Stable Cosmic Vortons in Field Theory

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In collaboration with Steven Cotterill and Jonathan Pearson
also with Paul Sutcliffe from 2009-2010

Stable cosmic vortons in Bosonic Field Theory, Physical Review Letters 127 (2021) 241601, arXiv:2111.07822
A detailed study of the stability of vortons, JHEP 04 (2022) 005, arXiv:2112.08066
Pinching instabilities in superconducting cosmic strings, to appear

Plan

- Superconducting strings and vortons in U(1)xU(1)
- Brandon's contribution: the thin string approximation and pert theory
- Previous attempts at constructing vortons
- Properties of constructed vortons
- Dynamical simulations – testing the thin string approximation
- Pinching instabilities
- Conclusions and perspectives

- Will concentrate on stability in field theory and presume that cosmological implications covered by Anne, Dani, Paul et al

Plan

Witten 1985

Davis and Shellard 1989

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(Witten 1985)

Superconducting strings in U(1)xU(1)

- Lagrangian $$\mathcal{L} = |D_\mu\Phi|^2 + |\partial_\mu\sigma|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(|\Phi|, |\sigma|)$$

Gauged complex scalar field coupling constant g

Complex scalar field

$$\mathcal{L} = \frac{1}{4}(|\Phi|^2 - 1)^2 + \frac{1}{4}\lambda_\sigma(|\sigma|^2 - \eta_\sigma^2)^2 + \beta|\Phi|^2|\sigma|^2$$

- Ansatz

$$\Phi(r,\theta) = \phi(r)\exp[i\theta] \qquad \sigma(r,z,t) = \sigma(r)\exp[i(\omega t - kz)]$$

(Witten 1985)

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4 parameters: $G = g/g_{BPS}$

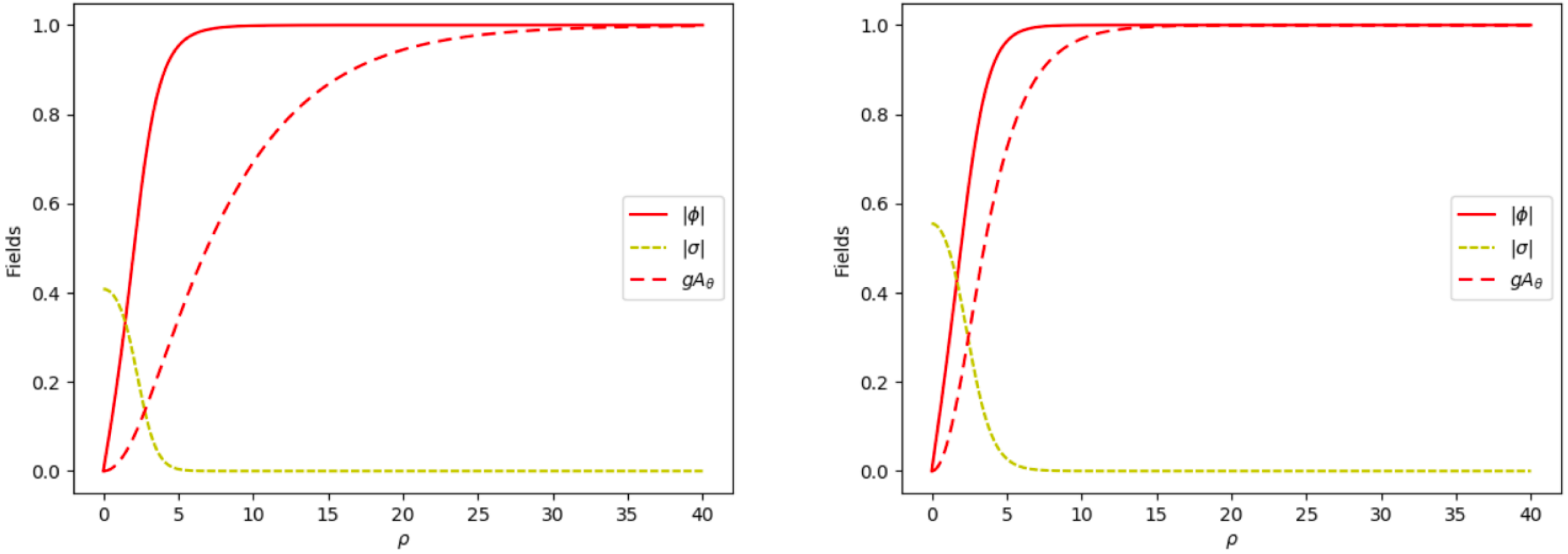
- Ansatz : $\chi = \omega^2 - k^2$ (=0 chiral, <0 magnetic, >0 electric)

$$\Phi(r,\theta) = \phi(r) \exp[i\theta] \qquad \sigma(r,z,t) = \sigma(r) \exp[i(\omega t - kz)]$$

(Witten 1985)

Superconducting strings in U(1)xU(1)

- Sample solutions



(a) $\eta_\sigma = 0.35$, $\lambda_\sigma = 36$, $\beta = 6.6$ and $G = 0.2$ (parameter set A) with $\chi = 1.074$.[1] This is an example of an electric string.

(b) $\eta_\sigma = 0.61$, $\lambda_\sigma = 10$, $\beta = 3$ and $G = 0.5$ (parameter set B) with $\chi = -0.01$. This string is mildly magnetic, but close to the chiral limit.

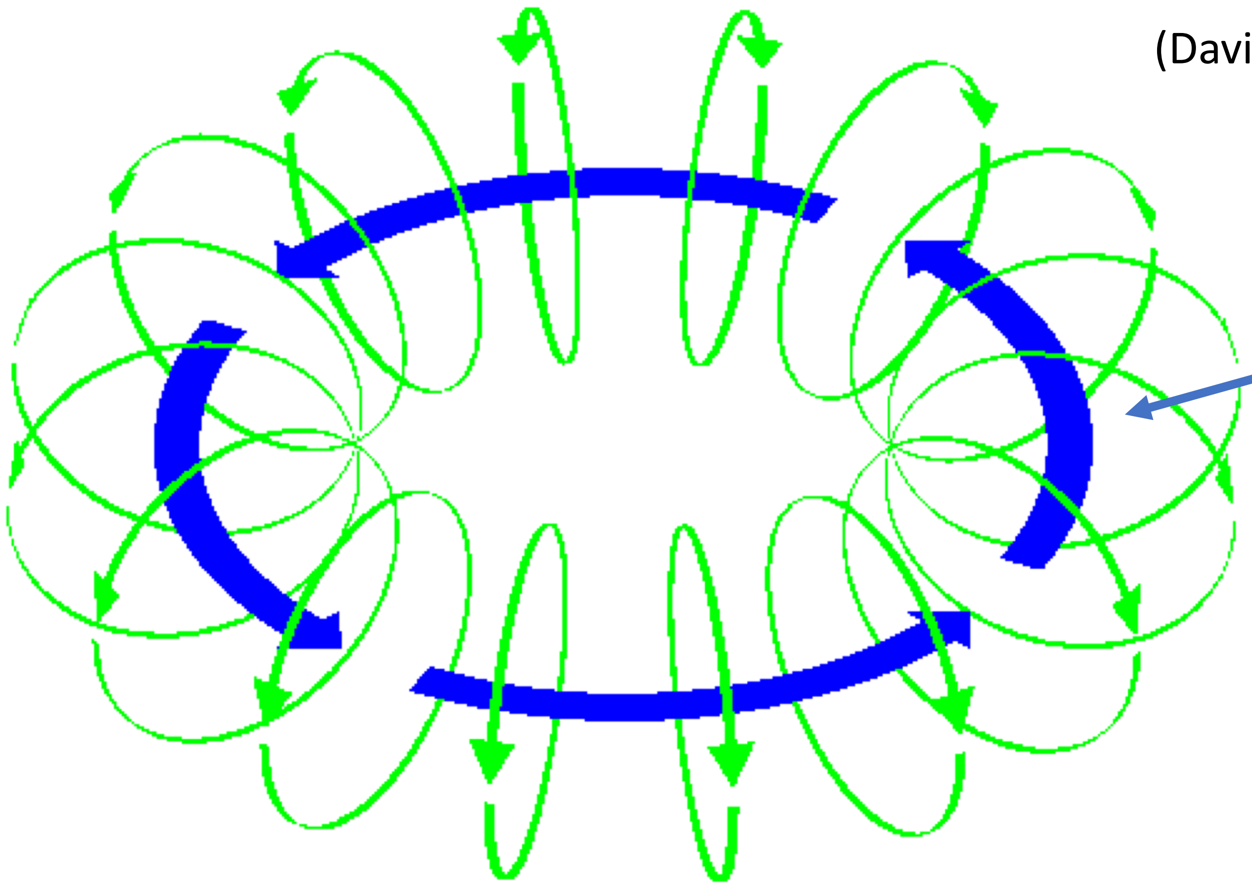
- NB not possible for all values of the parameters

(Davis and Shellard 1989)

Vortons

Solutions labelled by

- Q – Noether charge
- N = kR
- R = radius
- Related to χ



Current flowing around the loop

$$E = \left(\mu - \frac{1}{4}\lambda_\sigma \Sigma_4\right) L + \frac{2Q^2}{\Sigma_2 L}$$

Energy per unit length

Integrals of ϕ

Balance between tension & centrifugal barrier due to current/angular mom

(Brandon plus others including Patrick)

Thin string approximation (TSA)

- Line-like action – integrate out heavy modes eg core width

- Energy-momentum tensor

$$T^{\mu\nu} = \int d\tau d\sigma \left(U \frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau} - T \frac{dX^\mu}{d\sigma}\frac{dX^\nu}{d\sigma} \right)$$

- Examples of equations of state

Equation of state : T(U)

  - Nambu $T=U=\mu=m^2$
  - Transonic : $TU=m^4$
  - “Wiggly” strings : renormalization of the tension

(Carter and Martin 2003)

Perturbations to vortons

- Sound speeds

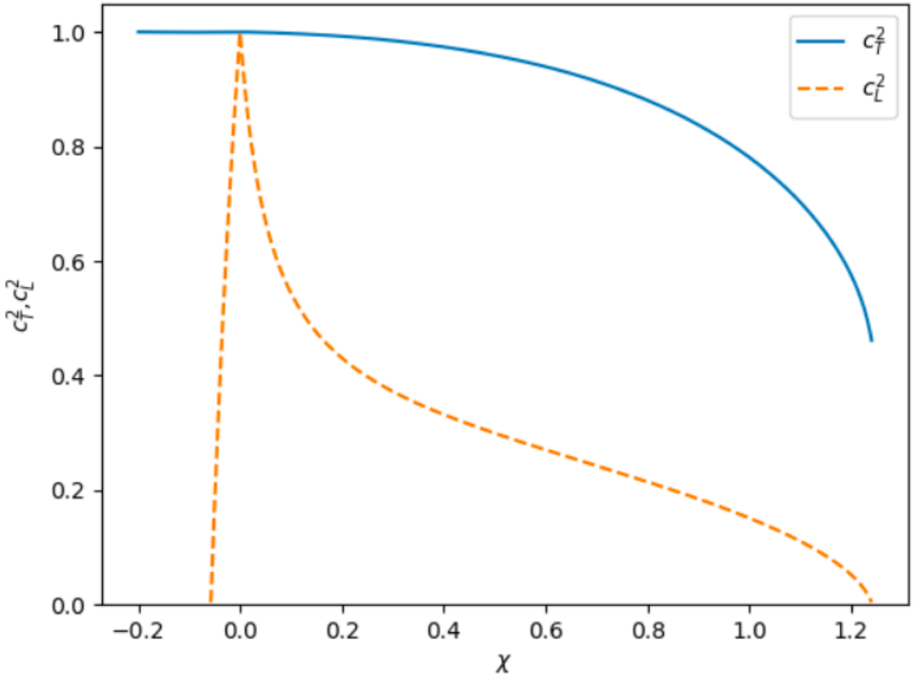
$$c_{\mathrm{T}}^2 = \frac{T}{U} \quad > \quad c_{\mathrm{L}}^2 = -\frac{dT}{dU}$$

- We have calculated semi-analytic expressions as a function of χ
- Perturbations

$$\delta r(t,\theta) = \sum_{m,j} A_{mj} \exp\left[i(\Omega_{mj}t - m\theta)\right]$$

- Solve eigenvalue problem for the frequencies

Sound speeds



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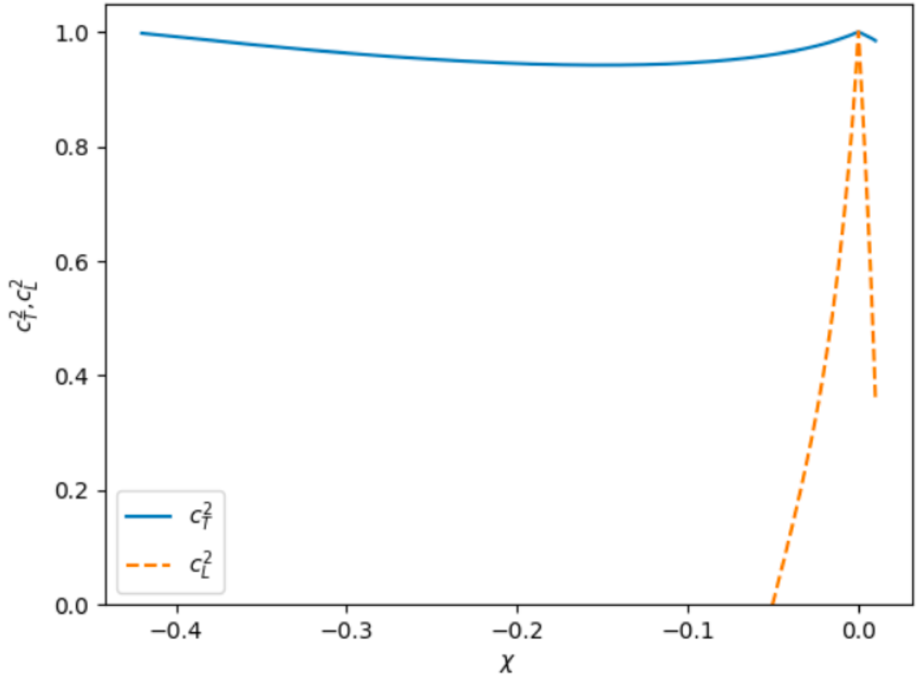
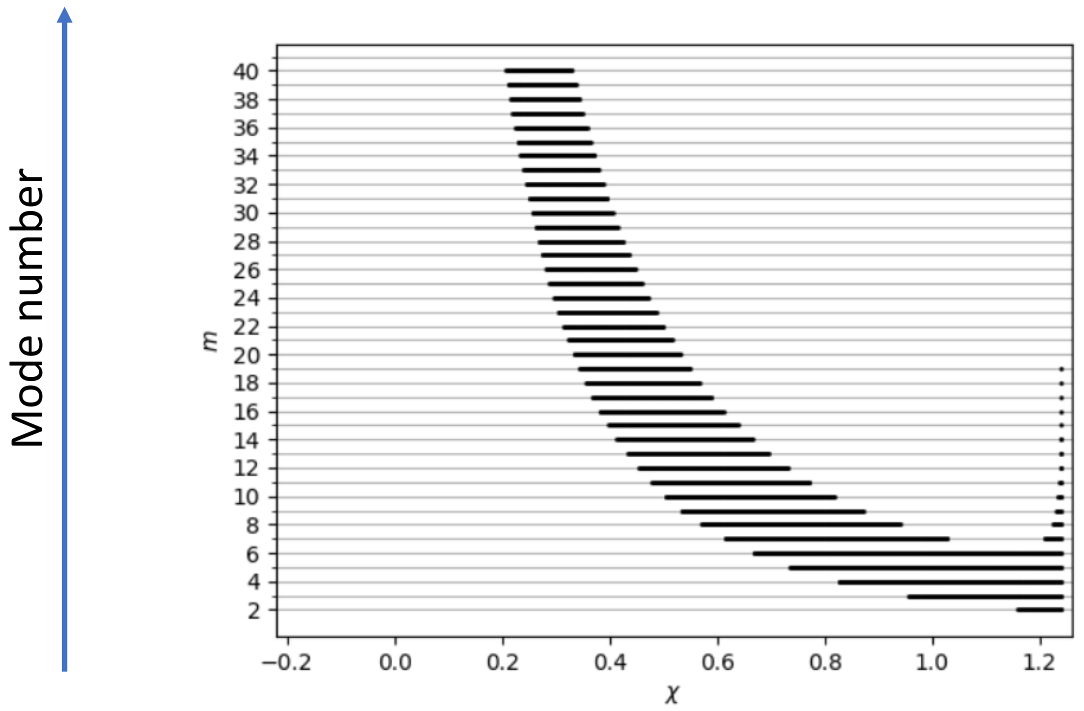


(b) $\eta_\sigma = 0.61$, $\lambda_\sigma = 10$, $\beta = 3$ and $G = 0.5$ (parameter set B).

Regions of stability as a function of χ



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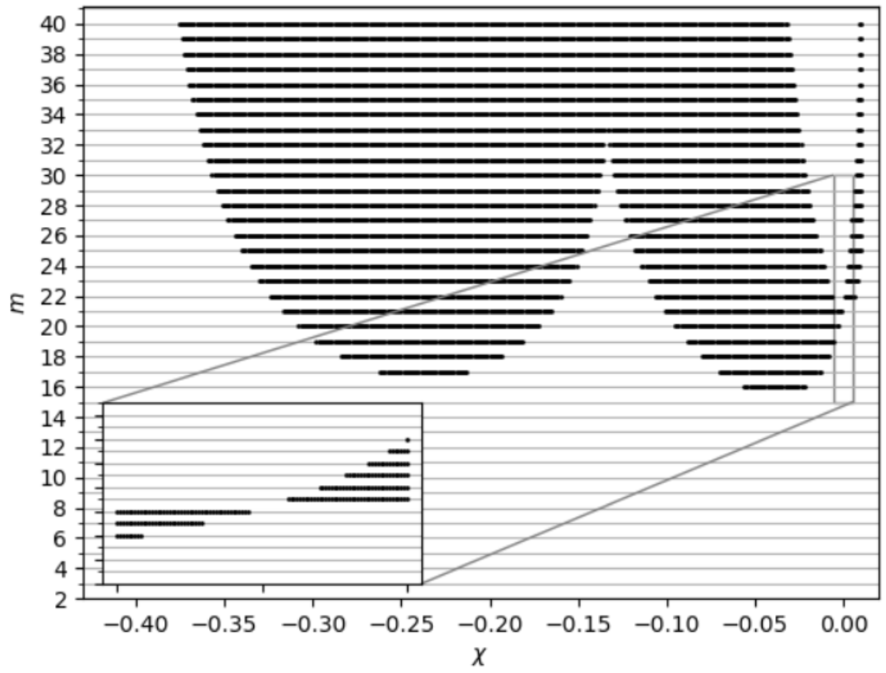


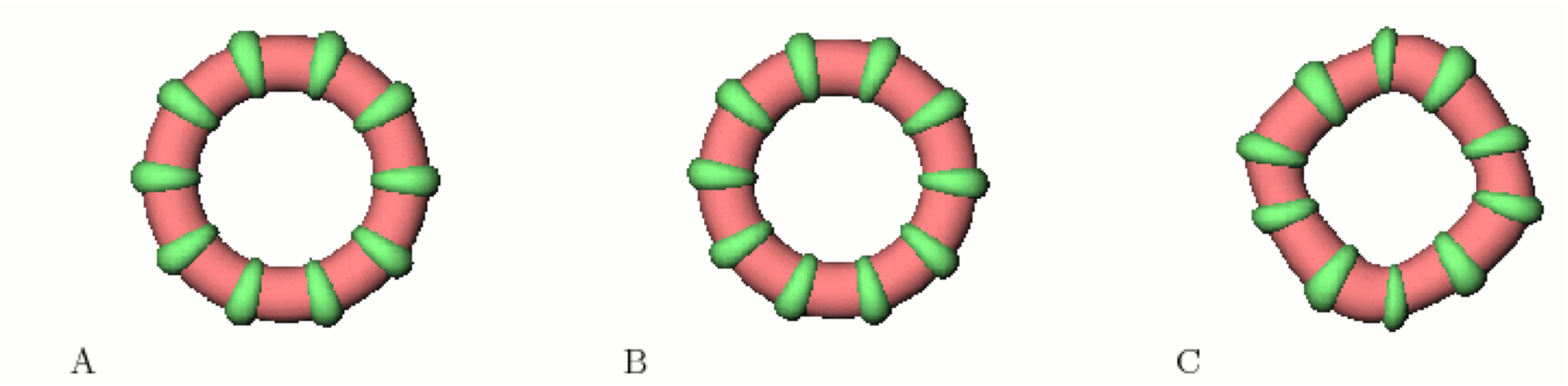
(b) $\eta_\sigma = 0.61$, $\lambda_\sigma = 10$, $\beta = 3$ and $G = 0.5$ (parameter set B).

Parameter set A – unstable
Electric regime

Parameter set B – stable around χ=0
ie Magnetic regim near chiral limit

(Battye and Sutcliffe, 2009)

Construction and dynamics of global vortons



time evolution

- Global vortons were numerically constructed – using similar methods to used here
- Dynamical simulations showed they were unstable, in particular to m=4 due to grid
- Thin string approximation failed to give sensible predictions for radius/stability

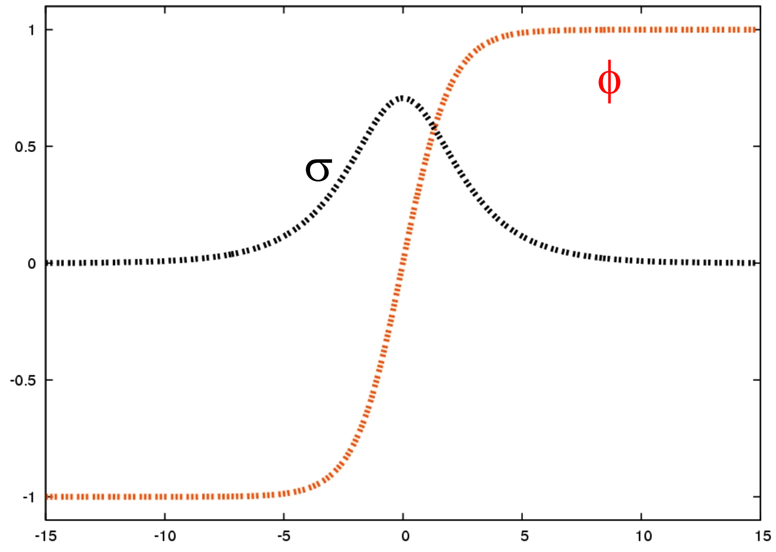
We now understand why!

Kinky vortons in $Z_2xU(1)$

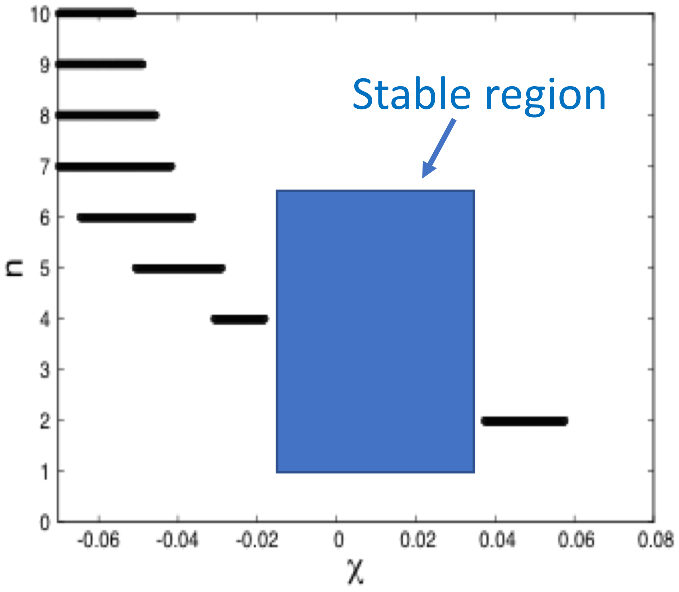
(Battye and Sutcliffe, 2008,2009)

- Make ϕ real, no gauge fields– string becomes a domain wall or `kink'
- Analytic solution for the superconducting wall solution for arbitrary χ
- Analytic expressions for equation of state and sound speeds

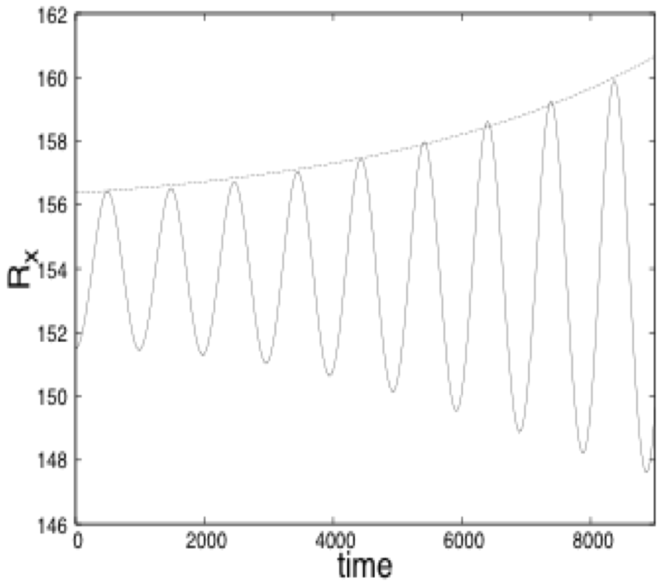
(adapting Hodges 1988)



Superconducting wall solution



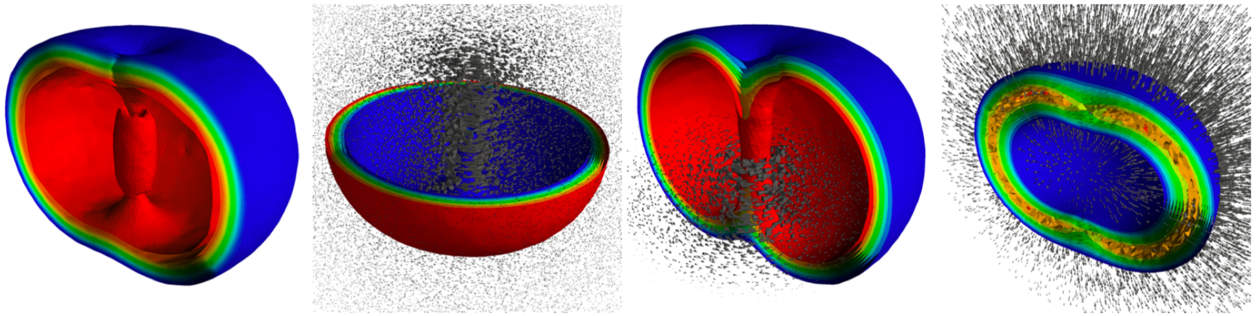
Domain of stability



Predicted increase

Work by others

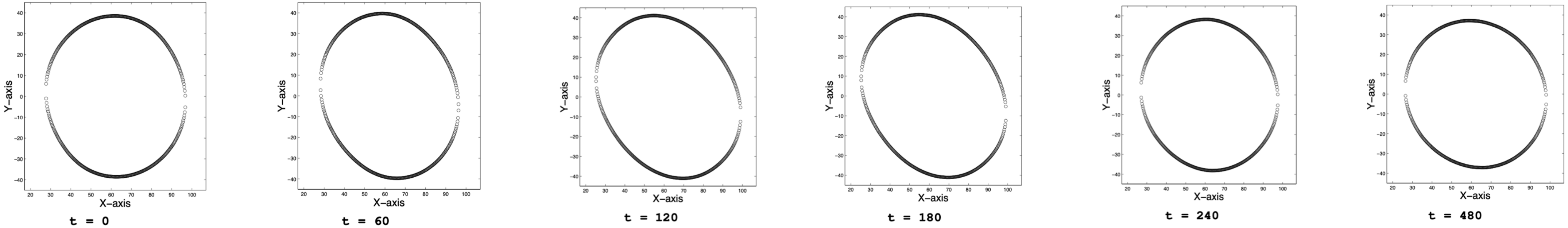
NB similar to vortons in 2 component BEC (see Battye, Cooper and Sutcliffe 2002)



- Radu and Volkov 2013
  - Small vortons : N, R small
  - Concluded that large vortons unstable to pinching instabilities
- Lemperiere and Shellard 2003

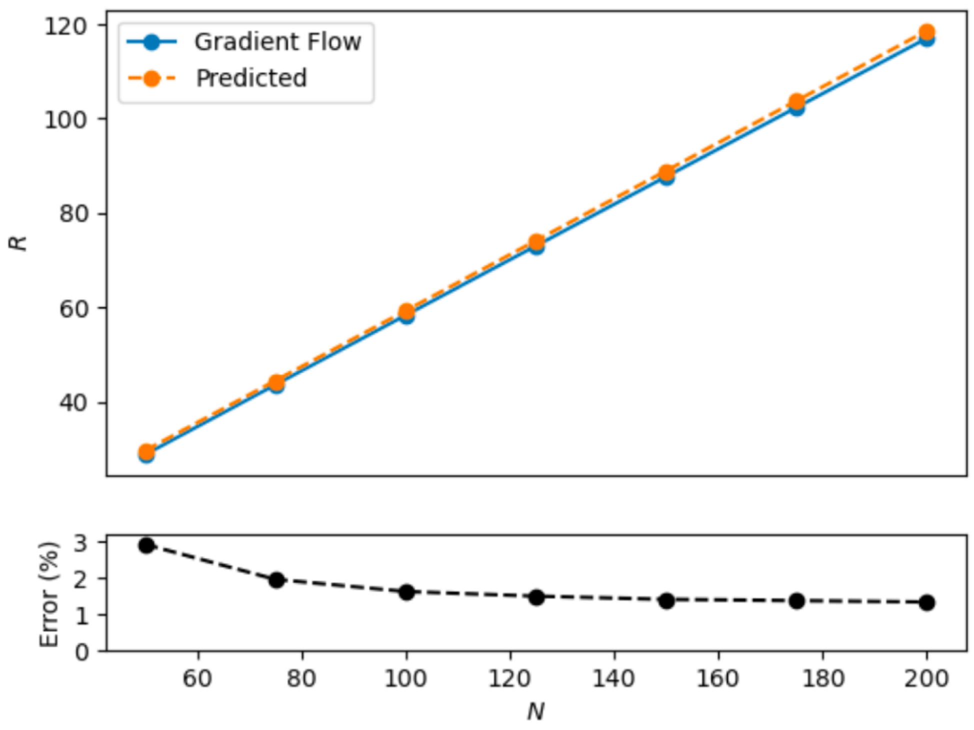
Stronger interaction potential : $V_{\text{int}} = \beta' |\phi|^6 |\sigma|^2$

NB our maximum simulation times will be x10

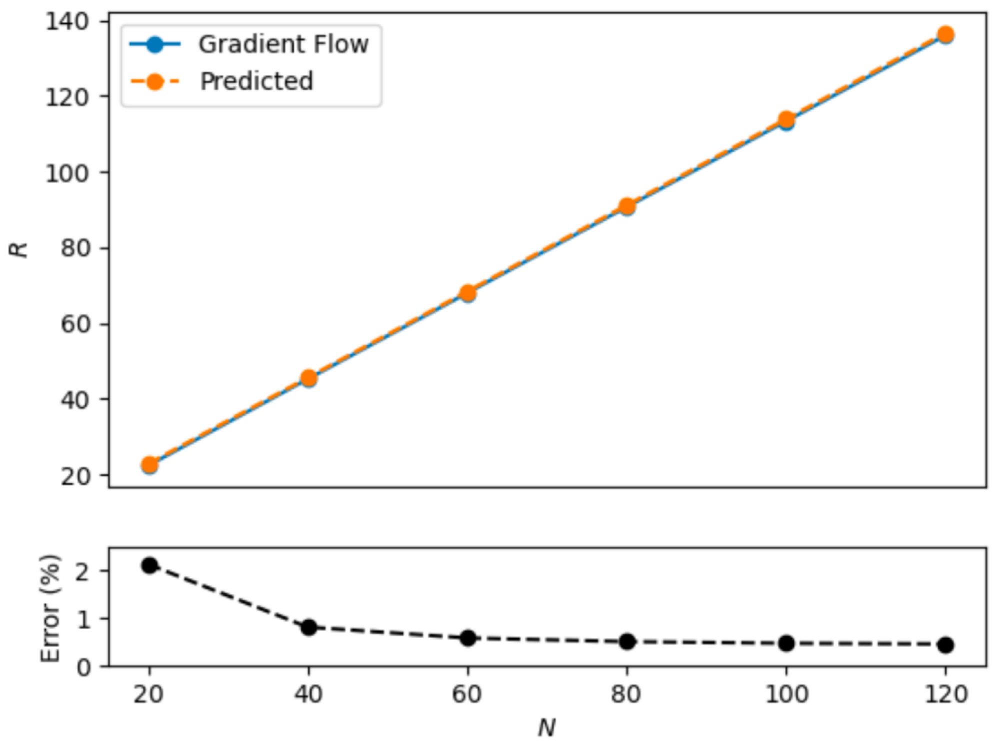


- Our objective was to repeat KV/global analysis for gauged vortons

Constructed vortons
– using cartoon method & gradient flow



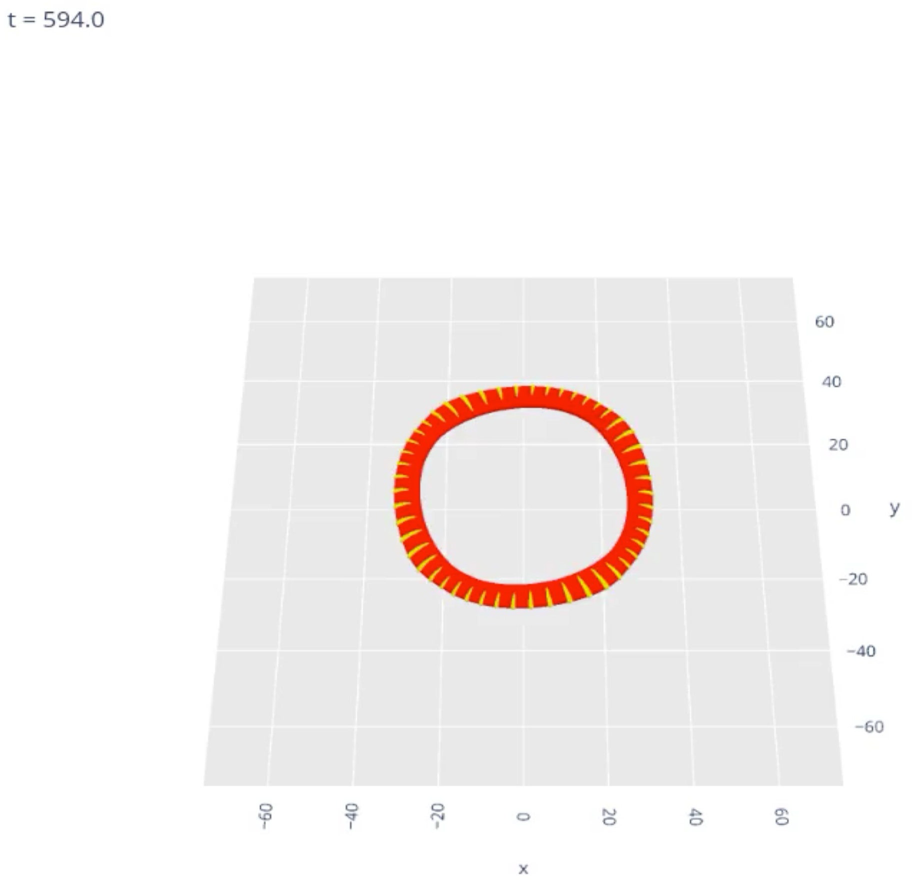
Parameter set A



Parameter set B

- Good agreement between TSA approximation (embedding cylindrical solns) and numerically generated solutions
- Visually the solutions are close to the straight string solutions
- Corrections seem to reduce like 1/R – ie curvature

Dynamical sims : parameter set A, N=50



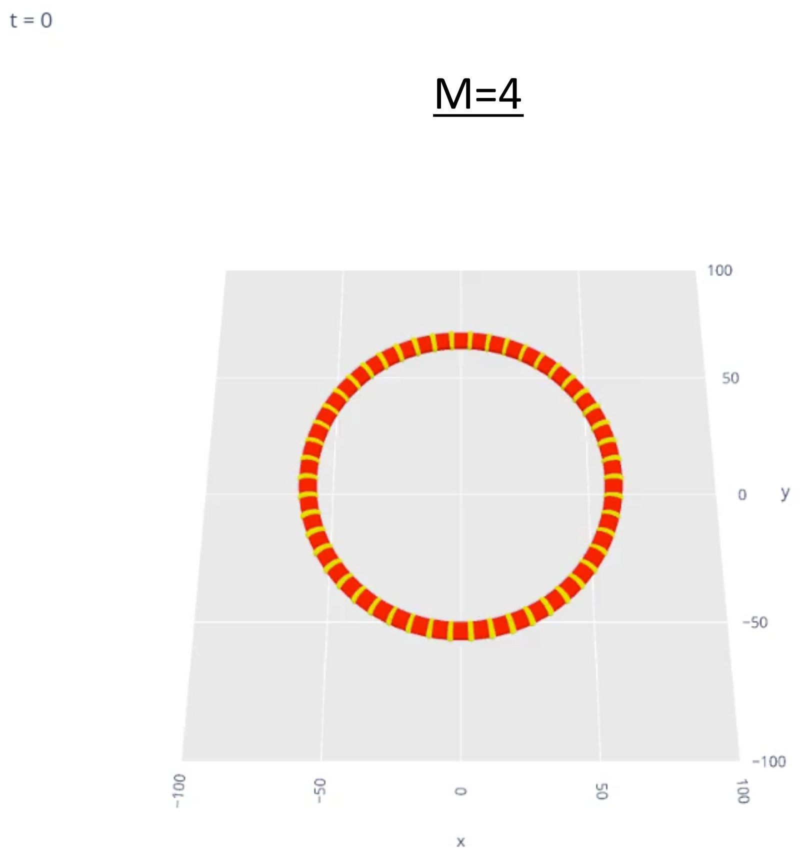
Predicted to be unstable
using the TSA!

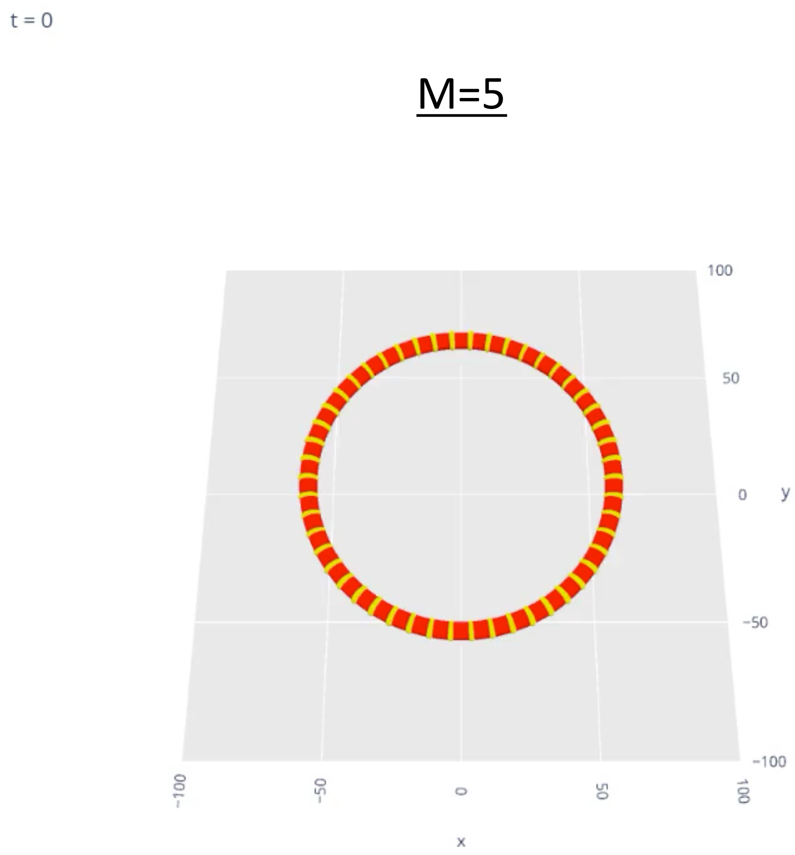
Red surface $|\phi| = 3/5$
- position of string

Yellow surface $Re(s)=\eta_\phi/5$
- illustrates current

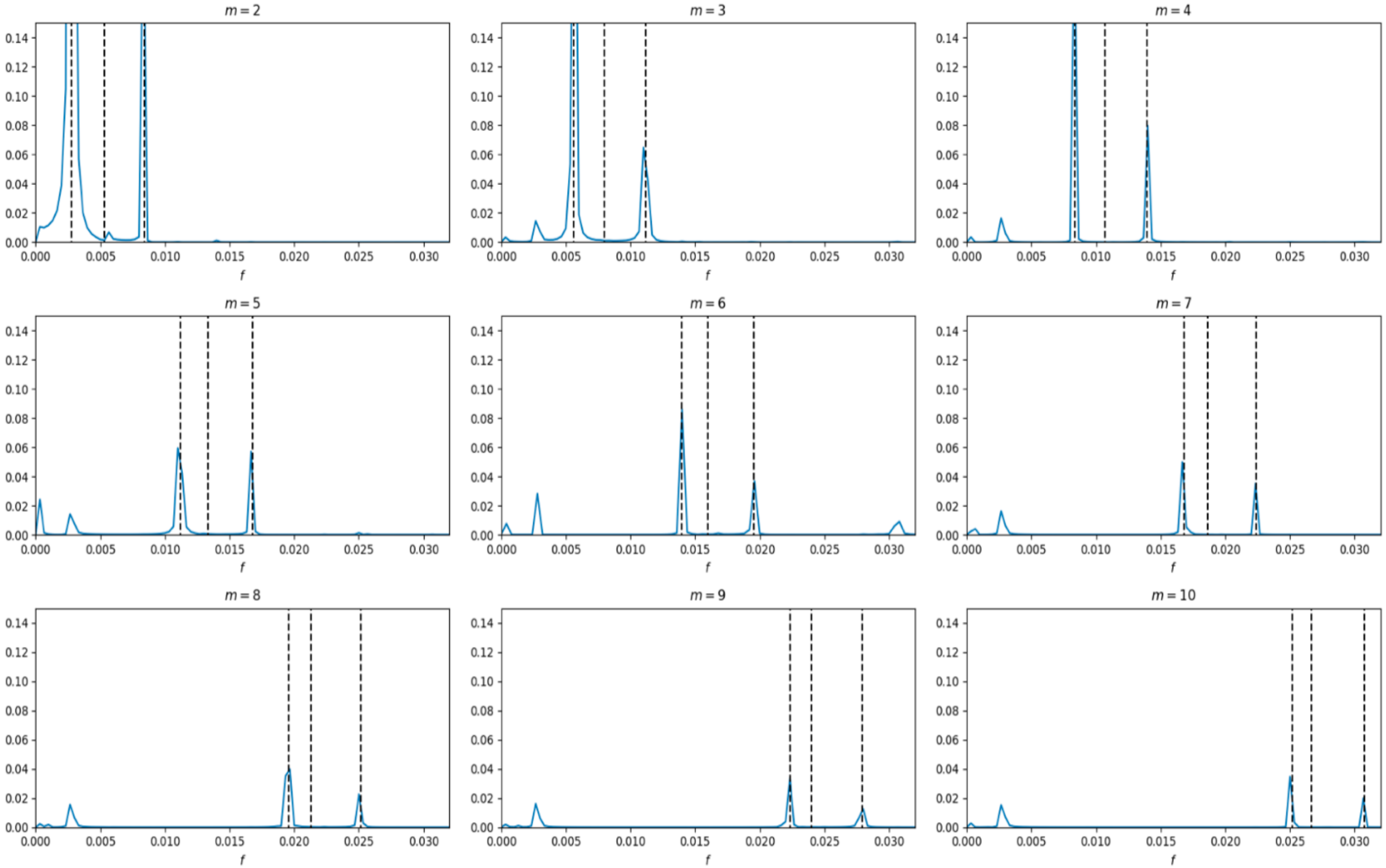
NB having a prediction of stability is crucial
- typical parameters plus Q, N will be unstable

Dynamical sims : parameter set B, N=50

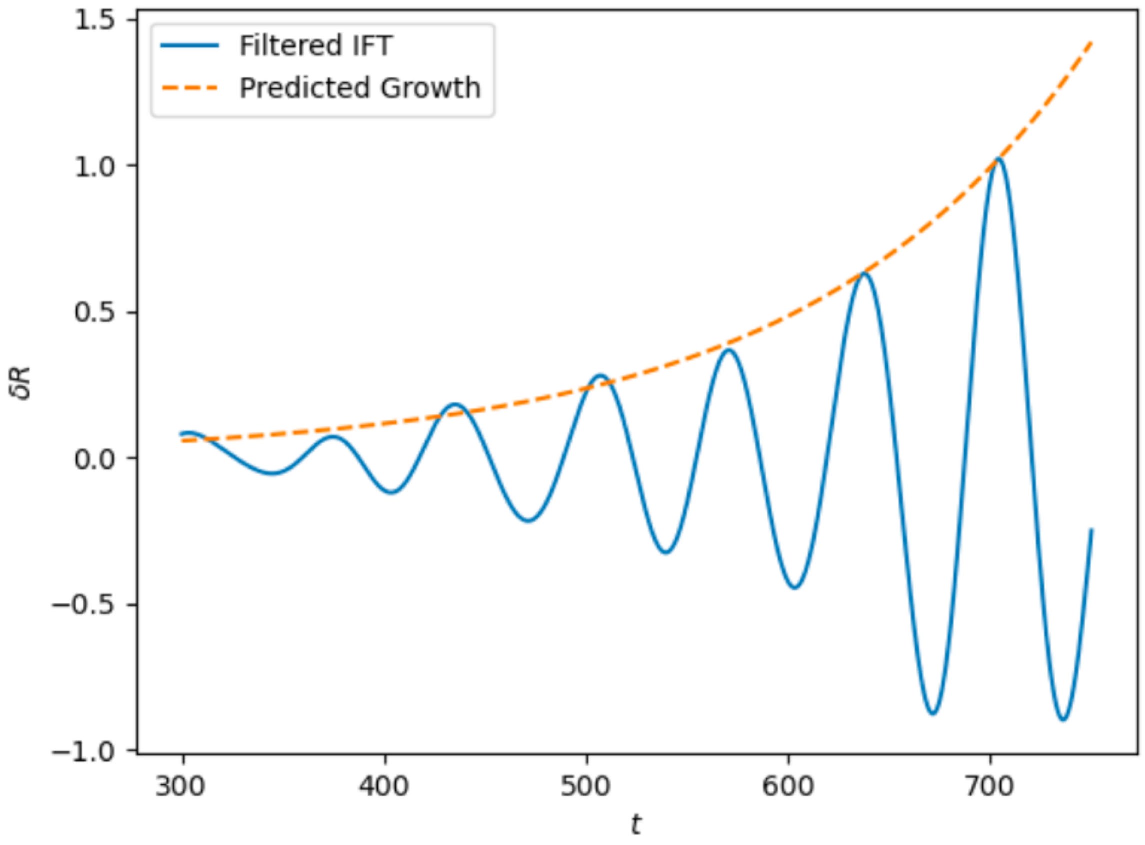




Measured frequencies in parameter set B



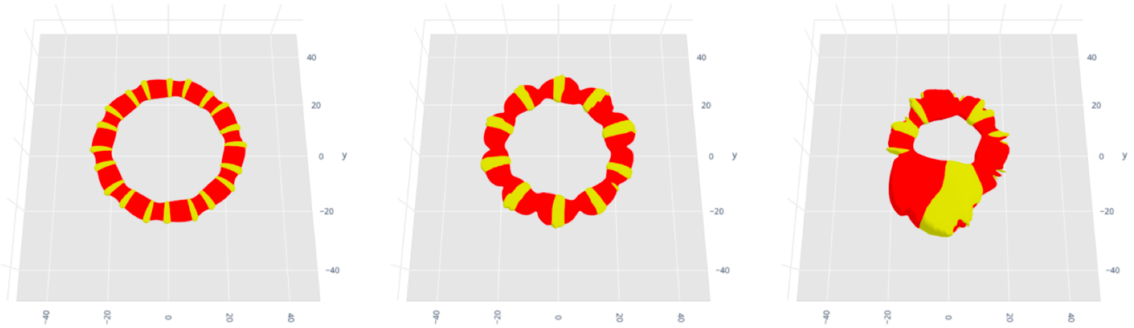
Growth of instability in parameter set A



Pinching instabilities

- due to core width (heavy) modes

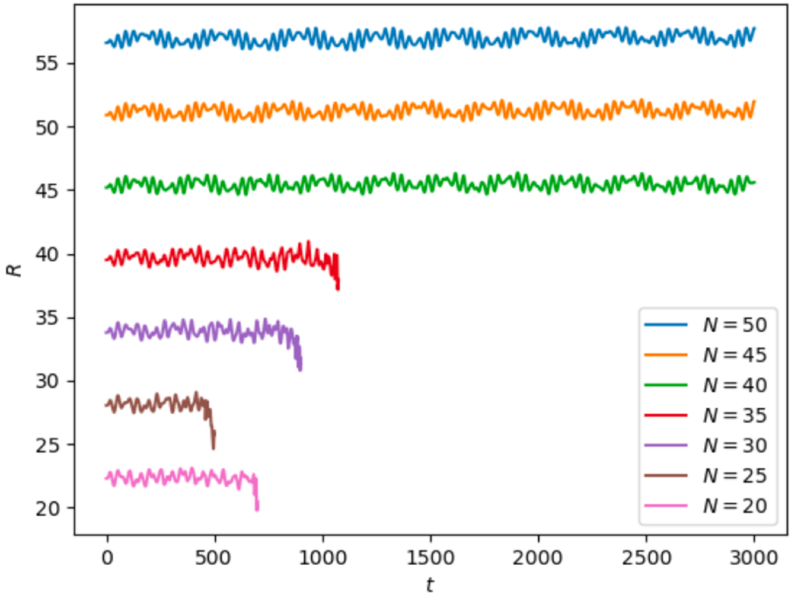
Eg Parameter set B for N=20



<u>Three types:</u>

- Numerical – increase dx or L or both
- Due to small R

  Recall : Small Vortons (Radu & Volkov 2013)



- Real instabilities for some parameters

Eg. found in Kinky Vortons (Battye & Sutcliffe 2009)

Periodic strings (Lemperiere & Shellard 2002, LS2002)

Straight string analysis – improving on LS2002

- This removes the instabilities due to the curvature
- Methodology:

Essentially the TSA plus core modes

Stationary ansatz + position dependent perturbations

Elliptic EOM solved using finite element method

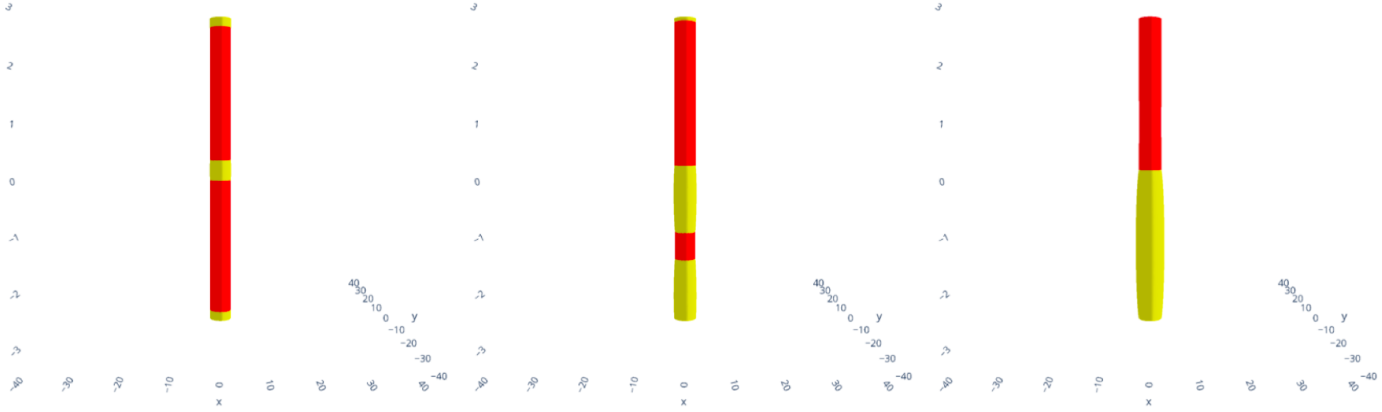
⬇

Predicts the collapse time & some range of parameters completely stable

- Conclusion: magnetic instability similar to LS2002, but improved also an instability in the electric regime
- Parameter set B predicted to be always stable!

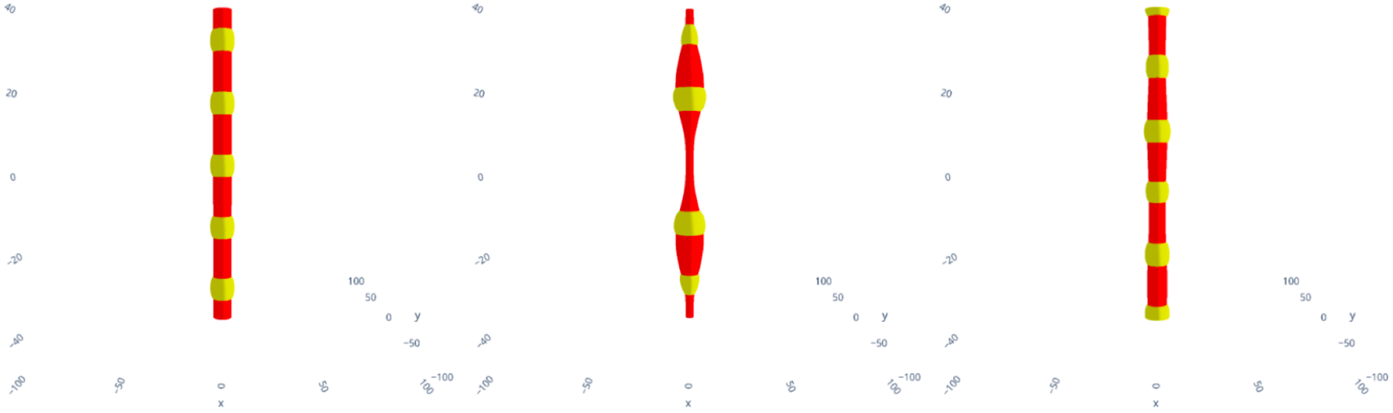
Magnetic and electric pinching instabilities

Magnetic



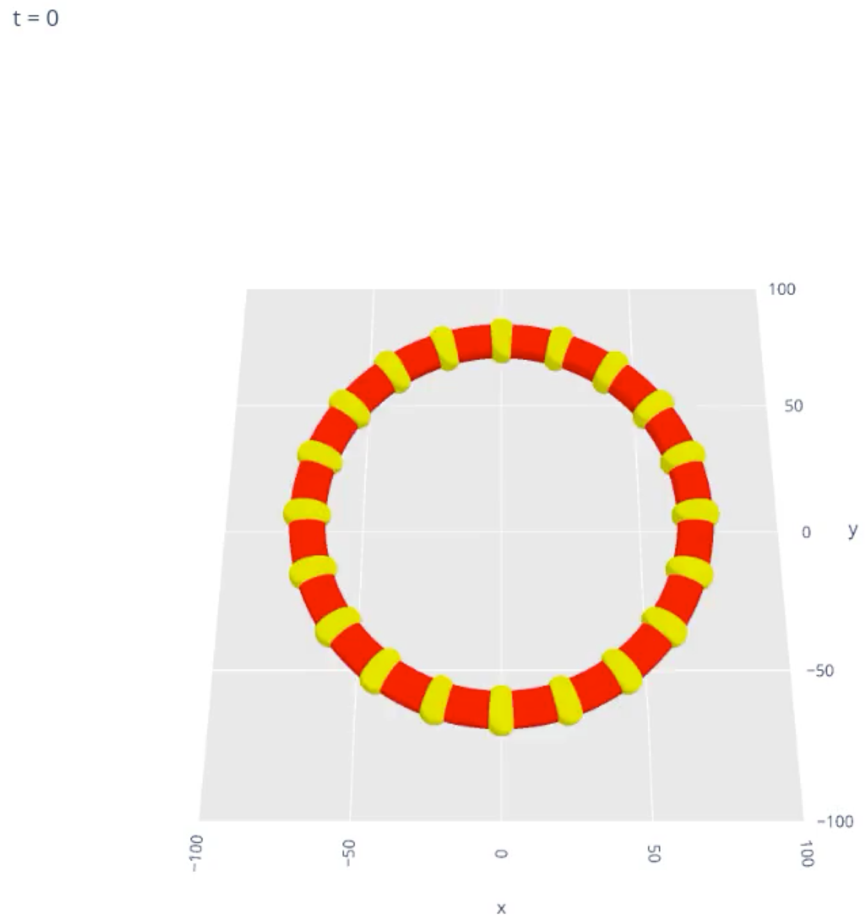
(a) $t = 0$ (b) $t = 10.4$ (c) $t = 12$

Electric



(a) $t = 0$ (b) $t = 630$ (c) $t = 900$

Vorton with m=8 pinching electric instability



Conclusions & Perspectives

- Have found vortons solutions – agree with simple energetic scaling arguments eg R v N to ~few % level
- Brandon's formalism for treating perturbations works well in describing dynamical properties – for the maximum time of sim
- Studied pinching instabilities – collapse time can be predicted
- Presently investigating formation in more realistic models

  - U(1)xU(1) requires possibly contrived parameters?
  - 2HDM allow U(1)xU(1) with the possibility of superconductivity
  - More generally extra symmetries in SM quite popular ATM
    eg dark photons