Constructing scalar tensor black holes from Kerr geodesics

Christos Charmousis

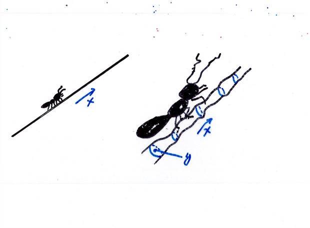
IJCLab-CNRS

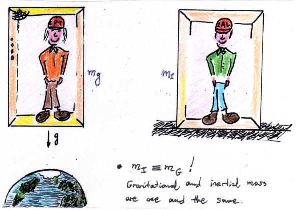
IAP Paris and LUTH Meudon

Carter Fest : Black holes and other cosmic systems

Influence of Brandon Carter

- Discussions and insightful remarks, explanations, critical comments, encouragement, cornerstone results derived from his papers
- self gravity of topological defects domain walls and strings and the limited role distributions can play
- Braneworld cosmology and the relation to Birkhoff's theorem
- codimension 2 cosmology in Lovelock theory
- constructing stationary black holes using Carter geodesics.





Black holes in scalar tensor theories

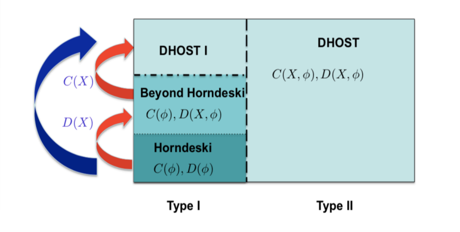
Simplest geometric modified gravity theory with one additional degree of freedom

BD theory,..., Horndeski,..., beyond Horndeski,..., DHOST theories [Noui, Langlois, Vernizzi, Crisostomi, Koyama et al]

- ST are limits of more complex fundamental theories (massive gravity, braneworld models, EFT from string theory, Lovelock theory etc.)
- Horndeski theory is parametrized by 4 functions of scalar and its kinetic energy, $G_i = G_i(\phi, X)$, $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
- General conformal and disformal map in an internal map in DHOST :

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$$

.



[Langlois, 2018]

- Example Horndeski theory [Babichev, CC]

$$S = \int d^4x \sqrt{-g} \left[R - 2\Lambda_b - \eta X + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right],$$

- $X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$
- One can find the general spherically symmetric and static solutions, $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, with $\phi = \phi(t, r)$,
- simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta} r^2$$

,

$$\phi = qt \pm \int dr \, \frac{q}{h} \sqrt{1 - h}$$

with secondary hair $q^2 = \frac{\eta + \Lambda_b \beta}{\beta \eta}$ relating the couplings.

Solutions of spherical symmetry in Horndeski theories

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with secondary hair $q^2=\frac{\eta+\Lambda_b\beta}{\beta\eta}$ relating the couplings.

- Interesting property $X=-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi=-\frac{1}{2}(-\frac{q^2}{h}+q^2\frac{f(1-h)}{h^2})=-\frac{q^2}{2}$ is constant [Kobayashi, Tanahashi].

Going beyond spherical symmetry?

We can find solutions of spherical symmetry

GR type solutions with X constant are generic in DHOST theories

- How do we implement rotation?
  How can we explicitly construct rotating black holes beyond GR?
- For a start : Can we construct stealth rotating solutions with non trivial hair?
- For spherical symmetry we have a GR metric and $X = -q^2$.
- Can we obtain a Kerr metric with a non trivial scalar, such that $X = -q^2$? This fails in Horndeski

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- We need to know the scalar field explicitly.
- The key is understanding what $X = -q^2$ signifies geometrically.

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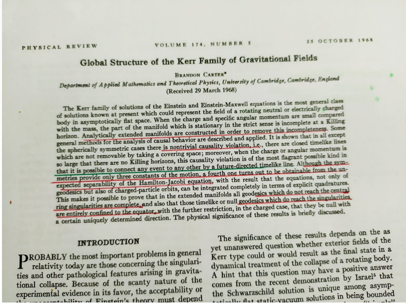

PHYSICAL REVIEW VOLUME 174, NUMBER 5 25 OCTOBER 1968

Global Structure of the Kerr Family of Gravitational Fields

BRANDON CARTER*
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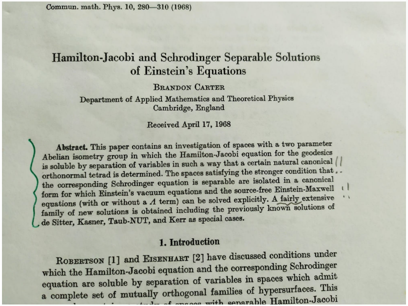
(Received 29 March 1968)

The Kerr family of solutions of the Einstein and Einstein-Maxwell equations is the most general class of solutions known at present which could represent the field of a rotating neutral or electrically charged body in asymptotically flat space. When the charge and specific angular momentum are small compared with the mass, the part of the manifold which is stationary in the strict sense is incomplete at a Killing horizon. Analytically extended manifolds are constructed in order to remove this incompleteness. Some general methods for the analysis of causal behavior are described and applied. It is shown that in all except the spherically symmetric cases there is nontrivial causality violation, i.e., there are closed timelike lines which are not removable by taking a covering space; moreover, when the charge or angular momentum is so large that there are no Killing horizons, this causality violation is of the most flagrant possible kind in that it is possible to connect any event to any other by a future-directed timelike line. Although the symmetries provide only three constants of the motion, a fourth one turns out to be obtainable from the unexpected separability of the Hamilton-Jacobi equation, with the result that the equations, not only of geodesics but also of charged-particle orbits, can be integrated completely in terms of explicit quadratures. This makes it possible to prove that in the extended manifolds all geodesics which do not reach the central ring singularities are complete, and also that those timelike or null geodesics which do reach the singularities are entirely confined to the equator, with the further restriction, in the charged case, that they be null with a certain uniquely determined direction. The physical significance of these results is briefly discussed.

INTRODUCTION

PROBABLY the most important problems in general relativity today are those concerning the singularities and other pathological features arising in gravitational collapse. Because of the scanty nature of the experimental evidence in its favor, the acceptability or

The significance of these results depends on the as yet unanswered question whether exterior fields of the Kerr type could or would result as the final state in a dynamical treatment of the collapse of a rotating body. A hint that this question may have a positive answer comes from the recent demonstration by Israel[5] that the Schwarzschild solution is unique among asymp-



Commun. math. Phys. 10, 280—310 (1968)

Hamilton-Jacobi and Schrodinger Separable Solutions of Einstein's Equations

BRANDON CARTER
Department of Applied Mathematics and Theoretical Physics
Cambridge, England

Received April 17, 1968

Abstract. This paper contains an investigation of spaces with a two parameter Abelian isometry group in which the Hamilton-Jacobi equation for the geodesics is soluble by separation of variables in such a way that a certain natural canonical orthonormal tetrad is determined. The spaces satisfying the stronger condition that the corresponding Schrodinger equation is separable are isolated in a canonical form for which Einstein's vacuum equations and the source-free Einstein-Maxwell equations (with or without a Λ term) can be solved explicitly. A fairly extensive family of new solutions is obtained including the previously known solutions of de Sitter, Kasner, Taub-NUT, and Kerr as special cases.

1. Introduction

ROBERTSON [1] and EISENHART [2] have discussed conditions under which the Hamilton-Jacobi equation and the corresponding Schrodinger equation are soluble by separation of variables in spaces which admit a complete set of mutually orthogonal families of hypersurfaces. This

The Kerr black hole and its properties

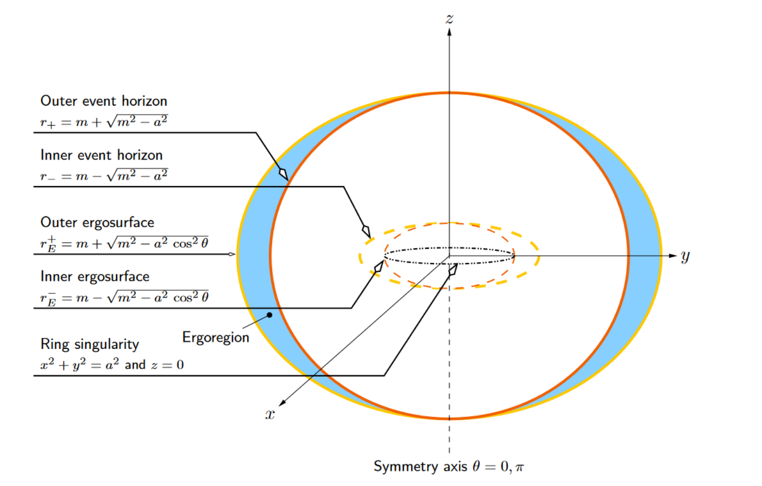
- Kerr black hole

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4aMr\sin^2\theta}{\rho^2} dt d\varphi + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right] d\varphi^2$$
$$+\frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2$$

where M is the mass, a is the angular momentum per unit mass, and

$$\rho^2 = r^2 + a^2\cos^2\theta,\ \Delta = r^2 + a^2 - 2Mr\ .$$

- Stationary and axisymmetric spacetime : two Killing vectors $\partial_t, \partial_\varphi$
- Spacetime is circular : $(-t, -\varphi) \leftrightarrow (t, \varphi)$
- Point singularity for $a = 0$ blows up to an equatorial ring singularity at $\rho = 0$



[Visser, 2007]

- ∂_t, $\partial_t + \omega\partial_\varphi$ define static and stationary observers.
- Kerr has a causal exterior as long as it is a black hole!
- Seperability properties will yield the geodesics but also a large class of type D solutions

Carter's solution : adding a cosmological constant to Kerr

- In the presence of a cosmological constant the rotating black hole is far more difficult to find.
- Brandon Carter using separability arguments found a very general type D class of exact solutions including the rotating solution

$$ds^2 = -\frac{\Delta_r}{\Xi^2\rho^2}\left[dt - a\sin^2\theta d\varphi\right]^2 + \rho^2\left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\right)$$
$$+ \frac{\Delta_\theta \sin^2\theta}{\Xi^2\rho^2}\left[a\, dt - \left(r^2 + a^2\right) d\varphi\right]^2 ,$$
$$\Delta_r = \left(1 - \frac{r^2}{\ell^2}\right)\left(r^2 + a^2\right) - 2Mr\,, \quad \Xi = 1 + \frac{a^2}{\ell^2}\,,$$
$$\Delta_\theta = 1 + \frac{a^2}{\ell^2}\cos^2\theta\,, \qquad \rho^2 = r^2 + a^2\cos^2\theta\,,$$

- Black hole parameters are $a, M, \Lambda = 3/l^2$ which describe a black hole with an inner, outer event and cosmological horizon for $\Lambda > 0$.
- Even if $M = 0$ the metric does not reduce to a trivial form of a de Sitter or anti de Sitter metric.

- Using Hamilton-Jacobi formalism we may, symmetries permitting, write geodesic eqs as a first order system. For that we need 4 constants of motion (for 4 dimensions).
- For Kerr we have 3 constants of motion : E, L_z, m. So in principle we would need to fix one of the dimensions in order to study geodesics in HJ fashion.
- The HJ equation reads :

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$

- where the HJ functional is $S = -Et + L_z\varphi + S(r,\theta)$. Brandon Carter showed that $S(r,\theta) = S_r(r) + S_\theta(\theta)$ is separable and the missing constant is $\mathcal{Q}$ Carter's constant

$$S_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr \,, \qquad S_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta} d\theta \,,$$

$$\begin{aligned} R(r) &= \Xi^2 \left[E \left(r^2 + a^2 \right) - a\, L_z \right]^2 \\ &- \Delta_r \left[\mathcal{Q} + \Xi^2 \left(a\, E - L_z \right)^2 + m^2 r^2 \right] \,, \\ \Theta(\theta) &= -\Xi^2 \sin^2\theta \left(a\, E - \frac{L_z}{\sin^2\theta} \right)^2 \\ &+ \Delta_\theta \left[\mathcal{Q} + \Xi^2 \left(a\, E - L_z \right)^2 - m^2 a^2 \cos^2\theta \right] \,. \end{aligned}$$

- Note we have $E, m, L_z, \mathcal{Q}$ parametrising the Energy at infinity, rest mass, angular momentum and Carter's separation constant.
- Θ and R are positive functions and their properties dictate many geodesic properties of Carter or Kerr solution

Carter's constant

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Stealth Kerr solution [CC, Crisostomi, Gregory and Stergioulas]

- in DHOST I theory (with $c_g = 1$) $X = -q^2$ and $R_{\mu\nu} = 0$ is solution under certain conditions.
- Metric is Kerr

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left[dt - a\sin^2\theta d\varphi\right]^2 + \rho^2\left(\frac{dr^2}{\Delta_r} + d\theta^2\right) + \frac{\sin^2\theta}{\rho^2}\left[a\,dt - \left(r^2 + a^2\right)d\varphi\right]^2 ,$$

$$\Delta_r = \left(r^2 + a^2\right) - 2\mu r\,, \quad \rho^2 = r^2 + a^2\cos^2\theta\,,$$

- and X is constant in Kerr.
- What is the scalar field painting this spacetime?

Carter found separable HJ potential $S = -Et + L_z\varphi + S_r(r) + S_\theta(\theta)$ such that

$\partial_\mu S\, \partial_\nu S\, g^{\mu\nu}_{Kerr} = -m^2 \longleftrightarrow \partial_\mu\phi\, \partial_\nu\phi\, g^{\mu\nu}_{Kerr} = -q^2$

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- The possible scalars for $\Lambda = 0$ read [Carter],

$$\phi(t,r,\theta) = -E\,t + L_z\varphi + \phi_r + \phi_\theta \textit{ with},$$

$$\phi_r = \pm\int \frac{\sqrt{R}}{\Delta_r}dr\,, \qquad \phi_\theta = \pm\int \sqrt{\Theta}d\theta\,,$$

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$$\phi(t,r) = q\,t + \int \frac{\sqrt{q^2(r^2+a^2)2Mr}}{\Delta_r} dr\,,$$

for $E = m = q$, $L_z = 0$, $Q = 0$

- Going to Kerr coords we see that scalar is regular at the event horizon. $v = t \pm \int dr\, \frac{r^2+a^2}{\Delta_r}$, $\quad \bar{\varphi} = \varphi \pm a \int \frac{dr}{\Delta_r}$
- For $a = 0$ we get spherical symmetry solution as before
- We have a Kerr solution of DHOST I theory with non trivial scalar
- What of $\Lambda \neq 0$

Rotating black hole with $\Lambda > 0$ [CC, Crisostomi, Gregory and Stergioulas]

- Scalar reads,

$$\phi(t,r,\theta) = -E\,t + \phi_r + \phi_\theta\,,$$

$$\phi_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr\,, \qquad \phi_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta} d\theta\,,$$

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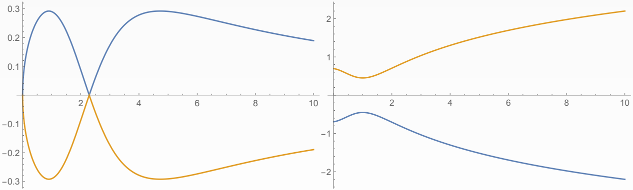
where $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$ where $\eta \leq 1$ for $\Theta > 0$.

- η_c is the limiting value of $R > 0$. ie., it is such that R has a double zero at $r_{EH} < r_0 < r_{CH}$
- we have $\eta_c < 1$ and as Λ increases η_c decreases
- We have two branches of solutions. Going to EF coords we see that one chart is regular at the EH while the latter at the CH but none are regular at both.



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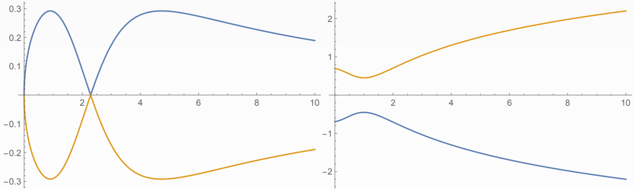
- η_c is the limiting value of $R > 0$. ie., it is such that R has a double zero at $r_{EH} < r_0 < r_{CH}$
- we have $\eta_c < 1$ and as Λ increases η_c decreases
- Fixing $\eta = \eta_c$ the two branches join with C_2 regularity at $r = r_0$!
- Then going from one branch to the other at $r = r0$, keeping $\phi - C^2$ differentiable, we have a regular scalar at the Event and Cosmological horizon.



- Starting from stealth Kerr and using disformal transformations we can construct stationary solutions of DHOST which are not stealth Kerr.
- In fact, the disformed Kerr metrics with X constant and therefore D constant are,

$$g^{Kerr}_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = g^{Kerr}_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$$

for given D. Rotation creates a solution which has similar characteristics but is completely distinct from the Kerr solution.

$$ds^2 = -\left(1 - \frac{2\tilde{M}r}{\rho^2}\right)dt^2 - \frac{4\sqrt{1+D}\tilde{M}ar\sin^2\theta}{\rho^2}dtd\varphi + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta\right]d\varphi^2$$
$$+ \frac{\rho^2\Delta - 2\tilde{M}(1+D)rD(a^2+r^2)}{\Delta^2}dr^2 - 2D\frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}dtdr + \rho^2d\theta^2 \ .$$

-For $D \neq 0$ and $a \neq 0$ not an Einstein metric!
-Mass, angular momentum effected by D
-For $D \neq 0$ we do not verify the GR no hair relation
-Disformed Kerr is a one parameter family of well defined admissible alternatives to Kerr

Geodesics not integrable spacetime not circular for $D \neq 0$

- Starting from stealth Kerr and using disformal transformations we can construct stationary solutions of DHOST which are not stealth Kerr.
- In fact, the disformed Kerr metrics with X constant and therefore D constant are,

$$g^{Kerr}_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = g^{Kerr}_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$$

for given D. Rotation creates a solution which has similar characteristics but is completely distinct from the Kerr solution.

$$\mathrm{d}s^2 = -\left(1-\frac{2\tilde{M}r}{\rho^2}\right)\mathrm{d}t^2 - \frac{4\sqrt{1+D}\tilde{M}ar\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta\right]\mathrm{d}\varphi^2$$

$$+\frac{\rho^2\Delta - 2\tilde{M}(1+D)rD(a^2+r^2)}{\Delta^2}\mathrm{d}r^2 - 2D\frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}\mathrm{d}t\mathrm{d}r + \rho^2\mathrm{d}\theta^2 .$$

-For $D \neq 0$ and $a \neq 0$ not an Einstein metric!
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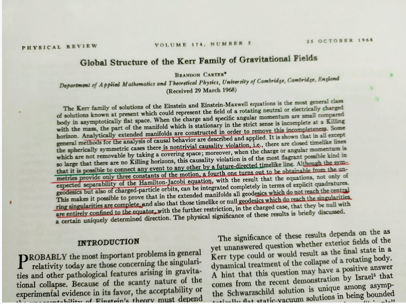
Geodesics not integrable spacetime not circular for $D \neq 0$

Conclusions

- Brandon Carter's insightful and important research helps out in unexpected ways future generations of researchers



PHYSICAL REVIEW VOLUME 174, NUMBER 5 25 OCTOBER 1968

Global Structure of the Kerr Family of Gravitational Fields

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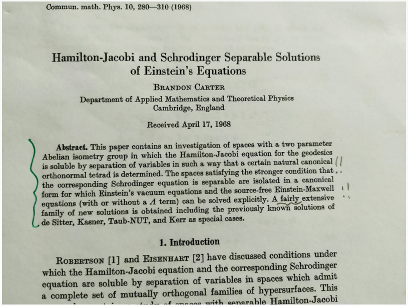
(Received 29 March 1968)

The Kerr family of solutions of the Einstein and Einstein-Maxwell equations is the most general class of solutions known at present which could represent the field of a rotating neutral or electrically charged body in asymptotically flat space. When the charge and specific angular momentum are small compared with the mass, the part of the manifold which is stationary in the strict sense is incomplete at a Killing horizon. Analytically extended manifolds are constructed in order to remove this incompleteness. Some general methods for the analysis of causal behavior are described and applied. It is shown that in all except the spherically symmetric cases there is nontrivial causality violation, i.e., there are closed timelike lines which are not removable by taking a covering space; moreover, when the charge or angular momentum is so large that there are no Killing horizons, this causality violation is of the most flagrant possible kind in that it is possible to connect any event to any other by a future-directed timelike line. Although the symmetries provide only three constants of the motion, a fourth one turns out to be obtainable from the unexpected separability of the Hamilton-Jacobi equation, with the result that the equations, not only of geodesics but also of charged-particle orbits, can be integrated completely in terms of explicit quadratures. This makes it possible to prove that in the extended manifolds all geodesics which do not reach the central ring singularities are complete, and also that those timelike or null geodesics which do reach the singularities are entirely confined to the equator, with the further restriction, in the charged case, that they be null with a certain uniquely determined direction. The physical significance of these results is briefly discussed.

INTRODUCTION

PROBABLY the most important problems in general relativity today are those concerning the singularities and other pathological features arising in gravitational collapse. Because of the scanty nature of the experimental evidence in its favor, the acceptability or

The significance of these results depends on the as yet unanswered question whether exterior fields of the Kerr type could or would result as the final state in a dynamical treatment of the collapse of a rotating body. A hint that this question may have a positive answer comes from the recent demonstration by Israel[5] that the Schwarzschild solution is unique among asymptotically flat static-vacuum solutions in being bounded



Commun. math. Phys. 10, 280—310 (1968)

Hamilton-Jacobi and Schrodinger Separable Solutions of Einstein's Equations

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Abstract. This paper contains an investigation of spaces with a two parameter Abelian isometry group in which the Hamilton-Jacobi equation for the geodesics is soluble by separation of variables in such a way that a certain natural canonical orthonormal tetrad is determined. The spaces satisfying the stronger condition that the corresponding Schrodinger equation is separable are isolated in a canonical form for which Einstein's vacuum equations and the source-free Einstein-Maxwell equations (with or without a Λ term) can be solved explicitly. A fairly extensive family of new solutions is obtained including the previously known solutions of de Sitter, Kasner, Taub-NUT, and Kerr as special cases.

1. Introduction

ROBERTSON [1] and EISENHART [2] have discussed conditions under which the Hamilton-Jacobi equation and the corresponding Schrodinger equation are soluble by separation of variables in spaces which admit a complete set of mutually orthogonal families of hypersurfaces. This