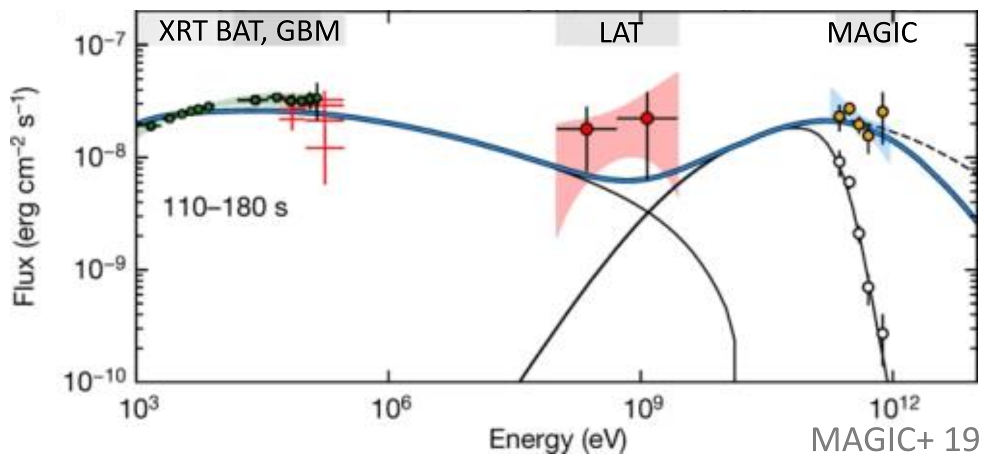
Particle acceleration at relativistic shock waves



GRB afterglows from unmagnetized collisionless shock waves

<u>GRB190114C:</u> an afterglow seen up to TeV energies…

… standard SED modeling gives:



MAGIC+ 19

$\epsilon_B \lesssim 10^{-4}$: self-generated turbulence, which decays through collisionless damping (+regeneration?)… e.g. Gruzinov 99, Rossi+Rees 03, ML 13, 15, 16, ML+ 13

$\epsilon_e \sim 0.1$: self-generated microturbulence preheats the electrons to average Lorentz factor ~ $\Gamma_{sh}\, m_p / m_e \sim 10^4 - 10^5$… e.g. Waxman 97, Spitkovsky 08, Martins+ 09, Sironi+ 13
… possibly through the generation of an electrostatic potential in the shock precursor, e.g. Vanthieghem+ 19, in prep, ML+ 19

<u>Expected maximal energies from microphysics:</u>

$E_{e,\,\mathrm{max}} \simeq 1 - 10\,\mathrm{TeV}$ from scattering in microturbulence, e.g. Kirk + Reville 10, Plotnikov+ 13

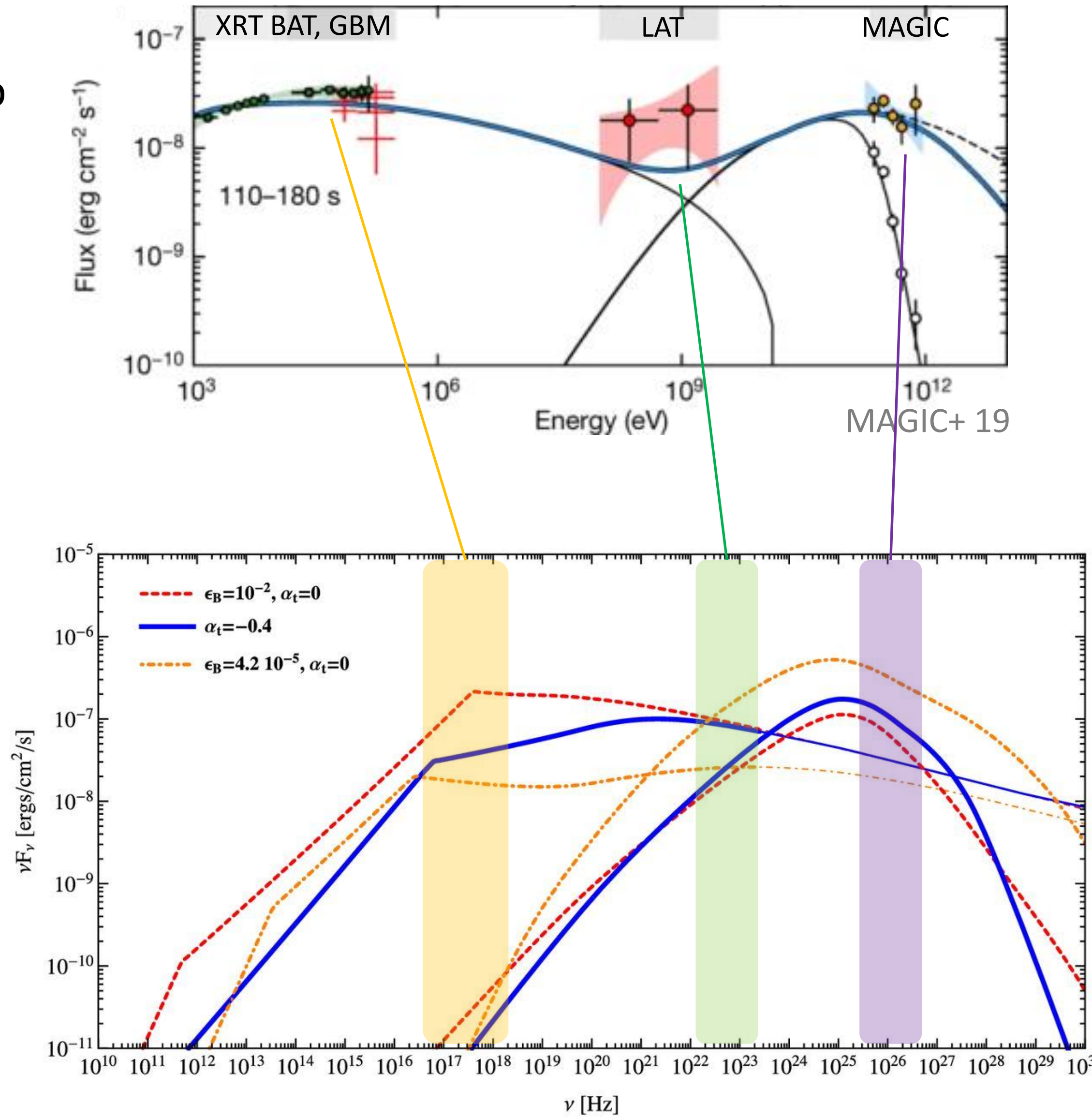
$\epsilon_{\gamma,\,\mathrm{syn\,max}} \sim 1\,\mathrm{GeV}$ e.g. Kirk + Reville 10, Plotnikov+ 13, Wang+ 13, Sironi+ 13

GRB afterglows from unmagnetized collisionless shock waves

<u>GRB190114C:</u> an afterglow seen up to TeV energies…

… *Compton dominance is a generic prediction of particle acceleration at unmagnetized relativistic collisionless shocks…* as $\epsilon_B \ll \epsilon_e$ from self-generation of microturbulence and subsequent dissipation…

… afterglow of blast waves with dissipative turbulence at large Y:
(ML 15)



note: SED with arbitrary parameters, at 30sec

Fermi acceleration

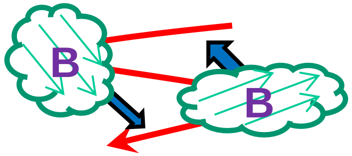
Ideal MHD:

$\rightarrow$ ***E*** field is 'motional', i.e. if plasma moves at velocity $\boldsymbol{\beta}_{\mathrm{p}}$: $\mathbf{E} = -\boldsymbol{\beta}_{\mathrm{p}} \times \mathbf{B}$

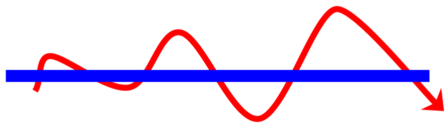
$\rightarrow$ need scattering to push particles across *B*

$\Rightarrow$ t_{acc} scales with the scattering time t_{scatt} (time needed to enter random walk)

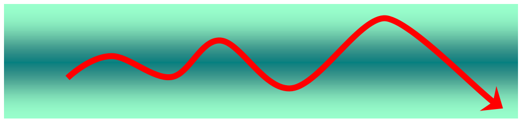
$\rightarrow$ examples: - turbulent Fermi acceleration



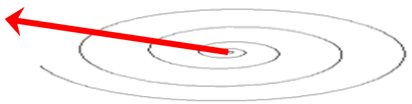
- Fermi acceleration at shock waves



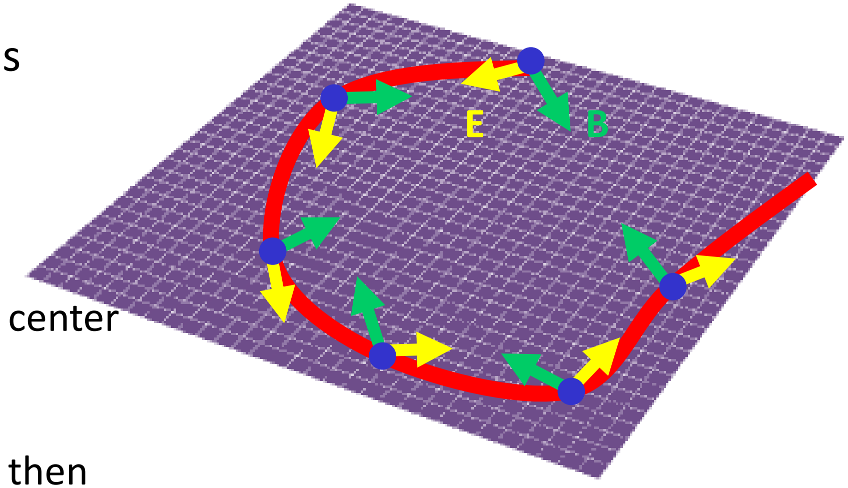
- acceleration in sheared velocity fields



- magnetized rotators



Characterization of Fermi acceleration



General problem: integrate trajectories of particles in a non-uniform **(E,B)** configuration…

→ various schemes depending on the situation…

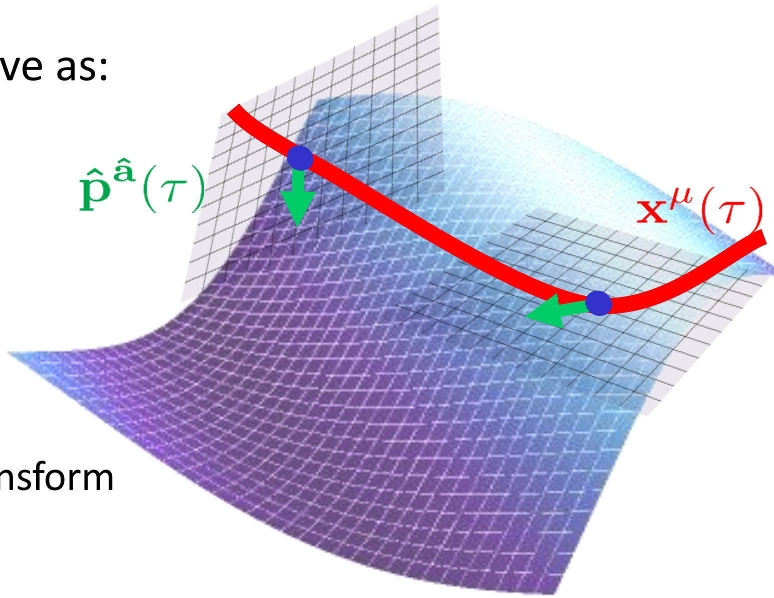
- explicit integration of trajectories with guiding center approximations in known E,B…
- in Fermi 1, go to local rest frame downstream, then upstream, then downstream…
- quasilinear theory: assume particles propagate on straight orbits and accumulate energy increments…
- in complex flows, derive transport equation and identify energy gain terms…

Present scheme: at each point along the particle trajectory, define: $\boldsymbol{\beta}_{\mathbf{u}} = \dfrac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}$

… and deboost by this velocity to go to the reference frame in which **E** vanishes to compute the force (elastic scattering on B!), model trajectory as random walk…
… here a local transform, hence a problem of general relativity …

Generalized Fermi acceleration

→ in the comoving locally inertial frame, momenta evolve as:



$$\frac{\mathrm{d}\hat{p}^{\hat{a}}}{\mathrm{d}\tau} = \underbrace{\frac{q}{m}\,\widehat{F}^{\hat{a}}{}_{\hat{b}}\,\hat{p}^{\hat{b}}}_{\boxed{\text{Lorentz force}}} - \underbrace{\widehat{\Gamma}^{\hat{a}}_{\hat{b}\hat{c}}\,\frac{\hat{p}^{\hat{b}}\hat{p}^{\hat{c}}}{m}}_{\boxed{\text{inertial correction}}}$$

$$\hat{p}^{\hat{a}} = e^{\hat{a}}{}_{\mu}\,p^{\mu} \quad \begin{cases} p^{\mu} & \text{: in lab frame} \\ e^{\hat{a}}{}_{\mu} & \text{: $\sim$ local Lorentz transform} \\ \hat{p}^{\hat{a}} & \text{: in comoving frame} \end{cases}$$

→ in particular: $$\frac{\mathrm{d}\hat{p}^{\hat{0}}}{\mathrm{d}\tau} = e^{\beta}{}_{\hat{b}}\,e^{\gamma}{}_{\hat{c}}\,\nabla_{\gamma}e^{\hat{0}}{}_{\beta}\,\frac{\hat{p}^{\hat{b}}\hat{p}^{\hat{c}}}{m}$$

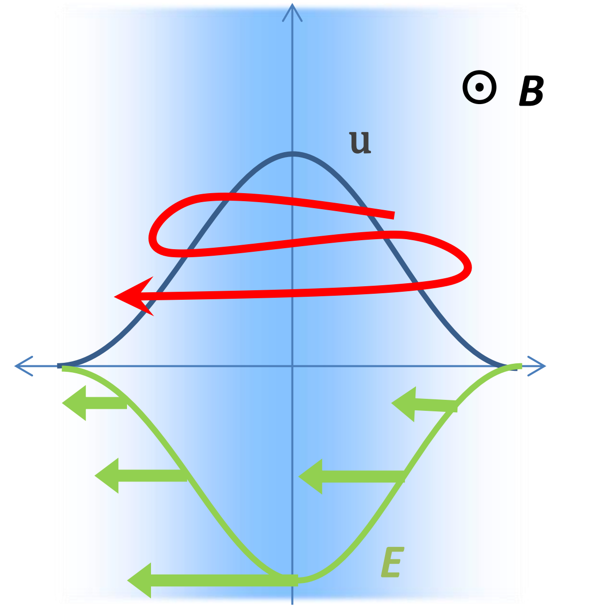
… and the vierbein $\approx$ u^{μ} : spacetime dependence of u is essential for Fermi acceleration… E and B have disappeared from the calculation…

→ **calculation scheme**:

(a) follow spatial coordinates in global frame, momenta in comoving frame…
(b) Lorentz force acts on angular part of momentum in comoving frame, not on energy, eventually leading to spatial diffusion with typical scattering time t_s.
(c) integrate over typical trajectory to obtain $\langle \Delta p^{\alpha}/\Delta t\rangle$ and $\langle \Delta p^{\alpha}\Delta p^{\beta}/\Delta t\rangle$, e.g.

$$\left\langle \frac{\Delta p^t}{\Delta t}\right\rangle = \lim_{\Delta t\to+\infty}\frac{1}{\Delta t}\left\langle e^{t}{}_{\hat{a}}(\Delta\tau)\int_0^{\Delta\tau}\mathrm{d}\tau_1\,\frac{\mathrm{d}\hat{p}^{\hat{a}}}{\mathrm{d}\tau_1}\right\rangle + \dots$$

Particle acceleration in sheared velocity flows



*particles with larger mean free paths explore larger gradient of **E** ⇒ faster acceleration…*

<u>Fermi shear acceleration:</u>

… the electric field in a sheared velocity flow cannot be boosted away globally: particles gain energy by exploring the shear gradient…
(e.g. Rieger+Duffy 04, 06, 08, Liu+ 17, Rieger 19, Webb+ 18,19, ML 19)

… acceleration timescale:

$$t_{\rm acc} \sim \frac{\Delta r^2}{t_{\rm scatt}} \frac{1}{\Delta u^2/\gamma_u^2}$$

⇒ inefficient at low energies, since $t_{scatt} \nearrow p$, requires a seed population of particles

… $t_{acc} \lesssim t_{esc} \sim \Delta r^2 / t_{scatt}$ if $\Delta u \gtrsim 1$: optimal for (mildly?) relativistic shear!

… if $L_{max} \sim \Delta r$, at confinement energy $r_g \sim \Delta r$, ⇒ $t_{\rm scatt} \sim r_g \sim \Delta r$ ⇒ $t_{\rm acc} \sim r_g$ for $\Delta u \sim u \sim 1$

⇒ reacceleration of a population of energetic CRs in mildly relativistic shear may reach confinement energy… (radiative signatures? beyond test-particle?)

Stochastic Fermi acceleration in BH environments

Stochastic gravito-centrifugo-shear acceleration near a (Schwarzschild) BH:

→ circular flow at angular velocity Ω(r): $u^\mu = \gamma_u(r)\,[1, 0, \Omega(r), 0]$

→ flow carries turbulence, which provides a source of scattering…
acceleration takes place through interaction with sheared flow…
(e.g. Rieger+Mannheim 02)

→ global frame: $\mathcal{R}_{\rm L}$= frame in which central object at rest..

→ at the equator,

$$D_{\hat p\hat p} = \hat p^2 \frac{\hat t_{\rm scatt}}{3r^2\left[1+\left(1-\frac{r_{\rm H}}{r}\right)\ell^2/r^2\right]^2} \times \left\{\left[\left(1-\frac{r_{\rm H}}{r}\right)^{3/2}\frac{\ell^2}{r^2} - \left(1-\frac{r_{\rm H}}{r}\right)^{-1/2}\frac{r_{\rm H}}{r}\right]^2 + \frac{\sqrt{2}}{6}\,r^4{\Omega_{,r}}^2\right\}$$

centrifugal term

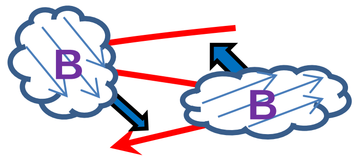
$$\ell = \frac{r^2\,\Omega}{1-\frac{r_{\rm H}}{r}}$$

gravitational term

shear term

Particle acceleration in relativistic magnetized turbulence

Fermi model for acceleration:



… particle interaction with random moving scattering centers…
… acceleration becomes stochastic with diffusion coefficient:

$$t_{\rm acc} \sim \frac{t_{\rm int}}{\beta_u^2}$$

… what are $t_{\rm int}$? $\beta_{\rm u}$?

… (quasilinear theory of) resonant wave-particle interactions in wave turbulence:

$t_{\rm int} \sim t_{\rm scatt}$, $\beta_{\rm u} \sim \beta_{\rm A}$… (Kennel + Engelmann 66… e.g. Schlickeiser 89)
[restricted to weak wave turbulence]

… however, in (modern) anisotropic turbulence, resonances disappear…
(Chandran 00, Yan+Lazarian 02, Lynn+ 14, Demidem+ 19)

$$t_{\rm acc} \sim \frac{L_{\rm max}}{\beta_{\rm A}^2}$$

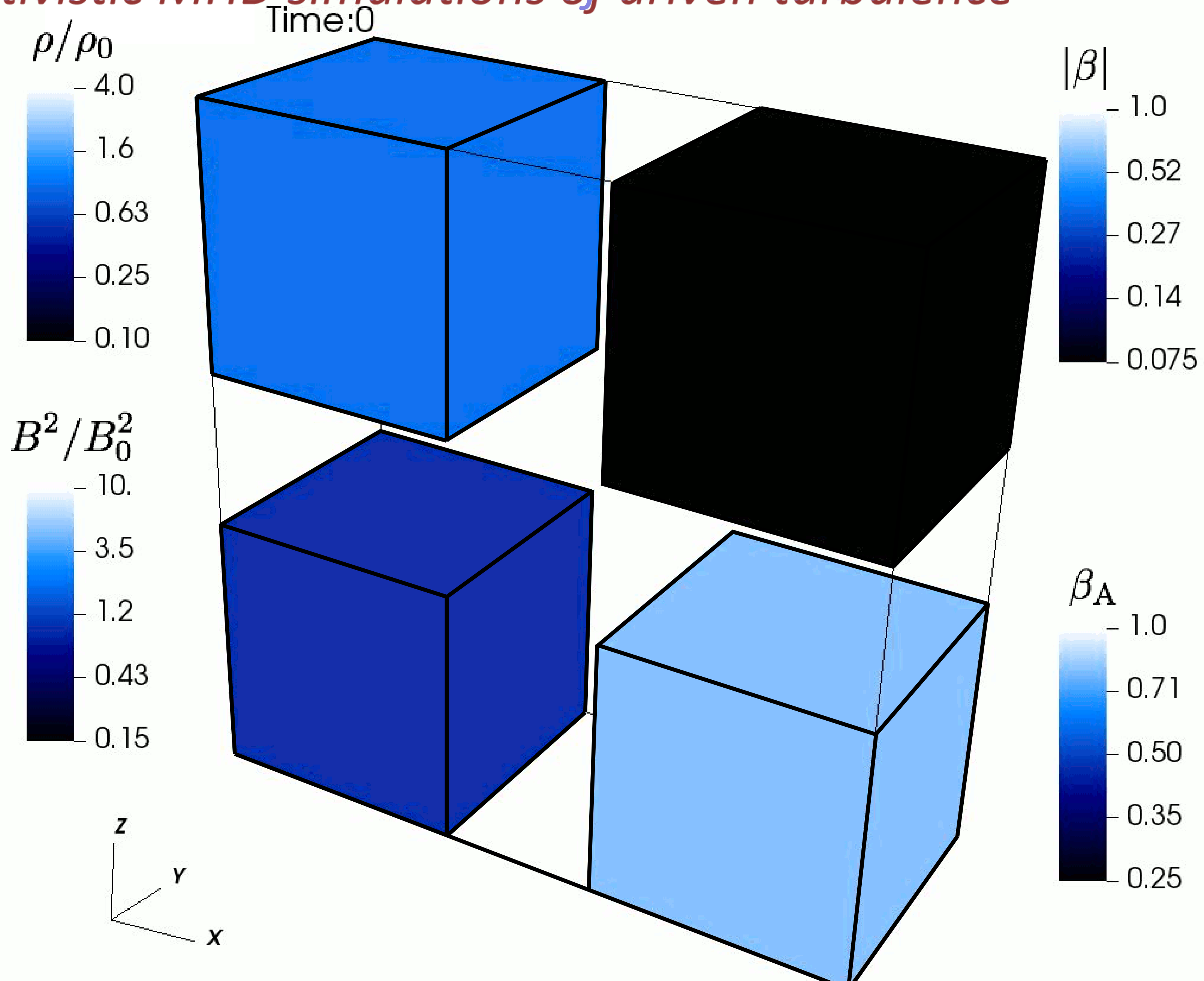
… in non-linear turbulence, composed of structures (~eddies) rather than waves:
(ML 19, Demidem+ 19)

$$t_{\rm acc} \sim \frac{L_{\rm max}}{\langle \delta u^2 \rangle}$$

⇒ slow at (low) energies $r_g \ll L_{\rm max}$, fast at high energy…

⇒ not efficient for leptons, but acceleration to close to confinement possible for nuclei

Relativistic MHD simulations of driven turbulence



Relativistic MHD simulation (256^3, σ_0=30, u_A~5) by Camilia Demidem (Demidem+ 19, in prep)

Stochastic acceleration in relativistic turbulence

<u>Non-perturbative description:</u> follow transport of particle in momentum space in a continuous sequence of (non-inertial) local plasma rest frames, where the electric field vanishes at each point… (M.L. 19)

… evolution of energy in local plasma rest frame
$$\frac{\mathrm{d}\hat{p}^{\hat{0}}}{\mathrm{d}\tau} = e^{\beta}{}_{\hat{b}}\, e^{\gamma}{}_{\hat{c}}\, \frac{\partial}{\partial x^{\gamma}} e^{\hat{0}}{}_{\beta}\, \frac{\hat{p}^{\hat{b}}\hat{p}^{\hat{c}}}{m}$$

with $e^{\hat{a}}{}_{\mu}$ vierbein to comoving plasma frame

… specify the statistics of the velocity field (→ statistics of the vierbein):

$$U^i = \cancel{\langle U^i \rangle} + u^i$$

u^i : here, of fixed magnitude, random orientation with coherence length and time k^{-1}

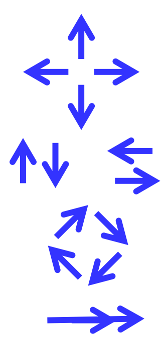
$$\frac{\partial u^i}{\partial x^{\alpha}} = u^i{}_{,\alpha} = \frac{1}{3}\theta\delta^i{}_{\alpha} + \sigma^i{}_{\alpha} + \omega^i{}_{\alpha} + a^i{}_{\alpha}$$

***θ*: expansion scalar**

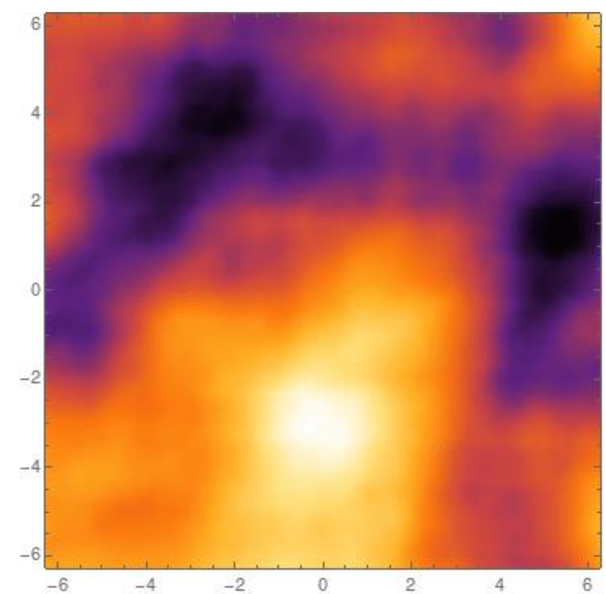
***σ*: shear tensor**

***ω*: vorticity tensor**

***a*: acceleration term**

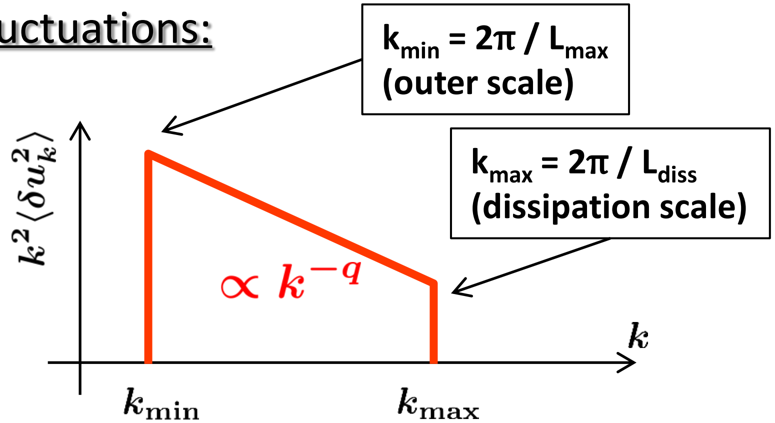


… each defined by a power spectrum of fluctuations



Particle acceleration in relativistic magnetized turbulence

<u>Interaction with a broadband turbulent spectrum of fluctuations:</u>



→ particle sees large-scale modes ($t_{scatt} \ll k^{-1}$), and small-scale modes ($t_{scatt} \gg k^{-1}$) ...

$$D_{pp}^{\rm non-res.} \simeq p^2 \,\langle\delta u^2\rangle \begin{cases} k_{\rm min}\,\left(t_{\rm scatt}k_{\rm min}\right)^{q-2} & \left(k_{\rm max}^{-1} \lesssim t_{\rm scatt} \lesssim k_{\rm min}^{-1}\right) \\ t_{\rm scatt}^{-1} & \left(k_{\rm min}^{-1} \lesssim t_{\rm scatt}\right) \end{cases}$$

[here, δu symbolizes the compressive/shear/vorticity/acceleration combination]

Demidem+ 19

→ at low turbulence amplitudes, hints for the scattering timescale from quasilinear theory ...

→ interestingly, for q ~ 2 and strongly magnetized particles: $t_{\rm acc} = \dfrac{p^2}{D_{pp}} \sim \dfrac{L_{\rm max}}{\langle\delta u^2\rangle}$

Particle acceleration in relativistic magnetized turbulence

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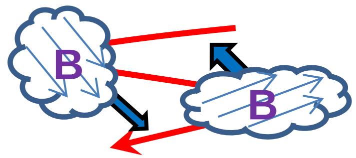

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... in non-linear turbulence, composed of structures (~eddies) rather than waves:
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$$t_{\rm acc} \sim \frac{L_{\rm max}}{\langle \delta u^2\rangle}$$ **⇒ slow at (low) energies $r_g \ll L_{max}$, fast at high energy...**

... in recent PIC simulations:
(Zhdankin+17, Comisso+ Sironi 18,19, Wong+ 19

$$t_{\rm acc} \sim \frac{L_{\rm max}}{\langle \delta u^2\rangle}$$

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Summary

A generalized scheme for Fermi acceleration:

→ follow the particle trajectory in mixed phase space coordinates:

$\mathbf{x}^{\mu}(\tau)$ lives in the lab (global) reference frame

$\hat{\mathbf{p}}^{\hat{\mathbf{a}}}(\tau)$ lives in a locally inertial frame that is comoving with the electromagnetic structure, so that **E** vanishes there…

→ non-perturbative calculation of energy gain as a function of scattering time t_{scatt} … in possibly non-trivial flow structures / non-trivial geometries…
(most often inaccessible to other schemes)

→ and its application to relativistic turbulence:

- **full characterization of non-resonant effects (compressive / shear / vorticity …)**
- **non-resonant acceleration timescale**

$$t_{\rm acc} \sim p^{\epsilon} \left\langle \delta u^2 \right\rangle^{-1} \eta^{-1} \left(k_{\rm min} c\right)^{-1}$$