Scaling of Global Defects

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Outline

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- In 4 spacetime dimensions the defects are cosmic strings for $N = 2$, monopoles for $N = 3$ and 'textures' for $N = 4$. Scalar fields with more than 4 components do not lead to defects in 4 spacetime dimensions.
- In the case of global defects, most of the field energy is actually in the gradient energy and not in the potential energy at the position of the defect and we can very well describe them as a non-linear sigma-model.

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Introducing $\beta = \phi/\eta$, the equation of motion of the non-linear σ-model is simply

$$\Box\beta - (\beta \cdot \Box\beta)\beta = 0, \qquad \beta^2 = 1\,, \qquad \Box = \partial_t^2 + 2\mathcal{H}\partial_t - \Delta\,.$$

$$(\beta \cdot \alpha) = \sum_{i=1}^{N} \beta_i \alpha_i\,.$$

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In the large N limit we may replace $(\beta \cdot \Box\beta) = -(\partial_\mu\beta \cdot \partial^\mu\beta)$ by its expectation value and in this way remove the non-linearity.
The field equation can then be solved exactly in Fourier space

$$\beta(k,t) = At^{3/2}\frac{J_\nu(kt)}{(kt)^\nu}\beta_{\rm ini}\,, \qquad \mathcal{H} = \frac{\nu - 1}{t}\,.$$

Effects of defects on structure formation

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They interact with the defect energy momentum tensor via gravity. This can be cast in the form of linear time dependent differential equations in Fourier space

$$\mathcal{D}X_j = M_j{}^i F_i$$

which can be solved via their Green's function,

$$X_j(\mathbf{k},t) = \int_{t_{\rm in}}^t G_j{}^i(t,t',k)F_i(\mathbf{k},t')dt' \,.$$

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Here the F_i are the stochastic components of the defect energy momentum tensor and the X_j are stochastic variables describing cosmic observables.
What we can compute are not these stochastic variables, but their correlation functions or power spectrum. For reasons of homogeneity and isotropy

$$\langle F_i(\mathbf{k},t)F_j(\mathbf{k}',t')\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')C_{ij}(k,t,t') \,.$$

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In principle, these functions of 3 variables (k,t,t') which are of fourth order in the field variable β,

$$T^{(s)}_{\mu\nu} = \frac{\eta^2}{a^2}[\partial_\mu\beta\cdot\partial_\nu\beta - \frac{1}{2}g_{\mu\nu}(\partial_\lambda\beta\cdot\partial^\lambda\beta)]$$

have to be determined with numerical simulations.
Considering that the defects form at very high energy, $T\sim 10^{15}$GeV hence $t\sim 10^{-28}t_0$. No computer simulation can reach a dynamical range of 28 orders of magnitude or similar... A new idea is needed :

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It is reasonable to assume that the unequal time correlation functions only depend on the dimensionless variables $x = kt$ and $x' = kt'$ or on the ratio $r = t'/t$. By definition they are symmetric in x and x' or under $r \to 1/r$.

Scaling

It makes sense to assume that the C_{ij} tend to a constant value on super horizon scales, $x = kt \to 0$ and inside the horizon the fields align and the C_{ij} tend to zero. We also expect them to be maximal for $t = t'$, i.e. $r = 1$ and to decay for $r \to \infty$ or $r \to 0$. We therefore only have to find the value of these correlations functions at $x = 0$, $r = 1$ and to determine the power law of the fall off. This can be done with good accuracy in a few 10^3 cubed simulations. Which, nowadays in very feasible with a moderate supercomputer.

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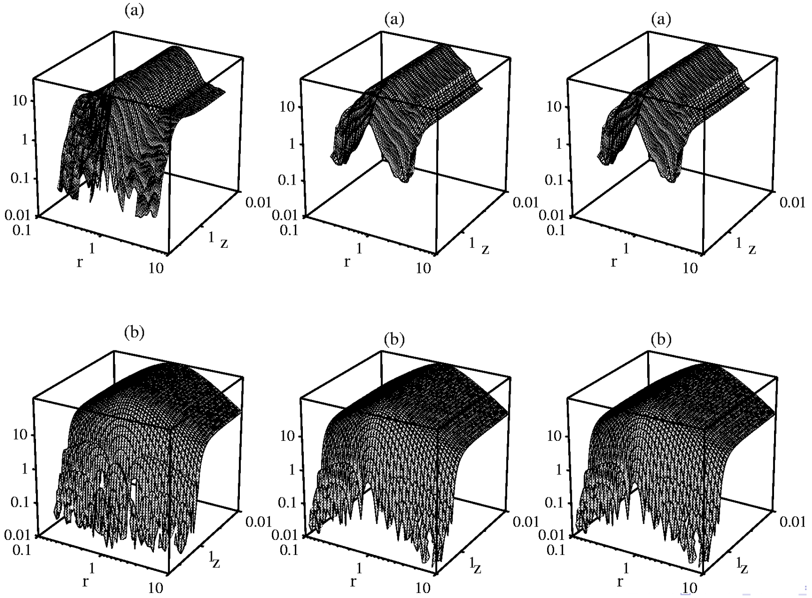
Symmetry under rotation, translation and parity allows to reduce the problem to 5 correlators.
2 from scalar degrees of freedom of $T^{(s)}_{\mu\nu}$ which can be cast in terms of the Bardeen potentials and one vector and one tensor correlator.

$$
\begin{aligned}
k^4\sqrt{tt'}\langle\Psi_s(\mathbf{k},t)\Psi_s^*(\mathbf{k}',t')\rangle &= (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\eta^4 C_1(x,r)\,,\\
k^4\sqrt{tt'}\langle\Phi_s(\mathbf{k},t)\Phi_s^*(\mathbf{k}',t')\rangle &= (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\,\eta^4 C_2(x,r)\,,\\
k^4\sqrt{tt'}\langle\Phi_s(\mathbf{k},t)\Psi_s^*(\mathbf{k}',t')\rangle &= (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\eta^4 C_3(x,r)\,.
\end{aligned}
$$

Scalar perturbations : the Bardeen potentials for 'texture' (a) and in the large N limit (b) (Durrer et al. 2002)



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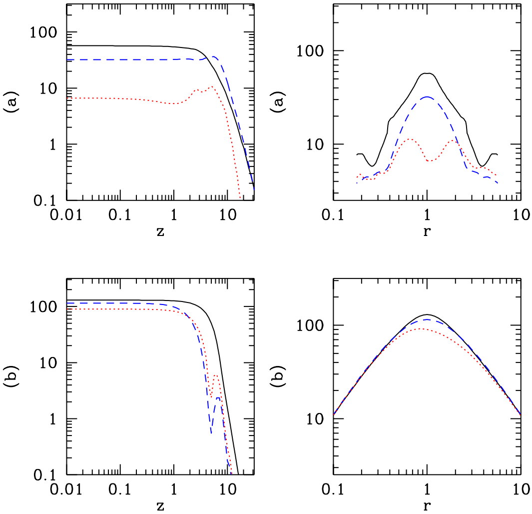
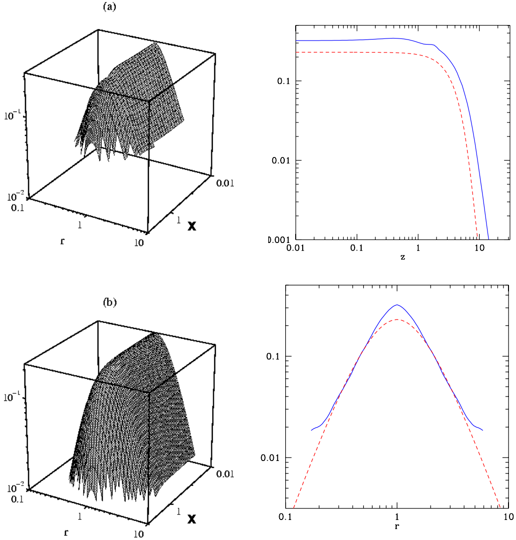


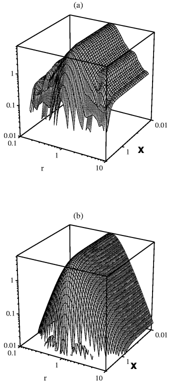
Cuts : $C_i(x, 1)$ (left) and $C_i(0, r)$ (right). Black the Φ-correlator, blue the Ψ-correlator and red the cross correlation.

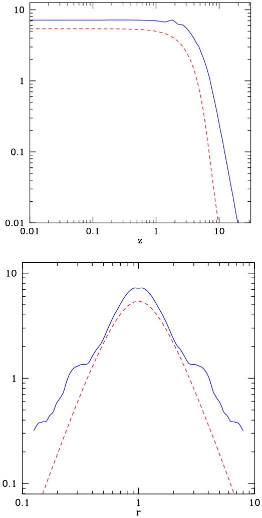
Vector perturbations (a) 'texture' (b) the large N limit (from Durrer et al. 2002)



Cuts : $W(x, 1)$ (top) and $W(0, r)$ (bottom). Solid the 'texture' simulation, dashed the large N limit.

Tensor perturbations (a) 'texture' (b) the large N limit (from Durrer et al. 2002)





Cuts : $T(x, 1)$ (top) and $T(0, r)$ (bottom). Solid the 'texture' simulation, dashed the large N limit.

These unequal time correlators can now be used to solve the linear perturbation equations. Usually one Diagonalized $C_i(k,t,t')$ in t,t',

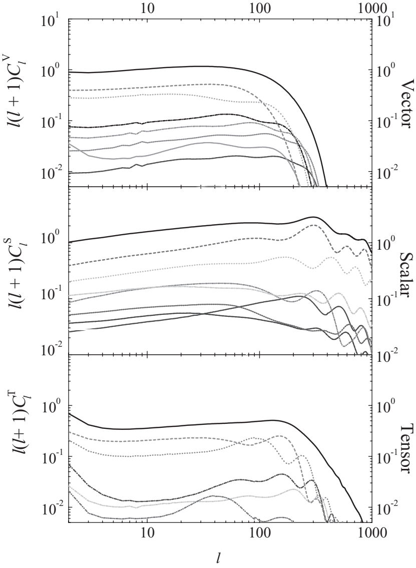
$$\int dt' C_{ij}(k,t,t')S_j^*(k,t',n) = \lambda_n(k)S_i(k,t,n)\,.$$

Here $\lambda_n(k)$ is an the eigenvalue and $S_i(k,t,n)$ an eigenvector of the unequal time correlator $C_i(k,t,t')$.

One can then solve the linear perturbation equations for the deterministic source terms $\sqrt{\lambda_n}S_i(n)$ and add the contributions from the 200 or so highest eigenvalues.

This program has been carried out for global defects, for the large N limit (of global defects) and for cosmic strings (Abelian Higgs model) for which the unequal time correlators also scale.

Results for CMB fluctuations from global texture



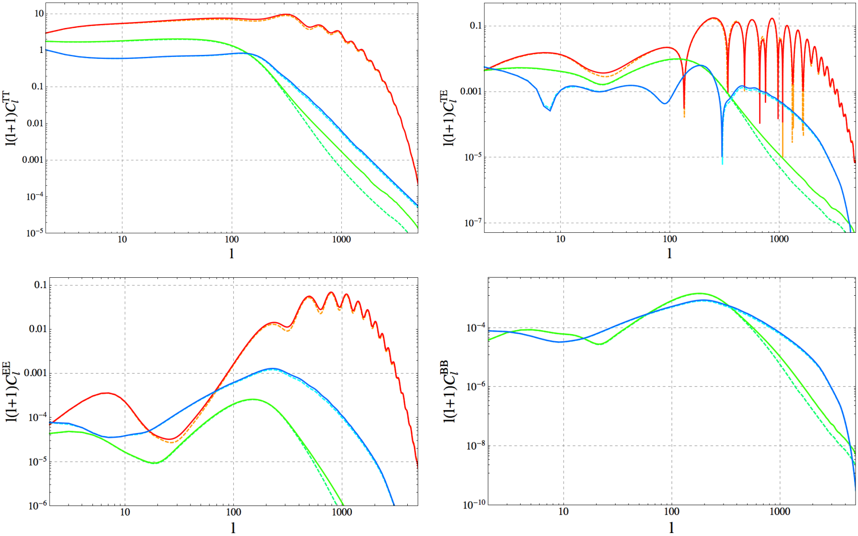
(Durrer et al. 2002)

The contributions from the highest eigenvalues to the CMB temperature anisotropies from global texture in units $(4\pi G\eta^2)^2 = \mu^2$.

Note that the scalar and vector perturbations have comparable amplitudes.

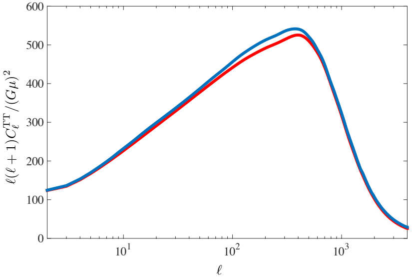
Also note the 'smearing out' of the acoustic peaks due to the sum over many eigenvalues $\Rightarrow$ decoherence.

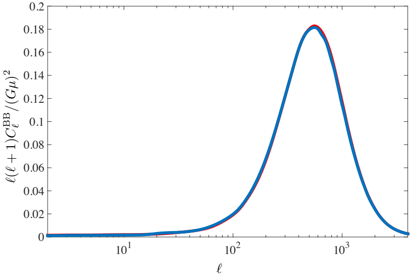
Results for CMB fluctuations from the large N limit (Fenu et al. 2014)



(Results from the sum over 200 eigenvectors.)

Results for CMB fluctuations from cosmic strings (Lizarraga et al. 2016)





Total CMB spectra for temperature and B-polarisation from Abelian Higgs model cosmic strings (field theory simulation).

Results obtained using 256 eigenvectors from $(4096)^3$ simulations (Daverio et al. 2015).

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Conclusions

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- Spectra from defects do not show a coherent sequence of acoustic peaks in the temperature anisotropies and are therefore not responsible for the observed signal. They can at best contribute a few % .
- Studying defects we have learned a lot about the very special nature of inflationary perturbations, which are coherent over scales much larger than the Hubble horizon after inflation.

Happy Birthday, Brandon !