Soft Hair

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Significance of black hole uniqueness theorems

- The no-hair theorems indicate that stationary black holes are uniquely specified by their electric charge, mass M and angular momentum.
- Higher multipole moments would radiate away.
- Black holes annihilate all information about their formation.
- Hawking radiation is thermal (apart from greybody factors).
- Black Hole completely evaporates leaving nothing on a timescale $\sim M^3$.
- An initial pure quantum state appears to turn into a mixed state.
- Violates the unitary time evolution laws of quantum mechanics.

Soft Electromagnetic Charges

Consider solutions of Maxwell's equations in Minkowski spacetime with field strength F_{ab}.

- On $\mathcal{I}^-_+$, the future endpoint of past null infinity, $\mathcal{F}^- = F_{rv}(\theta, \phi)$.
- On $\mathcal{I}^+_-$, the past endpoint of future null infinity, $\mathcal{F}^+ = F_{ru}(\pi - \theta, \phi + \pi)$.
- Lienard and Wiechert showed that $\mathcal{F}^- = \mathcal{F}^+$, the antipodal matching condition.
- Define charges, $Q^+_\epsilon = \int_{\mathcal{I}^+_-} \mathcal{F}^+ \epsilon$ for any spherical harmonic ϵ.

  and $Q^-_\epsilon = \int_{\mathcal{I}^-_+} \mathcal{F}^- \epsilon$.
- Conservation law $Q^-_\epsilon = Q^+_\epsilon$.

Flux Formula

- Define $Q^+_\epsilon(u_0)$ as the charge on any section of future null infinity at retarded time u_0.
- $Q^+_\epsilon(u_0) - Q^+_\epsilon = \int_{u=-\infty}^{u_0} \quad d\epsilon * F + \epsilon J$
  where J is the electric current.
- The first term is the flux of soft charge and the second is the change in the hard charge.
- There are similar soft magnetic charges found by replacing F by $*F$ even though there are no hard charges.

Quantum Theory of Soft Charges

Let z be the complex stereographic coordinate on a section of future null infinity, then

$$[Q_\epsilon, A_z] = i\partial_z \epsilon$$

The charge induces a "large" gauge transformation on the potential.

"Large" because it is does not die off at infinity.

Now consider a vacuum $|0\rangle$.

$|0'\rangle = Q_\epsilon|0\rangle$ is a new vacuum state orthogonal to $|0\rangle$. $|0'\rangle$ contains one more or one less soft photon than $|0\rangle$ and its angular momentum differs by $0, \pm 1$.

The vacuum is infinitely degenerate. Different vacua differ by containing different numbers of soft photons.

In scattering calculations when these soft photons are ignored, there are infrared divergences. When taken into account, there are no infrared divergences.

Black Holes

These charges appear at the boundary of any region through which lightlike degrees of freedom can pass.

Consider now a spacetime in which a black hole forms.

There are similar charges on the horizon. They can be found by extending the surface on which they are defined from just being on future null infinity to being the union of the future null infinity with the horizon.

The conservation law is now

$$Q_\epsilon^- = \int_{\text{Future null infinty}} \dots + \int_{horizon} \dots$$

Black holes states then are also infinitely degenerate distinguished by the number of soft photons that have fallen in. These give rise the soft hair.

Gravitation

Similar phenomena exist for gravitation. Metric near future null infinity in an asymptotically flat spacetime

$$ds^2 = -(1 - \frac{2m_B}{r})du^2 - 2dudr + (r^2\gamma_{z\bar{z}} + rC_{z\bar{z}})dzd\bar{z} + \ldots$$

m_B is the Bondi mass aspect and $C_{z\bar{z}}$ contains information about gravitational radiation. The Bondi news function $Nz\bar{z} = \frac{\partial C_{z\bar{z}}}{\partial u}$. Consider a BMS supertranslation f. The corresponding diffeomorphism is

$$f\partial_u - \frac{1}{r}D^A f\partial_A + \frac{1}{2}D^2 f\partial_r + \ldots$$

Supertranslation charge at the past endpoint of future null infinity

$$Q_f^+ = \frac{1}{4\pi}\int \sqrt{\gamma}\, dz\, d\bar{z}\ f\ m_B$$

There is also a flux formula

$$\Delta Q = \frac{1}{4\pi}\int du\sqrt{\gamma}\ dz\, d\bar{z}\ f\left(T_{uu} - \frac{1}{4}D^A D^B N_{AB}\right)$$

The first term in the integrand is the flux of hard charge coming from matter and gravitational radiation passing through null infinity. The second term is the soft charge.
The BMS group is the semi-direct product of these supertranslations and the Lorentz group. The extended BMS group includes Killing vector fields and conformal Killing vector fields with prescribed singularities.
Again, there are similar magnetic charges.

Horizon Charges

There are charges on the black hole horizon. It is difficult to evaluate these except for the Schwarzschild case, and the variation of the charge is easy to understand. Suppose the metric of Schwarzschild is g_{ab}. Then by processes involving either soft or hard components the variation of the metric will be h_{ab} and the variation of the flux formula is

$$\delta Q_f^{(E)} = \frac{1}{16\pi M}\int dv\sqrt{\gamma}\,dz\,d\bar{z}\,D^A D^B f \sigma_{AB}$$

where $\sigma_{AB} = \frac{1}{2}\partial_v h_{AB}$ and v is the advanced time on the horizon. There is a magnetic analog

$$\delta \tilde{Q}_f^{(M)} = \frac{1}{16\pi M}\int dv\sqrt{\gamma}\,dz\,d\bar{z}\,D^A D^B f\ \epsilon_A{}^C \sigma_{CB}$$

Charge Algebra

Expect the algebra of charges to close.

$$[Q^{(E)}, Q^{(E)}] \sim Q^{(E)}$$

$$[Q^{(M)}, Q^{(M)}] \sim Q^{(E)}$$

$$[Q^{(E)}, Q^{(M)}] \sim Q^{(M)} + \text{central term}$$

The presence of a central term indicates that diffeomorphism invariance is violated since the algebra of diffeomophisms should close.

The anomaly can be cancelled by supposing the horizon is described by a holographic Chern-Simons theory on the horizon with gauge group $SL(2, \mathbb{C})$.

This holographic theory might describe the internal states of the black hole that give rise to its entropy.

It would not be required to cancel the anomaly for observers falling into the black hole as the boundary of that they can observe is not the horizon. This observer dependence seems typical of black hole physics.

The Information Paradox

- Too much hair - there is an infinite amount?
- Wrong kind of hair - Does it describe the quantum state of the matter forming a black hole?
- Not enough hair - Species problem.
- Entropy calculation?
- How does the information get recorded?
- How does the information get retrieved?
- No cloning theorem and the singularity?
- The precise nature of the holographic theory on the horizon.