Superfluid Dynamics in Neutron Stars

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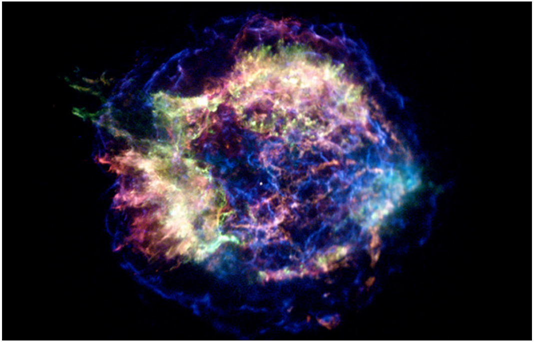
Carter's Fest, 5 July 2022

Neutron stars: laboratories for dense matter

Formed in gravitational core-collapse supernova explosions, neutron stars are the **most compact stars** in the Universe.



Nuclear physics:

$M \sim 1 - 2M_\odot$
$R \sim 10$ km
$\Rightarrow \rho \sim 10^{15}$ g cm^{-3}

Energy scale: MeV

"cold" $\lesssim 10^{10}$ K $\lesssim$ "hot"

Neutron stars are initially very hot ($\sim 10^{12}$ K) but cool down to $\sim 10^9$ K within days by releasing neutrinos.

Their dense matter is thus expected to undergo various phase transitions, as observed in terrestrial materials at low-temperatures.

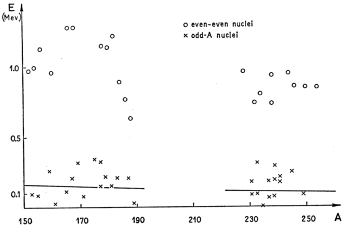
Nuclear superfluidity and superconductivity

The implications of the BCS theory (published in January 1957) for atomic nuclei were first discussed by A. Bohr, B. R. Mottelson, and D. Pines during the Summer of 1957.

D. Pines in "BCS: 50 Years" (World Scientific, 2011), pp.85-105

Bohr, Mottelson, and Pines speculated that nuclear pairing might explain the **energy gap** in the excitation spectra of nuclei.

Phys. Rev. 110, 936 (1958)

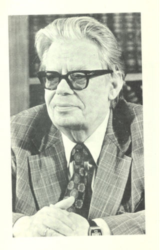


They also anticipated that nuclear pairing could explain **odd-even mass staggering, and the reduced moments of inertia** of nuclei.

"**The present data are insufficient to indicate the limiting value for the gap in a hypothetical infinitely large nucleus.**"
Bohr, Mottelson, Pines.

Superfluidity and superconductivity in neutron stars

In the 1960's, several superconductors had been found but ^{4}He was the only superfluid known (the superfluidity of ^{3}He below 2.5 mK was discovered by Osheroff in 1971).



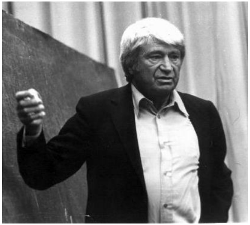
Bogoliubov, who developed a microscopic theory of superfluidity and superconductivity, was the first to explore its application to nuclear matter.

Dokl. Ak. nauk SSSR 119, 52 (1958)

Neutron-star superfluidity was predicted by Arkady Migdal in 1959, and first studied by Ginzburg & Kirzhnits in 1964 **before the discovery of pulsars** in 1967.

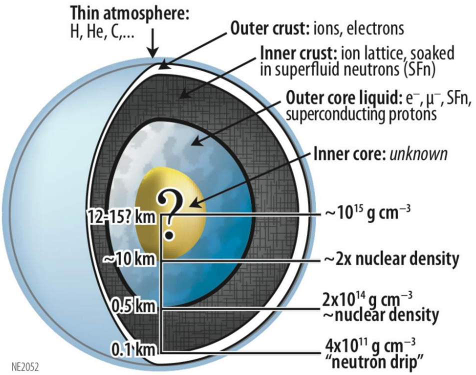
Migdal, Nucl. Phys. 13, 655 (1959)

Ginzburg & Kirzhnits, Zh. Eksp. Teor. Fiz. 47, 2006 (1964)



Superstars

The huge gravity of neutron stars produces the highest-T_c and largest superfluids and superconductors known in the Universe!



Neutron stars	**$\sim 10^{10}$ K**
⋮	⋮
CH_8S	288 K
Cuprates	$1 - 130$ K
Electrons in ordinary metals	$1 - 25$ K
Helium-4	2.17 K
Helium-3	2.491×10^{-3} K
Bosonic condensates	$\sim 10^{-6}$ K
Fermionic condensates	$\sim 10^{-8}$ K

For an overview: *Chamel, J. Astrophys. Astr. 38, 43 (2017)*

Classical vs superfluid hydrodynamics

A superfluid cannot be described using classical hydrodynamics (it can flow without resistance, does not boil, flows from cool to hot regions, etc).

Laszlo Tisza and later Lev Landau showed that **two distinct components coexist:**

- a superfluid that carries no entropy
- a normal viscous fluid.

Tisza, Nature 141, 913 (1938); Landau, Phys. Rev. 60, 356 (1941)

Neutron stars contain **three distinct components**:

- a neutron superfluid in the crust and in the core,
- a proton superconductor in the core,
- a normal viscous fluid.

Relativistic multifluid hydrodynamics is required for modelling superfluid neutron stars.

Carter's contribution to superfluid hydrodynamics


credit: M. Lorenzo

Brandon Carter developed an elegant **variational formalism based on differential forms and Cartan's exterior calculus** for describing relativistic superfluid mixtures as in the core of neutron stars.

Carter in "Relativistic fluid dynamics" (Springer-Verlag, 1989), pp.1-64
Carter, Lect. Notes Phys. 578 (Springer, 2001), 54.

This formalism relies on an **action integral** $\mathcal{A} = \int \Lambda\{n_{\rm x}^{\mu}\}\,{\rm d}\mathcal{M}^{(4)}$ over the 4-dimensional manifold $\mathcal{M}^{(4)}$.

The **Lagrangian density or "master function"** Λ depends on the **4-currents** $n_{\rm x}^{\mu}$ of the different fluids X.

This formalism allows for a rigorous and systematic derivation of hydrodynamic equations and conservation laws.

Covariant nonrelativistic superfluid hydrodynamics

The formalism was adapted to the more intricate Newtonian theory

- Because Carter's formalism relies on exterior calculus, the **4D covariant dynamical equations take the same form!**
- The difference lies in the underlying spacetime structure (absence of a spacetime metric, Galilean gauge symmetry)

Carter&Chamel, Int.J.Mod.Phys.D13,291(2004); ibid. D14,717(2005); D14,749(2005)

Note: the 3+1 version was developed by Reinhard Prix.

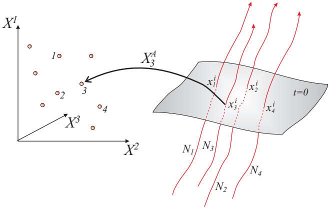
Prix, Phys. Rev. D 69, 043001 (2004); Phys. Rev. D71, 083006 (2005)

Why a fully 4D covariant formulation?

- direct comparison with relativistic theory
- conservation laws and identities can be more easily derived using mathematical concepts from differential geometry!
- matching between local (nonrelativistic) and global dynamics

Variational formulation of superfluid hydrodynamics

The hydrodynamic equations can be expressed in a very simple form.



picture from Andersson&Comer

Using the action principle and considering variations of the fluid particle trajectories yield

$$n^\mu_{\mathrm{x}} \varpi^{\mathrm{x}}_{\mu\nu} + \pi^{\mathrm{x}}_\nu \nabla_\mu n^\mu_{\mathrm{x}} = f^{\mathrm{x}}_\nu$$

4-momentum 1-form $$\pi^{\mathrm{x}}_\mu = \frac{\partial \Lambda}{\partial n^\mu_{\mathrm{x}}}$$

4-vorticity 2-form $$\varpi^{\mathrm{x}}_{\mu\nu} = 2\nabla_{[\mu}\pi^{\mathrm{x}}_{\nu]} = \nabla_\mu \pi^{\mathrm{x}}_\nu - \nabla_\nu \pi^{\mathrm{x}}_\mu$$

4-force density 1-form $$f^{\mathrm{x}}_\nu$$

π^{x}_μ and n^μ_{x} are mathematically different objects: the first is a *covector*, while the second is a *vector*. This distinction is fundamental in Newtonian spacetime (no metric to raise or lower indices!).

Hydrodynamical helicity

Introduced by Jean-Jacques Moreau and Robert Betchov in 1961, helicity was rediscovered by Keith Moffatt in 1969 and first discussed in superfluids by Zbigniew Peradzynski in 1990.

$$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t} = 0\,, \qquad \mathcal{H} = \int \mathrm{d}^3 r\, \boldsymbol{v} \cdot \boldsymbol{\nabla} \times \boldsymbol{v}$$

The demonstration is rather cumbersome.

Moreau, CR Hebd. Acad. Sci. 252, 2810; Betchov, Phys. Fluids 4, 925 (1961)

Moffat, J. Fluid Mech. 35, 117 (1969); Peradzynski, Int. J. Theo. Phys. 29, 1277 (1990)

The conservation of helicity arises naturally in 4D!

Introducing the **helicity 4-current** $\eta^\mu_{\mathrm{x}} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\varpi^{\mathrm{x}}_{\rho\sigma}\pi^{\mathrm{x}}_{\nu}$,

$$\nabla_\mu \eta^\mu_{\mathrm{x}} = \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}\varpi^{\mathrm{x}}_{\mu\nu}\varpi^{\mathrm{x}}_{\rho\sigma} = 0$$

since $\varpi^{\mathrm{x}}_{\mu\nu}$ is of rank-2 (Euler's eq. $n^\mu_{\mathrm{x}}\varpi^{\mathrm{x}}_{\mu\nu} = 0$).

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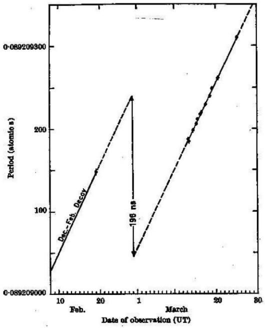
In 3+1 spacetime decomposition, the helicity conservation for fluid mixtures reads

$$\frac{\mathrm{d}\mathcal{H}^{\mathrm{x}}}{\mathrm{d}t} = 0\,, \qquad \mathcal{H}^{\mathrm{x}} = \int \mathrm{d}^3 r\, \boldsymbol{\pi}^{\mathbf{x}} \cdot \nabla \times \boldsymbol{\pi}^{\mathbf{x}}$$

Pulsar frequency glitches and superfluidity

Pulsars are spinning very rapidly with **extremely stable periods** $\dot{P} \gtrsim 10^{-21}$, outperforming the best atomic clocks.

Milner et al., Phys. Rev. Lett. 123, 173201 (2019)



Still, some pulsars have been found to **suddenly spin up** (in less than a minute).

664 glitches have been detected in 208 pulsars.

`http://www.jb.man.ac.uk/pulsar/glitches.html`

Recent review: *Manchester, Proc. IAU 13 (2017)*

The first glitch was detected in Vela in 1969.
Radhakrishnan&Manchester, Nature 222, 228 (1969)

Reichley&Downs, ibid. 229

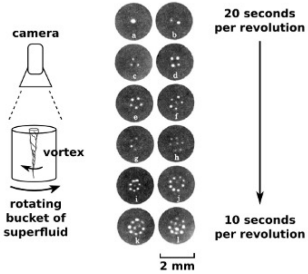
The very **long spin-down relaxation** (up to years) provided the first evidence for superfluidity.

Baym, Pethick, Pines, Nature 224, 673 (1969)

Vortex dynamics in neutron stars

A rotating superfluid is threaded by **quantized vortex lines**, each of which carries an angular momentum $\hbar$.

Similarly, a rotating neutron star is threaded by $\sim 10^{18}$ vortices, as pointed out by Ginzburg & Kirzhnits in 1964.



Yarmchuk et al., PRL43, 214 (1979)

In 1975, it was proposed that giant glitches are triggered by the sudden **unpinning of vortices** in neutron-star crust.
Anderson&Itoh, Nature 256, 25 (1975)

This scenario found support from **laboratory experiments** on He II.
J. S. Tsakadze & S. J. Tsakadze, J. Low Temp. Phys. 39, 649 (1980)

Postglitch relaxation can be explained by **vortex creep**.
Pines & Alpar, Nature 316, 27(1985)

Glitches and the superfluid inertia

Giant glitches are thus interpreted as **sudden transfers of angular momentum between the superfluid and the rest of star**.

The fractional moment of inertia of the superfluid component can be inferred from **pulsar-timing observations**:

$$\frac{I_s}{I} \geq \mathcal{G} \, , \qquad \mathcal{G} = 2\tau_c A_g$$

$\tau_c = \dfrac{\Omega}{2|\dot{\Omega}|}$ is the **characteristic age**,

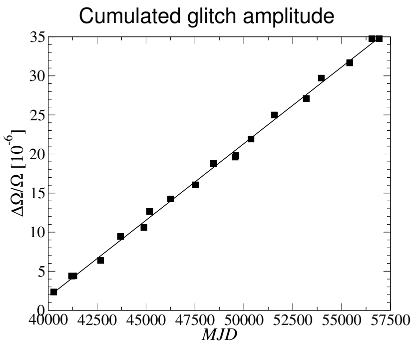
$A_g = \dfrac{1}{t}\sum_i \dfrac{\Delta\Omega_i}{\Omega}$ is the **glitch activity**.

Link, Epstein, Lattimer, Phys. Rev. Lett. 83, 3362 (1999)

Further information can be gained from individual glitches but more model dependent.

Pulsar glitch constraint

Since 1969, 24 glitches have been regularly detected in Vela. The latest one occurred in July 2021.



The cumulated glitch amplitude increases almost linearly:

$$\sum_i \frac{\Delta\Omega_i}{\Omega} = A_g t$$

where $A_g \simeq 2.25 \times 10^{-14}\ \mathrm{s}^{-1}$

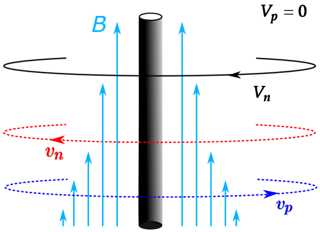
$$\Rightarrow \mathcal{G} = 2\tau_c A_g \simeq 1.62\%$$

The analysis of other glitching pulsars leads to $\mathcal{G} \lesssim 2\%$.

Neutron-star cores are expected to be superfluid. Why is $\mathcal{G}$ so small?

Entrainment and dissipation in neutron-star cores

Neutrons and protons are **mutually entrained**: mass currents $\boldsymbol{\rho}_{\mathbf{X}} = \sum_{\mathrm{Y}} \rho_{\mathrm{XY}} \boldsymbol{V}_{\mathbf{Y}}$ are not aligned with superfluid "velocities" $\boldsymbol{V}_{\mathbf{x}} \equiv \boldsymbol{\pi}^{\mathbf{x}} / m^{\mathrm{x}}$



Neutron vortices thus carry a **fractional magnetic quantum flux**

$$\Phi_{\star} = \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{\ell} = k\Phi_0 \,, \quad k = \frac{\rho_{pn}}{\rho_{pp}}, \Phi_0 \equiv \frac{hc}{2e}$$

Sedrakyan&Shakhabasyan, Astrofizika 8, 557 (1972); ibid. 16, 727 (1980)

Due to electrons scattering off the magnetic field of the vortex lines, **the core superfluid is strongly coupled to the crust**.

Alpar, Langer, Sauls, ApJ 282, 533 (1984)

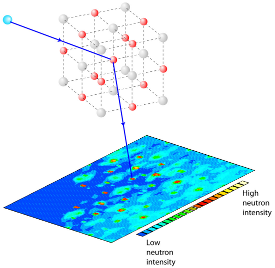
At the scale of the star, general relativity leads to additional fluid couplings due to frame-dragging effects!

B. Carter, Ann. Phys. 95, 53 (1975); Sourie et al., MNRAS 464, 4641(2017)

Bragg scattering of neutrons in neutron-star crusts

Carter's insight: neutrons in the crust do not flow freely due to Bragg diffraction by analogy with electrons in solids.

Carter, Chamel, Haensel, Nucl. Phys. A748, 675 (2005); IJMP D15, 777 (2006)



Neutrons can be diffracted by a crystal. This is routinely used to explore the structure of materials.

Bragg reflection means no flow in crust frame: neutrons are entrained by the crust!

However, Bragg's law not satisfied for all neutrons due to Pauli principle.

Band-structure calculations for neutrons showed that

$$\boldsymbol{\rho_n} = m_n n_n^s \boldsymbol{V_n} \text{ with } n_n^s \ll n_n \text{ independently of BCS pairing}$$

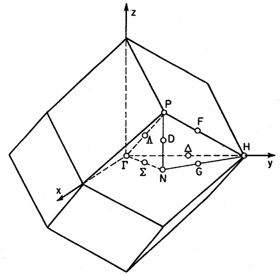
Chamel, Nucl. Phys A747, 109 (2005)

Carter, Chamel, Haensel, Nucl. Phys. A759, 441 (2005)

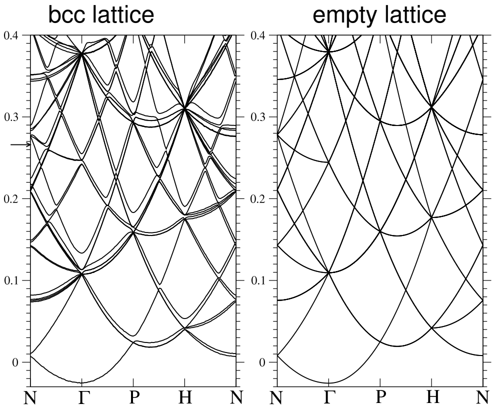
Neutron superfluid fraction in shallow region

Neutron band structure (s.p. energy in MeV vs $\boldsymbol{k}$) in a body-centered cubic (bcc) lattice at $\bar{n} = 0.0003\ \mathrm{fm}^{-3}$ ($Z = 50, A = 200$):

First Brillouin zone:



Chamel,Phys.Rev.C85,035801(2012)

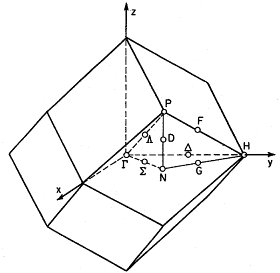


The spectrum is similar that of free neutrons: $n_n^s/n_n = 83\%$.

Neutron superfluid fraction in deep region

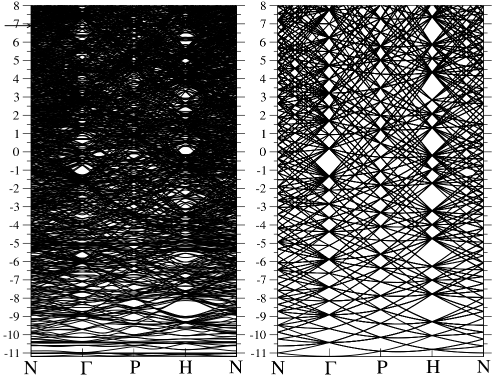
Neutron band structure (s.p. energy in MeV vs $\boldsymbol{k}$) in a body-centered cubic (bcc) lattice at $\bar{n} = 0.03$ fm^{-3} ($Z = 40, A = 1590$):

First Brillouin zone:



Chamel,Phys.Rev.C85,035801(2012)

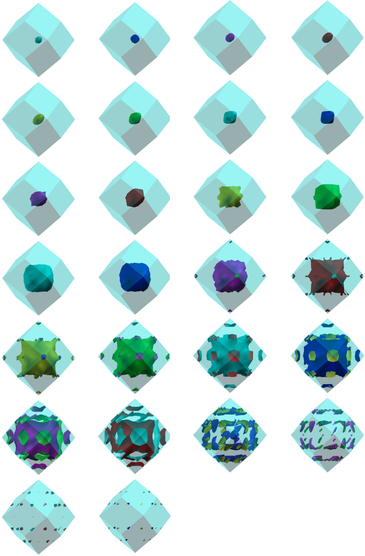
bcc lattice | empty lattice



The spectrum is very different: $n_n^s/n_n = 7\%$. Neutron superfluidity is **almost entirely suppressed**!

Neutron Fermi surface

Example at $\bar{n} = 0.03$ fm^{-3} (reduced zone scheme)



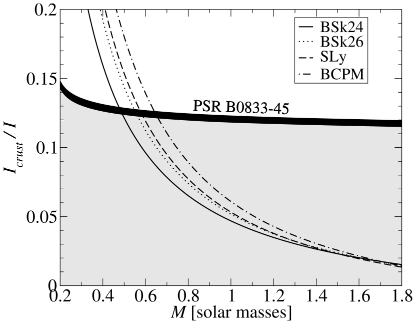
Superfluid reservoir and giant pulsar glitches

The depletion of the superfluid reservoir in the crust leads to a very **stringent pulsar glitch constraint**.
Chamel&Carter,MNRAS368,796(2006)

The inferred mass of Vela is much lower than expected from supernova simulations and known neutron-star masses.

At such central densities ($\bar{n} \approx 0.23 - 0.33$ fm^{-3}), the EoS is fairly well constrained by laboratory experiments.

Delsate,Chamel,Gürlebeck et al., PRD94, 023008(2016)

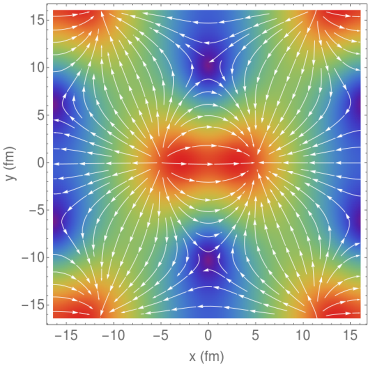


The superfluid in **the crust does not carry enough angular momentum**. Some superfluid in the core must be also involved.
Andersson et al., PRL 109, 241103; Chamel, PRL 110, 011101 (2013)

Local hydrodynamics of neutron superfluid

The local neutron flow in the crust was studied in the **strong coupling limit** adopting a purely **classical hydrodynamical** treatment.

- Superfluid "velocity": $\boldsymbol{V_n} = \frac{\hbar}{2m_n}\nabla\Phi$
- Incompressible superfluid flow: $\nabla \cdot \boldsymbol{V_n} = 0$
- Spherical clusters (obstacles) with sharp surfaces.



Classical potential flow $\Delta\Phi = 0$

The neutron mass current is

$$\boldsymbol{\rho_n} \equiv n_n m_n \boldsymbol{v_n} = \frac{1}{\mathcal{V}_{\text{cell}}} \int_{\text{cell}} n_n(\boldsymbol{r}) \nabla\Phi(\boldsymbol{r})$$

$$= n_n^s m_n \boldsymbol{V_n}$$

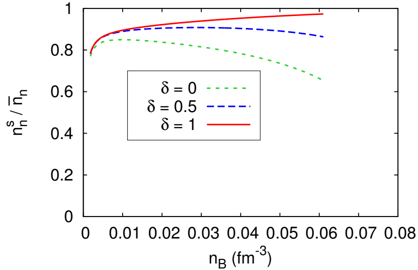
n_n^s is the **superfluid density**
$\boldsymbol{v_n}$ is the true velocity

Martin&Urban,Phys.Rev.C94, 065801(2016)

Classical potential flow past obstacles

Permeability of the clusters:
$\delta = 0$ no superfluidity,
$\delta = 1$ superfluidity everywhere.
Martin&Urban,Phys.Rev.C94,065801(2016)

Added perturbations from different clusters are negligible.
Epstein, ApJ333, 880 (1988)



The potential flow past a single cluster can be solved analytically:

$$\frac{n_n^s}{n_n} = 1 + 3\frac{\mathcal{V}_{\rm cl}}{\mathcal{V}_{\rm cell}}\frac{\delta-\gamma}{\delta+2\gamma} \Rightarrow 1 - \frac{3}{2}\frac{\mathcal{V}_{\rm cl}}{\mathcal{V}_{\rm cell}} \leq \frac{n_n^s}{n_n} \leq 1 + 3\frac{\mathcal{V}_{\rm cl}}{\mathcal{V}_{\rm cell}}$$

Magierski&Bulgac,Act.Phys.Pol.B35,1203(2004); Magierski, IJMPE13, 371(2004)
Sedrakian, Astrophys.Spa.Sci.236, 267(1996); Epstein, ApJ333, 880 (1988)

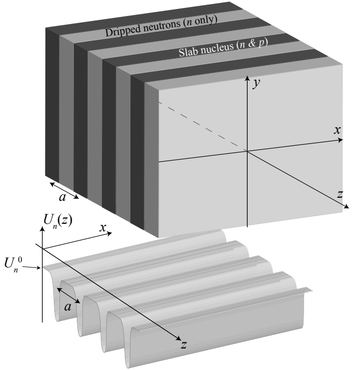
The superflow is found to be only **weakly perturbed** by clusters.
However, the strong coupling regime is usually not reached.

Suppression of band structure effects by pairing?

Recent band-structure calculations suggest that n_n^s is **less suppressed** when pairing is taken into account within the Hartree-Fock-Bogoliubov method

Watanabe&Pethick, PRL 119,062701(2017)
Minami & Watanabe, arXiv:2205.10742
Kashiwaba&Nakatsukasa, PRC100, 035804 (2019)
Sekizawa et al., PRC105, 045807 (2022)

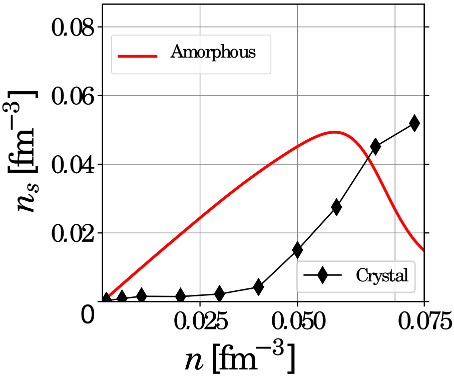
But **simplified 1D models**. Do the conclusions still hold in realistic 3D models?



Fully self-consistent 3D band structure calculations with pairing included still remain extremely challenging!

Role of disorder

Adapting statistical 3D models of uncorrelated random impurities that have been widely studied in the context of superconductivity in metallic alloys:

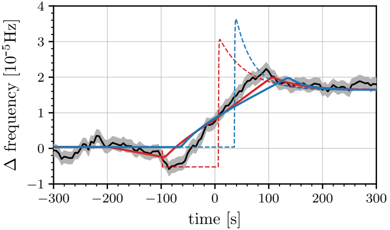


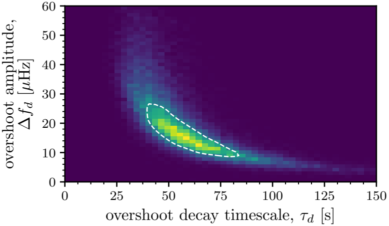
Sauls,Chamel,Alpar, arXiv:2001.09959

Glitch rise

Timing of the Crab and Vela pulsars have recently revealed very peculiar evolutions of their spin frequency during the rise of a glitch.

- Analyses of a Vela glitch in 2016 suggest a rotational-frequency **overshoot and a fast relaxation ($\sim$ min)** following the glitch.





Ashton, Lasky, Graber, Palfreyman, Nature Astronomy 3, 1143 (2019)

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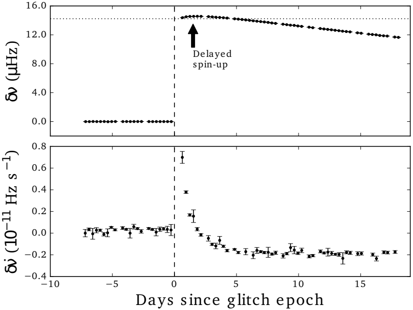
  Ashton, Lasky, Graber, Palfreyman, Nature Astronomy 3, 1143 (2019)

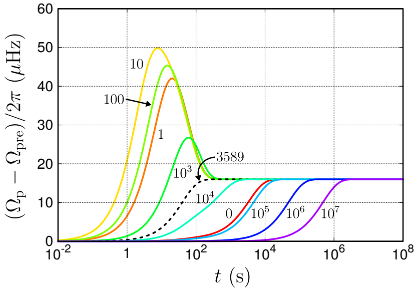
- A **delayed spin-up** has been detected in the 1989, 1996 and 2017 Crab glitches.



Shaw et al., MNRAS, 478, 3832 (2018)

Role of vortex pinning to fluxoids

These differences can be interpreted from the interactions between superfluid vortices and proton fluxoids in neutron-star cores.



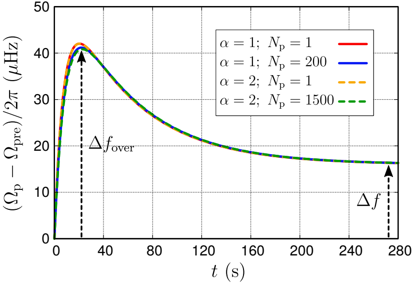
The number $N_{\rm p}$ of fluxoids attached to vortices turns out to be a key parameter governing the global dynamics of the star:

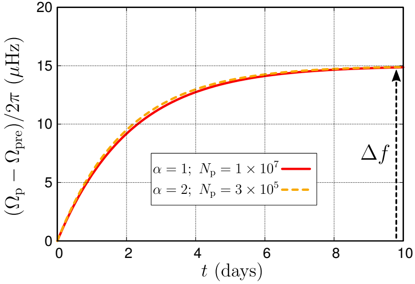
- $N_{\rm p} < N_{\rm p}^{\rm crit}$: overshoot $\Delta\Omega_{\rm over} < \Delta\Omega/(1 - I_{\rm n}^{\rm free}/I)$,
- $N_{\rm p} > N_{\rm p}^{\rm crit}$: smooth spin-up on a longer timescale.

Sourie&Chamel, MNRAS 493, L98 (2020)

Role of vortex pinning to fluxoids

The behavior of Vela and Crab glitches can be reproduced:





However, this neutron-star model remains very simplified:

- Newtonian approach
- physical reason for different N_p remains to be investigated
- N_p^{crit} depends on poorly-known mutual friction.

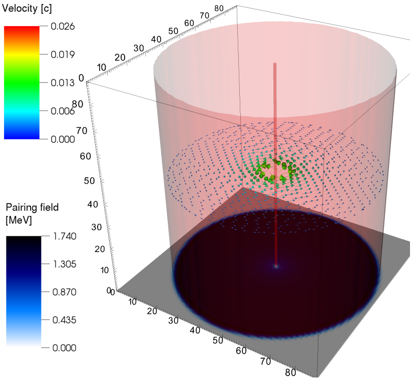
Alternative explanations:

Haskell et al., MNRAS 481, L146 (2018)

Gügercinoğlu&Alpar, MNRAS 488, 2275 (2019)

Microscopic dynamics of a vortex

To make progress, the dynamics of individual vortices at the smallest scale (~ 1 fm $= 10^{-15}$ m!) needs to be better understood.



Fully self-consistent dynamical quantum calculations using HFB method requires supercomputers.

Piz Daint Supercomputer

Peçak, Chamel, Magierski, Wlazlowski, Phys. Rev. C 104, 055801 (2021)

Cold atoms offer another venue to study neutron-star superfluidity in the lab.

Summary

Carter developed a very elegant formalism to describe the dynamics of superfluids neutron stars (including vortices), both in GR and in Newtonian theory.

His formalism has proved to be extremely powerful for extending the models and deriving conservation laws.

Carter's insights have lead to far-reaching astrophysical implications!

However, some of the microscopic parameters are still not very well determined.

The main challenge is to relate the local nonrelativistic dynamics of vortices and fluxoids at the nuclear scale (~ 1 fm $= 10^{-15}$ m) to the global general-relativistic dynamics of the star (~ 10 km).