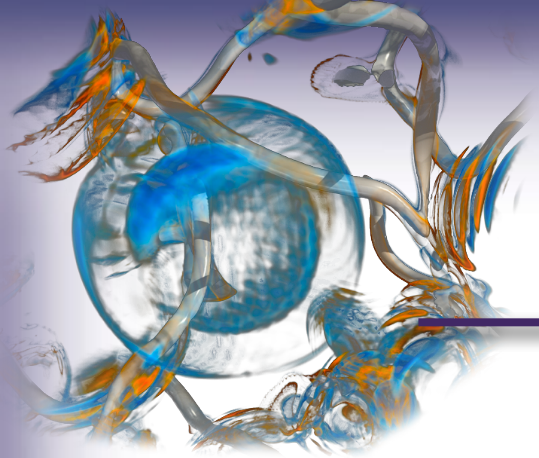
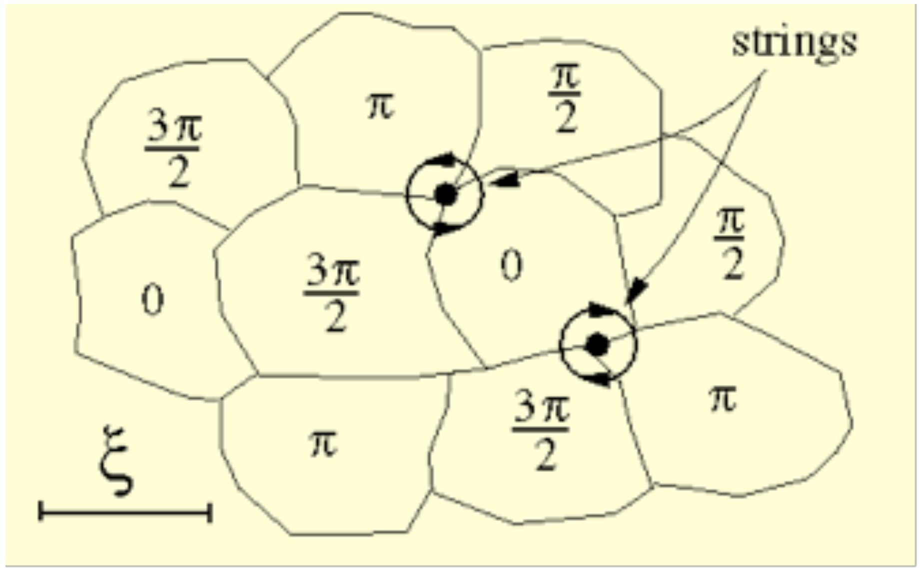
Cosmic Strings with Currents–the CVOS model

Paul Shellard
Centre for Theoretical Cosmology, DAMTP, Cambridge
Carlos Martins (Porto), Patrick Peter (IAS/CNRS) and Ivan Rybak (Porto)
also Amelia Drew (Cambridge), Andrei Lazanu (ENS) and José Ricardo Correia (Porto)

Brandon Carter 80 Fest: BHs and other Cosmic Systems
Institut d'Astrophysique de Paris (IAP) and Observatoire de Paris (Meudon)
4th July 2022



Cosmic string disorder



Cosmic strings are a natural consequence of symmetry breaking in phase transitions

- *arguably 'generic' in realistic GUT theories and brane inflation models*
- *topology implies existence through the Kibble mechanism*

Observable through their gravitational effects related to $G\mu \sim (\eta/M_{Pl})^2$

- Cosmic microwave sky 'line-like' anisotropies and B-mode polarisation

  Planck CMB constraint $G\mu < 1.5 \times 10^{-7}$. *[Planck 2015]*

  Improves further with Simons and CMBS4

- Gravitational wave stochastic backgrounds or 'burst' signatures

  Uncertainty about loop production size/history

  Standard model A: $G\mu < 1 \times 10^{-8}$

  New model B: $G\mu < 4 \times 10^{-15}$. *[LIGO 2021]*

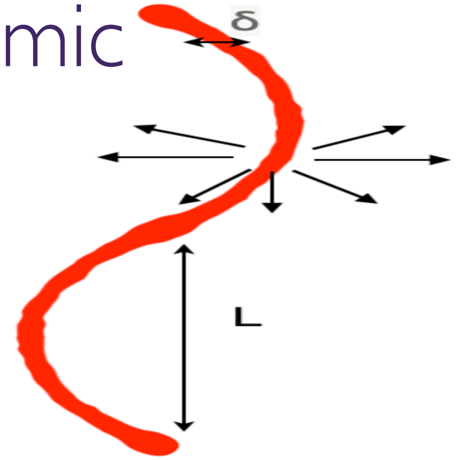
  Also pulsar constraints and LISA prospects…

- Axion mass prediction uncertainty 25-500 μeV for ADMX etc searches

Numerical advances: AMR

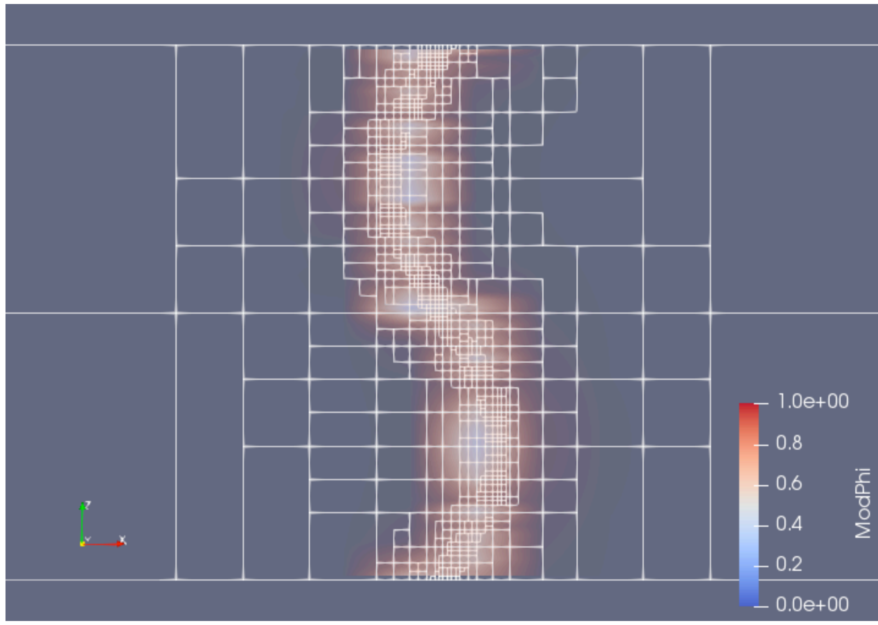
Challenge: to separate string scales from microscopic to cosmic

Amelia Drew and EPS (arXiv:1910.01718)

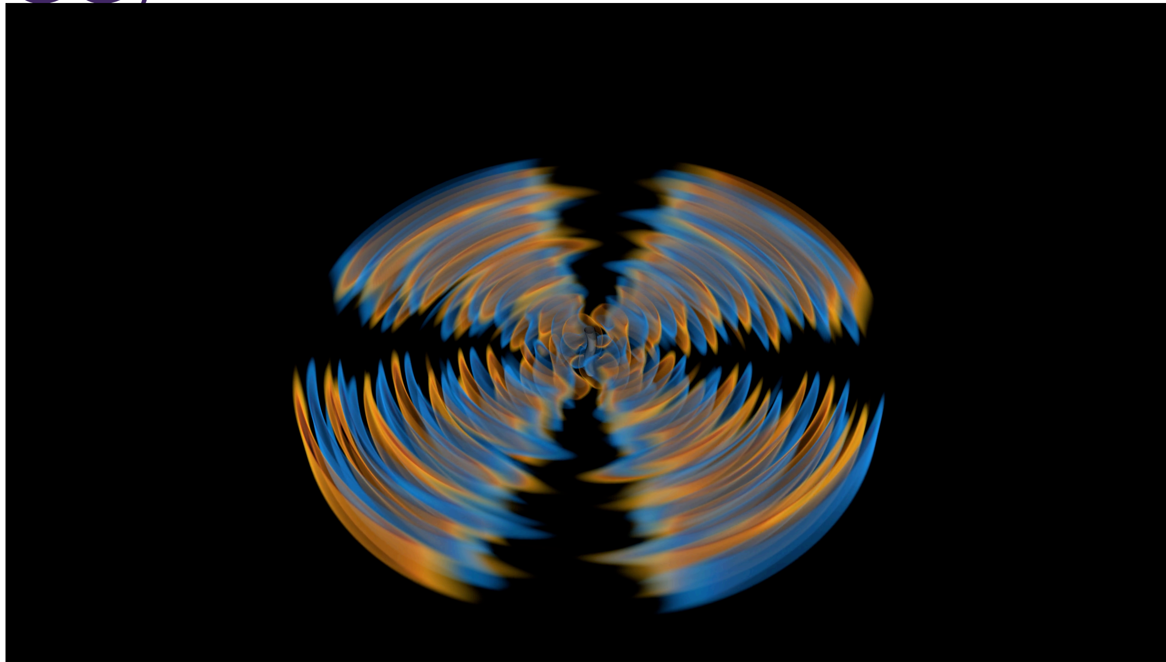


Fully Adaptive Mesh Refinement dynamically adapts solution grid to scale of the problem, so allows for accurate evolution across different scales

[see also Buschmann, et al 2022]

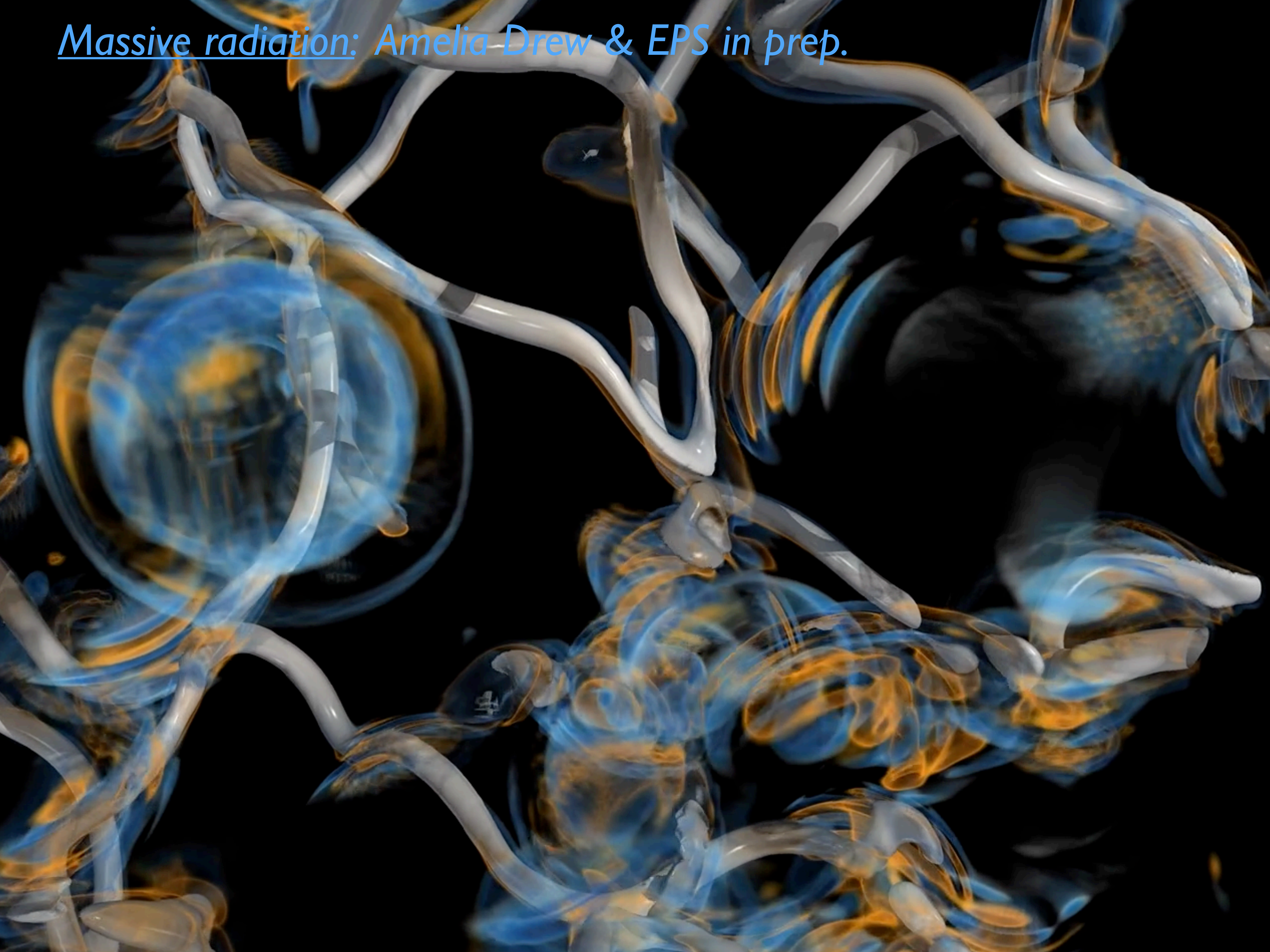


- GRChombo: Dynamically tag cells according to chosen gradient criterion
- HPC OpenMP/MPI parallelism (Intel IPCC)
- Refinement levels structured into boxes distributed over processors
- Advanced in-situ visualization with Intel OSPRay (in Paraview)

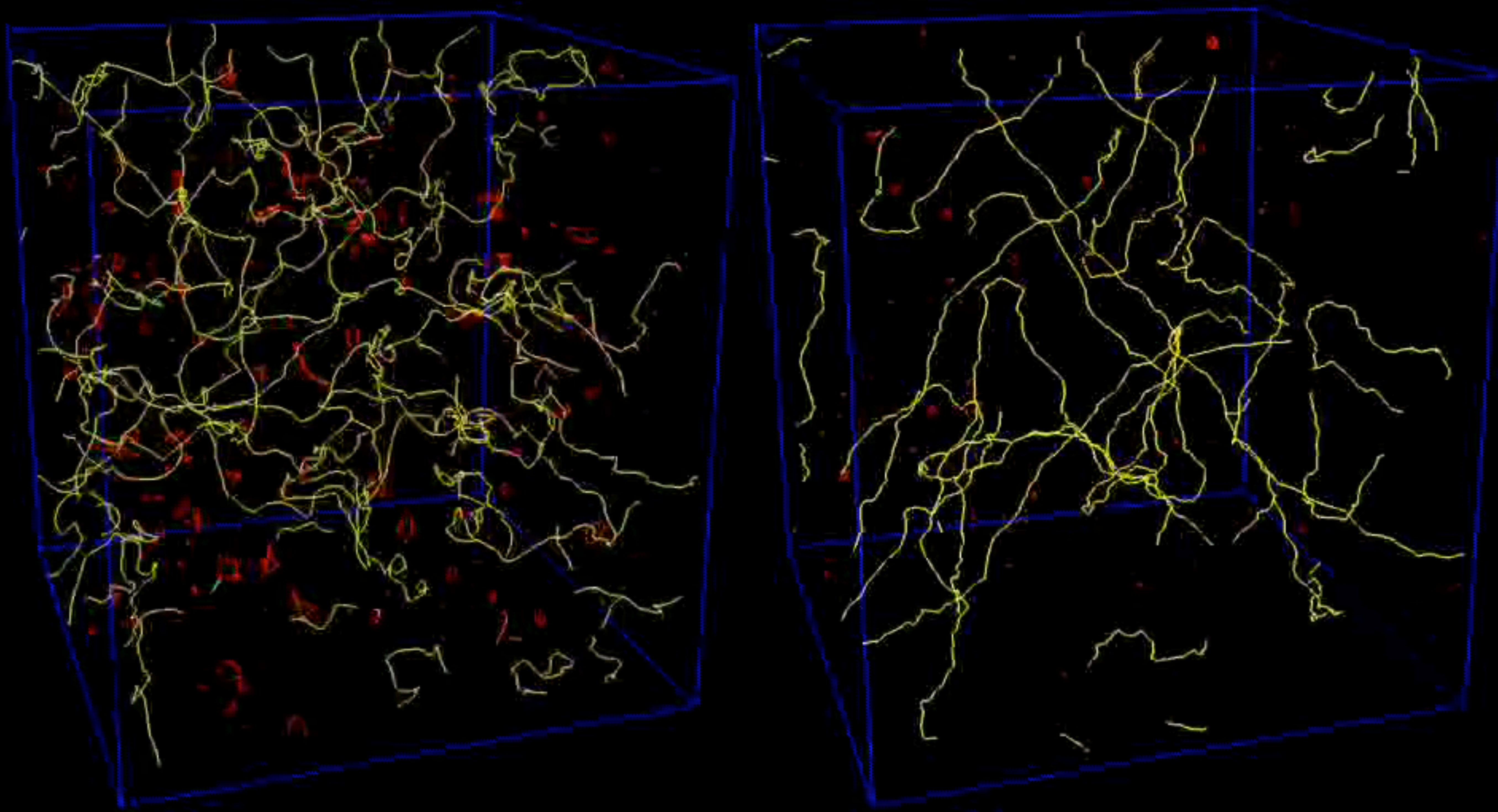


Massless radiation: *Amelia Drew & EPS arXiv:1910.01718*

Massive radiation: Amelia Drew & EPS in prep.

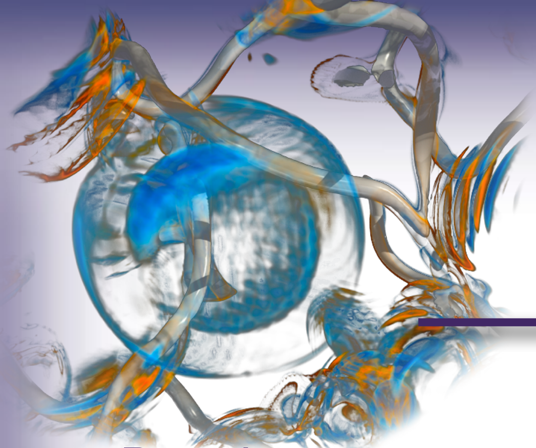


Nambu-Goto strings: Allen & EPS ’90 [also Bennett, Bouchet, ’89; Olum et al, ’95-, etc]



Radiation era

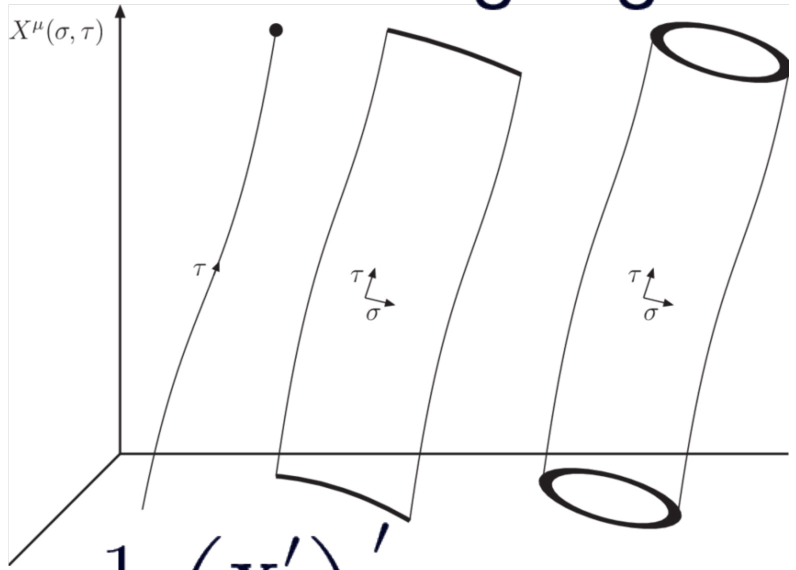
Matter era



Velocity one-scale model

Preliminaries: Uniformly expanding universe, the string EoM are:

Using conformal time $\sigma^0 = \tau$ and the transverse gauge



$$\dot{\mathbf{x}} \cdot \mathbf{x}' = 0$$

the string equations become

acceleration

$$\ddot{\mathbf{x}} + \left(2\frac{\dot{a}}{a} + \frac{a}{\ell_{\rm f}}\right)\left(1 - \dot{\mathbf{x}}^2\right)\dot{\mathbf{x}} = \frac{1}{\epsilon}\left(\frac{\mathbf{x}'}{\epsilon}\right)',$$

curvature

$$\dot{\epsilon} + \left(2\frac{\dot{a}}{a} + \frac{a}{\ell_{\rm f}}\right)\dot{\mathbf{x}}^2\epsilon = 0\,,$$

damping

where ϵ is the energy per unit σ is $\epsilon^2 = \frac{\mathbf{x}'^2}{1 - \dot{\mathbf{x}}^2}$,

Velocity One-scale Model (VOS)

Thermodynamic approach: Total network energy

$$E = \mu a(\tau) \int \epsilon d\sigma \, ,$$

Time derivative - energy conservation equation

$$\frac{d\rho}{dt} + \left(2H\left(1+v^2\right) + \frac{v^2}{\ell_{\rm f}}\right)\rho = 0 \, .$$

add loop production

where the averaged rms velocity is defined by

$$v^2 \equiv \langle \dot{\mathbf{x}}^2 \rangle = \frac{\int \dot{\mathbf{x}}^2 \epsilon d\sigma}{\int \epsilon d\sigma} \, .$$

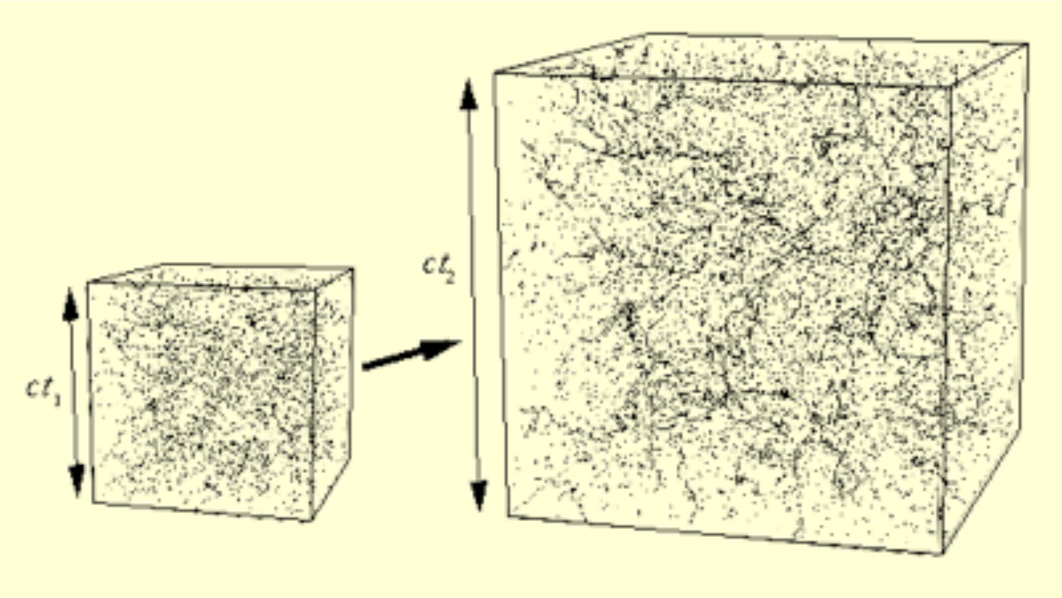
Kibble 1980; Vilenkin 1982

Martins & EPS 1995,6

Brownian network 'correlation length'

1 string μ*L* per correlation volume L^3

$$\rho_\infty \equiv \frac{\mu}{L^2} \, .$$



The VOS Model

Governs large-scale network evolution over cosmic history

Martins & EPS 1995,6

$$2\frac{dL}{dt} = 2HL(1+v_\infty^2) + \frac{Lv_\infty^2}{\ell_{\mathrm{f}}} + \tilde{c}v_\infty \,.$$

Hubble stretching & redshifting *damping* *loop creation*

$$\frac{dv}{dt} = (1-v^2)\left[\frac{k}{R} - v\left(2H + \frac{1}{\ell_{\mathrm{f}}}\right)\right] .$$

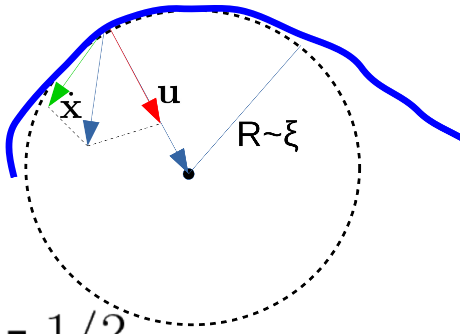
acceleration *curvature* *Hubble damping* *friction*

where the curvature term is

NB: only one free parameter ***c***

$$k(v) = \frac{2\sqrt{2}}{\pi}\,\frac{1-8v^6}{1+8v^6}$$

$$k(v) \equiv \frac{\left\langle (1-\dot{\boldsymbol{X}}^2)(\dot{\boldsymbol{X}}\cdot\boldsymbol{u})\right\rangle}{v(1-v^2)}$$



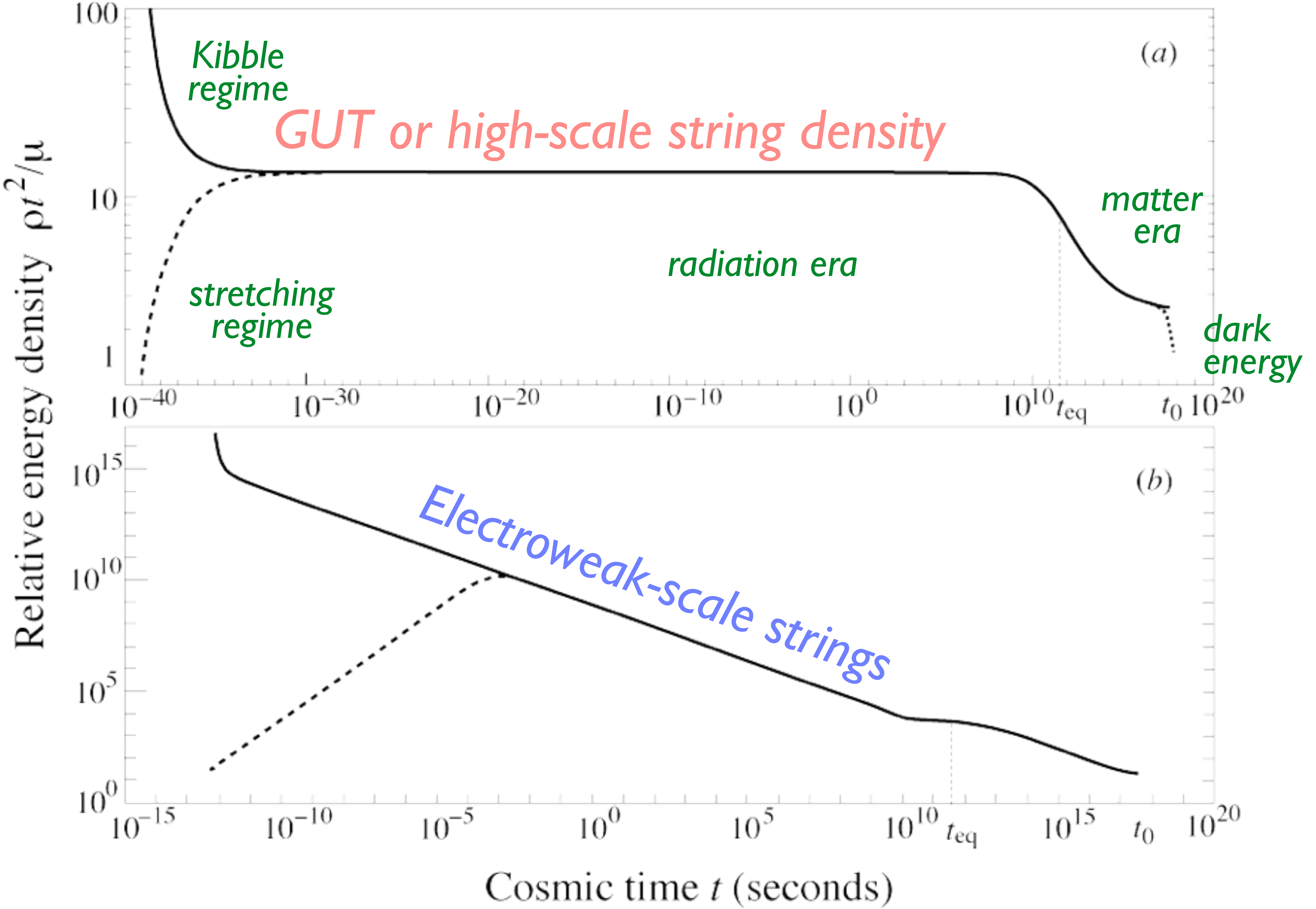
For scale factor $a(t) \propto t^\beta$ we have

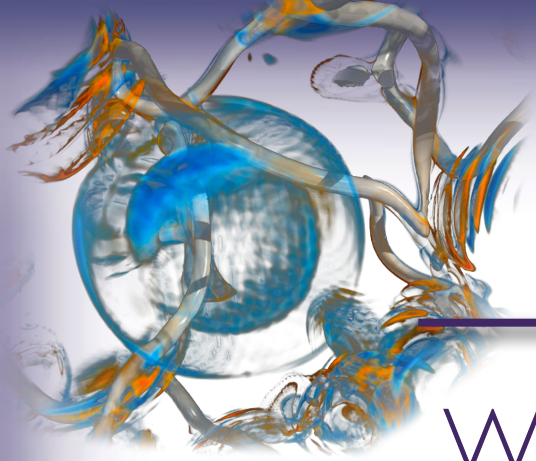
Solution has linear scaling:

$$L = \left[\frac{k(k+\tilde{c})}{4\beta(1-\beta)}\right]^{1/2} t, \qquad v = \left[\frac{k(1-\beta)}{\beta(k+\tilde{c})}\right]^{1/2}$$

that is, $L \propto t$ and $v = \mathrm{const.}$ with $\rho_\infty \propto t^{-2}$.

Complete history of strings



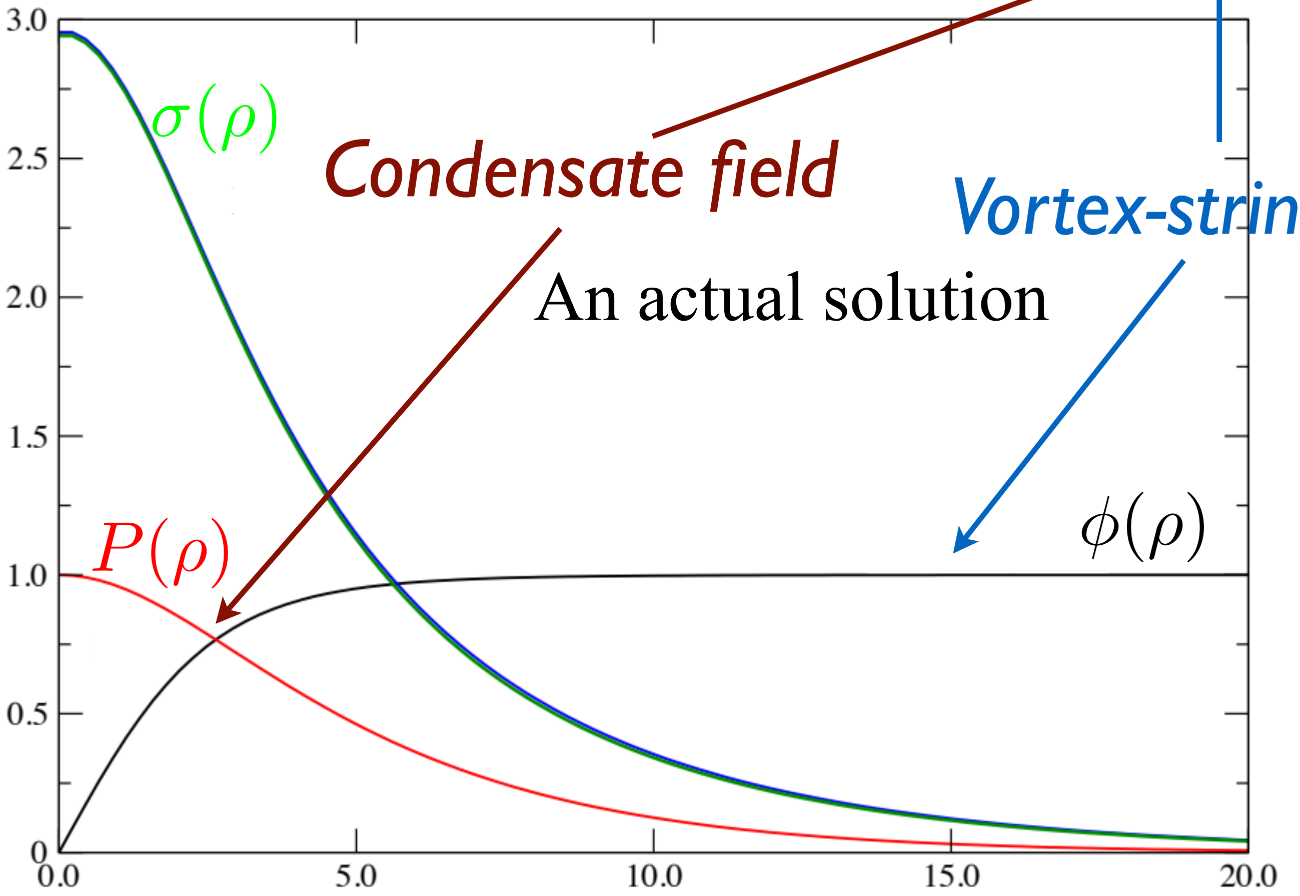


Superconducting strings

Witten superconducting strings model with $U(1)$x$U(1)$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}D_\mu\Phi\left(D^\mu\Phi\right)^* + \frac{1}{2}D_\mu\Sigma\left(D^\mu\Sigma\right)^* - V(\Phi,\Sigma)$$

Cylindrical coordinates (t, r, θ, z)



Condensate field

Vortex-string field

An actual solution

<u>Ansatz</u>

$$B_\mu \mathrm{d}x^\mu = \frac{1}{e_\phi}\left[n - P(r)\right]\mathrm{d}\theta$$

$$\Phi(r,\theta) = \eta\phi(r)\mathrm{e}^{\mathrm{i}n\theta}$$

$$A_\mu \mathrm{d}x^\mu = A_z(r)\mathrm{d}z + A_t(r)\mathrm{d}t$$

$$\Sigma(t,r,z) = \eta\sigma(r)\mathrm{e}^{\mathrm{i}(\omega t - kz)}$$

$$\rho \equiv \sqrt{\lambda_\phi}\eta r$$

PP, *Phys. Rev.* **D45**, 1091 (1992)

The String equation of state

Finding the correct EoS for *U*(1)x*U*(1) strings from microphysics

Peter, *Phys. Rev.* **D46**, 3335 (1992)
Phys. Rev. **D47**, 3169 (1993)

Stress energy tensor $T^{\mu\nu} = -2\frac{\delta\mathcal{L}}{\delta g_{\mu\nu}} + g^{\mu\nu}\mathcal{L}$

Integrated for macroscopic treatment

$$\begin{aligned}\mathcal{T}^{\mu\nu} &\equiv \int T^{\mu\nu}\mathrm{d}^2x^\perp \\ &= Uu^\mu u^\nu - Tv^\mu v^\nu \\ &= (U-T)u^\mu u^\nu - T\eta^{\mu\nu}\end{aligned}$$

$$U(\nu) \qquad T(\nu)$$

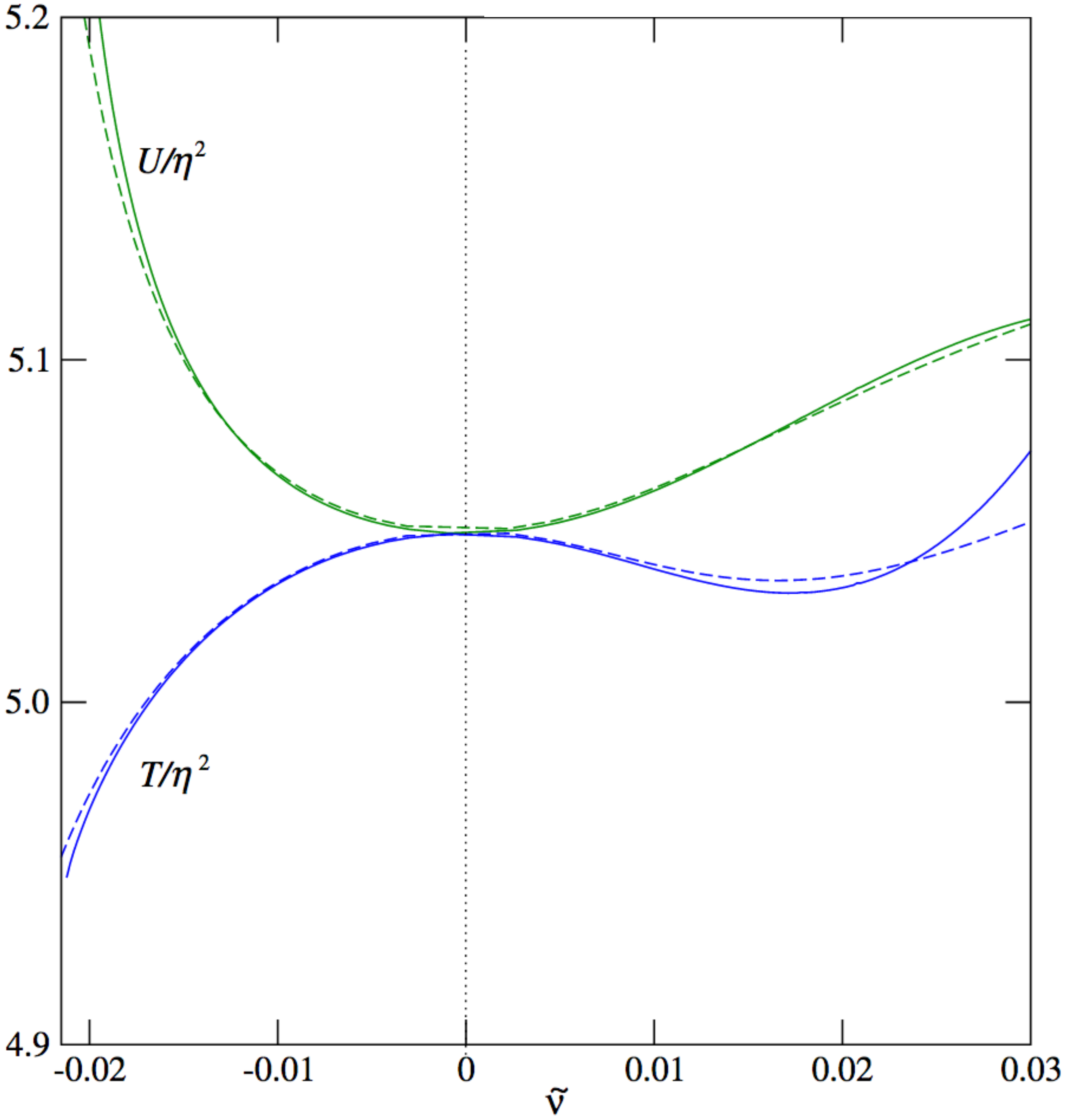
$$U = T + \mu\nu$$

$$\mu = \frac{\mathrm{d}U}{\mathrm{d}\nu}$$

$$\nu = -\frac{\mathrm{d}T}{\mathrm{d}\mu}$$

$$\boxed{U - T = \nu\mu}$$

Legendre transform $\Longrightarrow$ dual formalism



String equation of state II

$$\boxed{U = T + \mu\nu}$$

Legendre transform $\Longrightarrow$ dual formalism

B. Carter & PP, *Phys. Rev.* **D52**, R1744 (1995)
B. Carter, PP & A. Gangui, *Phys. Rev.* **D55**, 4647 (1997)
B. Carter, Phys Lett. B224, 61; B228, 466 (1989)

Macroscopic formalism

State parameter $w \Longrightarrow$ worldsheet lagrangian $\mathcal{L}(w)$ and $w = \kappa_0 \gamma^{ab} \partial_a \varphi \partial_b \varphi$

$$\nu^2 = |w|$$

$$\mathcal{S}_\mathcal{L} = -m^2 \int \mathrm{d}^2\xi \sqrt{-\gamma} \mathcal{L}(w)$$

$$\mu^2 = |\chi|$$

Master function (dual to lagrangian) $\Lambda(\chi)$: set $\chi = \tilde{\kappa}_0 \gamma^{ab} \partial_a \psi \partial_b \psi$

→ 2 conserved (orthogonal:$\gamma_{ab} n^a z^a = 0$) currents

$$z^a = -\frac{\partial \mathcal{L}}{\partial \partial_a \varphi} \qquad n^a = -\frac{\partial \Lambda}{\partial \partial_a \psi}$$

Equivalent alternative dynamical description

$$\mathcal{S}_\mathcal{L} \Longleftrightarrow \mathcal{S}_\Lambda = -m^2 \int \mathrm{d}^2\xi \sqrt{-\gamma} \Lambda(\chi)$$

Equation of state applications

Many applications for formalism $c_{\rm T}^2 = \frac{T}{U} > 0 \qquad c_{\rm L}^2 = -\frac{\mathrm{d}T}{\mathrm{d}U} = \frac{\nu}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}\nu} > 0$

B. Carter, Phys Lett. B224, 61; B228, 466 (1989)

- Nambu-Goto (structureless) $\mathcal{L} = -m^2 \implies U = T$
- Small field expansion $\mathcal{L} = -m^2 + \frac{w}{2} \implies \Lambda = -m^2 + \frac{\chi}{2}$ self-dual

Subsonic equation of state $U + T = 2m^2 \implies c_{\rm T} < c_{\rm L} = 1$

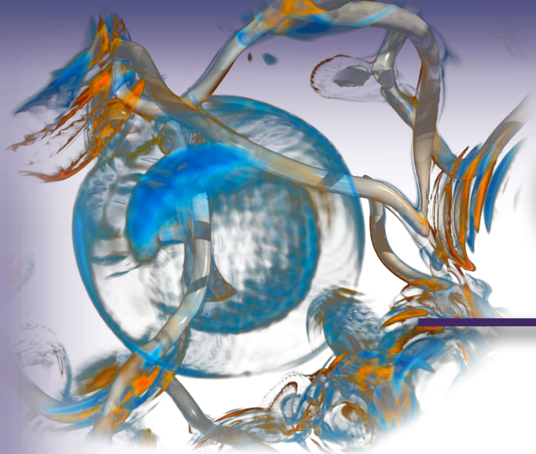
- Kaluza-Klein $\mathcal{L} = -m\sqrt{m^2 - w} \implies \Lambda = -m\sqrt{m^2 - \chi}$ self-dual

Transonic equation of state $UT = m^4 \implies c_{\rm T} = c_{\rm L} \leq 1$

- Small field expansion (higher order) $\mathcal{L} = -m^2 + \frac{w}{2} + \frac{w^2}{4m_*^2} + \mathcal{O}(w^6)$
- Witten magnetic model $\mathcal{L} = -m^2 + \frac{w}{2}\left(1 - \frac{w}{m_*^2}\right)^{-1}$
- Witten electric model $\mathcal{L} = -m^2 - \frac{m_*^2}{2}\ln\left(1 - \frac{w}{m_*^2}\right)$

$\}$ Supersonic equation of state $c_{\rm L} \leq c_{\rm T} \leq 1$

Peter, *Phys. Rev.* **D46**, 3335 (1992)
Phys. Rev. **D47**, 3169 (1993)



Current/Charge VOS Model

Martins, Peter, Ryback, EPS, ,arXiv:2011.09700

Use formalism to obtain the superconducting string equations of motion

State parameter $\kappa=\frac{\dot{\varphi}^2}{a^2\left(1-\dot{\boldsymbol{X}}^2\right)}-\frac{\varphi'^2}{a^2\boldsymbol{X}'^2}\equiv q^2-j^2$

Equations of motion

$$\partial_\tau\left(\epsilon\bar{U}\right)+\frac{\dot{a}}{a}\epsilon\left[\left(\bar{U}+\bar{T}\right)\dot{\boldsymbol{X}}^2+\bar{U}-\bar{T}\right]=\partial_\sigma\Phi$$

$$\ddot{\boldsymbol{X}}\epsilon\bar{U}+\frac{\dot{a}}{a}\epsilon\left(\bar{U}+\bar{T}\right)\left(1-\dot{\boldsymbol{X}}^2\right)\dot{\boldsymbol{X}}=\partial_\sigma\left(\frac{\bar{T}}{\epsilon}\boldsymbol{X}'\right)+2\Phi\dot{\boldsymbol{X}}'+\boldsymbol{X}'\left(\dot{\Phi}+2\frac{\dot{a}}{a}\Phi\right)$$

$$\partial_\tau\left(f_\kappa a\sqrt{q^2\boldsymbol{X}'^2}\right)=\partial_\sigma\left[f_\kappa a\sqrt{j^2(1-\dot{\boldsymbol{X}}^2)}\right]$$

$$\boxed{\epsilon^2=\frac{\boldsymbol{X}'^2}{1-\dot{\boldsymbol{X}}^2}}$$

Averaging with string currents

Thermodynamic approach: Total network energy now including current/charges

$$E = a\mu_0 \int \bar{U}\epsilon\, \mathrm{d}\sigma$$ Energy

or just the strings

$$E_0 = a\mu_0 \int \epsilon\, \mathrm{d}\sigma$$ Bare energy

Total charge $Q^2 \equiv \langle q^2 \rangle$ and current $J^2 \equiv \langle j^2 \rangle$

RMS velocity $v \equiv \sqrt{\langle \dot{\boldsymbol{X}}^2 \rangle}$

Integrated state parameter

$$K = Q^2 - J^2$$

Averaging assumptions:

Uncorrelated variables

$$\langle \mathcal{F}(\mathcal{O}) \rangle \approx \mathcal{F}(\langle \mathcal{O} \rangle)$$

\+ Brownian string network $E = \dfrac{\mu_0 V}{L_c^2 a^2} \Longleftrightarrow E_0 = \dfrac{\mu_0 V}{\xi_c^2 a^2}$

$$E = E_0 \langle f - 2q^2 f_\kappa \rangle \Longrightarrow \frac{E}{E_0} = F - 2Q^2 F'$$

$$F(K) \equiv \langle f(\kappa) \rangle \quad F' \equiv \langle f_\kappa \rangle \quad F'' \equiv \langle f_{\kappa\kappa} \rangle$$

CVOS evolution equations

Equations of motion from formalism with general master equation (EoS)

$$\frac{\mathrm{d}L_{\mathrm{c}}}{\mathrm{d}\tau}=\frac{\dot{a}}{a}\frac{L_{\mathrm{c}}}{F-2Q^{2}F'}\left\{v^{2}\left[F-\left(Q^{2}-J^{2}\right)F'\right]-\left(Q^{2}+J^{2}\right)F'\right\}$$

$$\frac{\mathrm{d}v}{\mathrm{d}\tau}=\frac{\left(1-v^{2}\right)}{F-2Q^{2}F'}\left\{\frac{k(v)}{L_{\mathrm{c}}\sqrt{F-2Q^{2}F'}}\left(F+2J^{2}F'\right)-2v\frac{\dot{a}}{a}\left[F-\left(Q^{2}-J^{2}\right)F'\right]\right\}$$

$$\frac{\mathrm{d}J^{2}}{\mathrm{d}\tau}=2J^{2}\left[\frac{vk(v)}{L_{\mathrm{c}}\sqrt{F-2Q^{2}F'}}-\frac{\dot{a}}{a}\right]$$

$$\frac{\mathrm{d}Q^{2}}{\mathrm{d}\tau}=2Q^{2}\frac{F'+2J^{2}F''}{F'+2Q^{2}F''}\left[\frac{vk(v)}{L_{\mathrm{c}}\sqrt{F-2Q^{2}F'}}-\frac{\dot{a}}{a}\right]$$

Chirality

$$K=Q^{2}-J^{2}$$

Charge

$$Y=\tfrac{1}{2}(Q^{2}+J^{2})$$

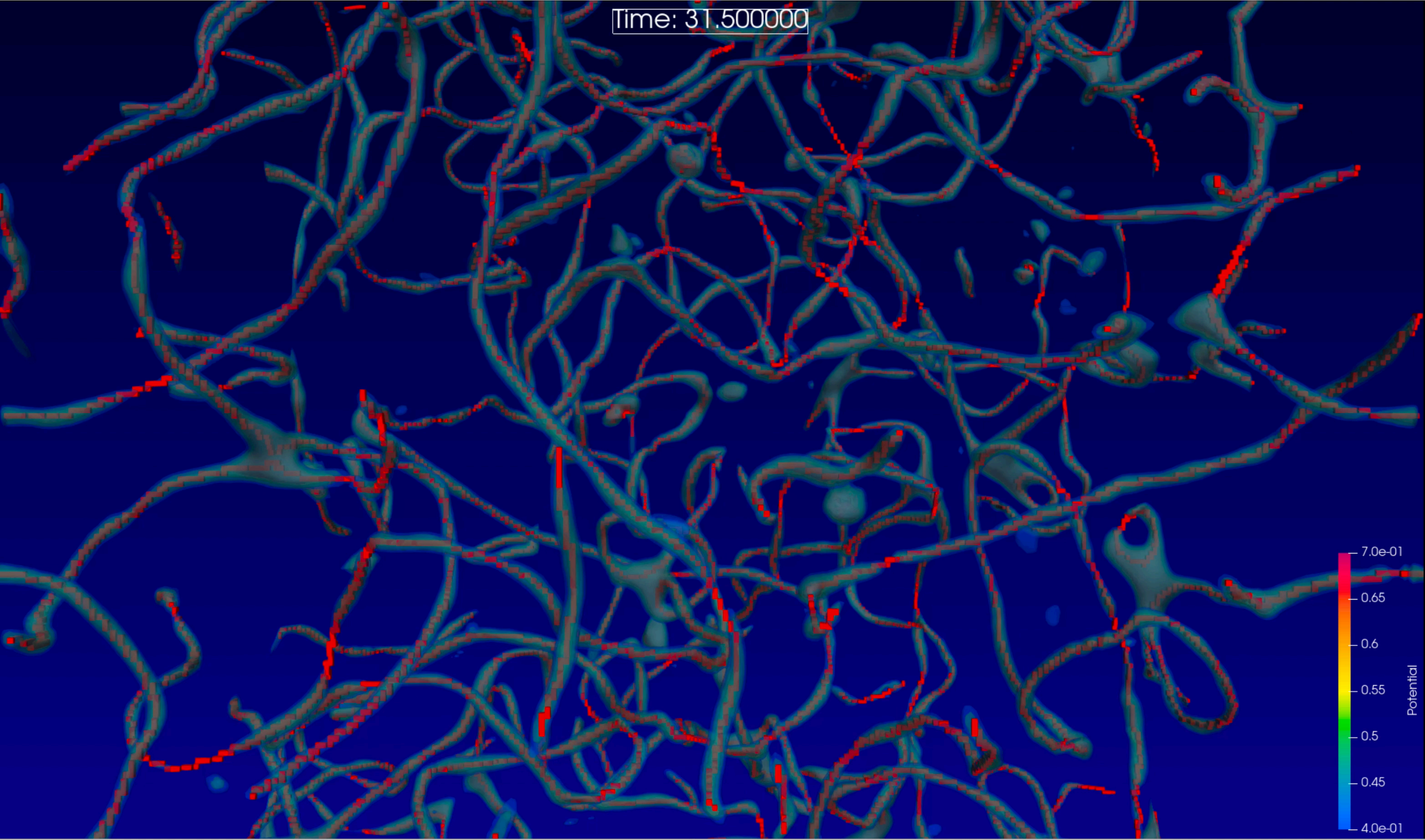
$$k_{\mathrm{NG}}(v)=\frac{2\sqrt{2}}{\pi}\frac{1-8v^{6}}{1+8v^{6}}$$

Martins, Peter, Ryback, EPS, ,arXiv:2011.09700

U(1)xU(1) superconducting strings: by permission José Ricardo Correia (Porto)

Preliminary: small resolution simulation for debug purposes

Radiation epoch, 256^3 dx=0.5 (half a light crossing time)



Energy loss mechanisms

Phenomenological parameter:

◆ loop chopping efficiency $$\left.\frac{\mathrm{d}E_0}{\mathrm{d}\tau}\right|_{\mathrm{loops}} = -\, c\, v \frac{E_0}{\xi_\mathrm{c}}$$

($c \simeq 0.5$ in ordinary “VOS” model)

◆ current chopping efficiency $$\left.\frac{\mathrm{d}E}{\mathrm{d}\tau}\right|_{\mathrm{loops}} = -\, g\, cv \frac{E}{\xi_\mathrm{c}}$$

◆ charge leakage $$\left.\frac{\mathrm{d}Y}{\mathrm{d}\tau}\right|_{\mathrm{leakage}} = -\, A\, \frac{Y}{\xi_\mathrm{c}} = -\, A\, \frac{Y}{L_\mathrm{c}\sqrt{F - 2Q^2F'}} \to \frac{Y}{L_\mathrm{c}\sqrt{1+Y}}$$

Universe expansion: $$a(\tau) = a_\mathrm{eq}\left[2\left(\frac{\tau}{\tau_\mathrm{eq}}\right) + \left(\frac{\tau}{\tau_\mathrm{eq}}\right)^2\right]$$

Linear CVOS Model

First attempt: Take simplified linear EoS for calculational simplicity

Linear regime $F(K) = 1 - \frac{\kappa_0}{2} K$ Martins, Peter, Ryback, EPS, arXiv:2108.03147

$$\boxed{\begin{aligned} \dot{\zeta}\tau &= \frac{v^2+Y}{1+Y}\frac{\dot{a}}{a}\zeta + \frac{gcv(1+Y)+AY}{2(1+Y)^{3/2}} - \zeta \\ \dot{v}\tau &= \frac{1-v^2}{1+Y}\left[\frac{k\,(1-Y)}{\zeta\sqrt{1+Y}} - 2v\frac{\dot{a}}{a}\right] \\ \dot{Y}\tau &= 2Y\left(\frac{vk}{\zeta\sqrt{1+Y}} - \frac{\dot{a}}{a}\right) - \frac{vc(g-1)}{\zeta}\sqrt{1+Y} - \frac{AY}{\zeta\sqrt{1+Y}} \end{aligned}}$$

$$\dot{K} = 2K\left(\frac{vk}{L_c\sqrt{1+Y}} - \frac{\dot{a}}{a}\right) - \frac{2(1-2\rho_A)AY}{L_c\sqrt{1+Y}} - 2\frac{v}{L_c}c\,(g-1)\,(1-2\rho)\sqrt{1+Y}$$

Scaling solution $L_c = \zeta\tau$ with $\dot{\zeta} = 0$

$$\dot{v} = 0$$

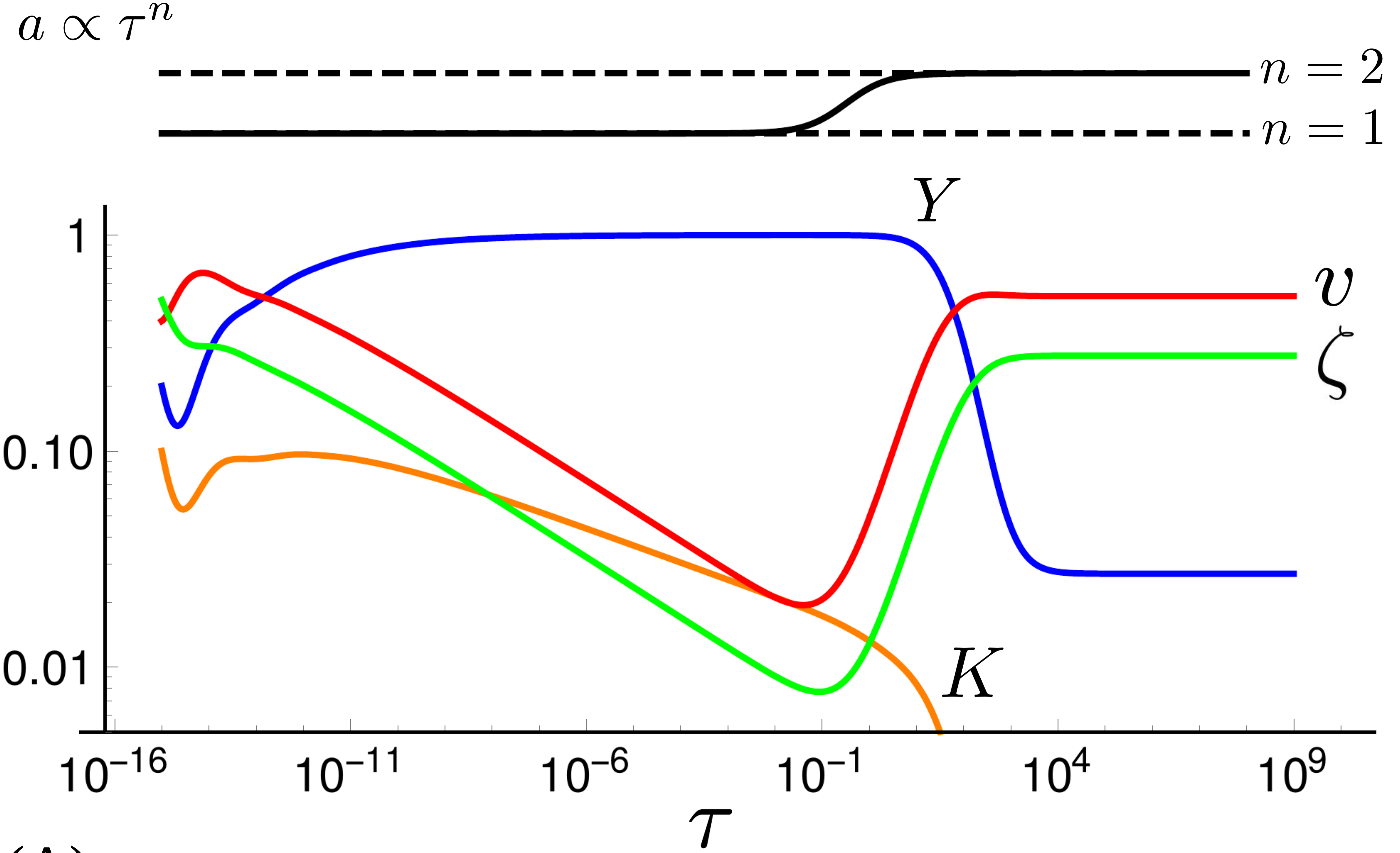
$$\dot{Y} = \dot{K} = 0$$

Charge/current domination (no leakage)

Dynamical solutions

No leakage

$g = g_o = 0.9 \quad c_o = 0.5 \quad k(v) = k_o = 0.6$



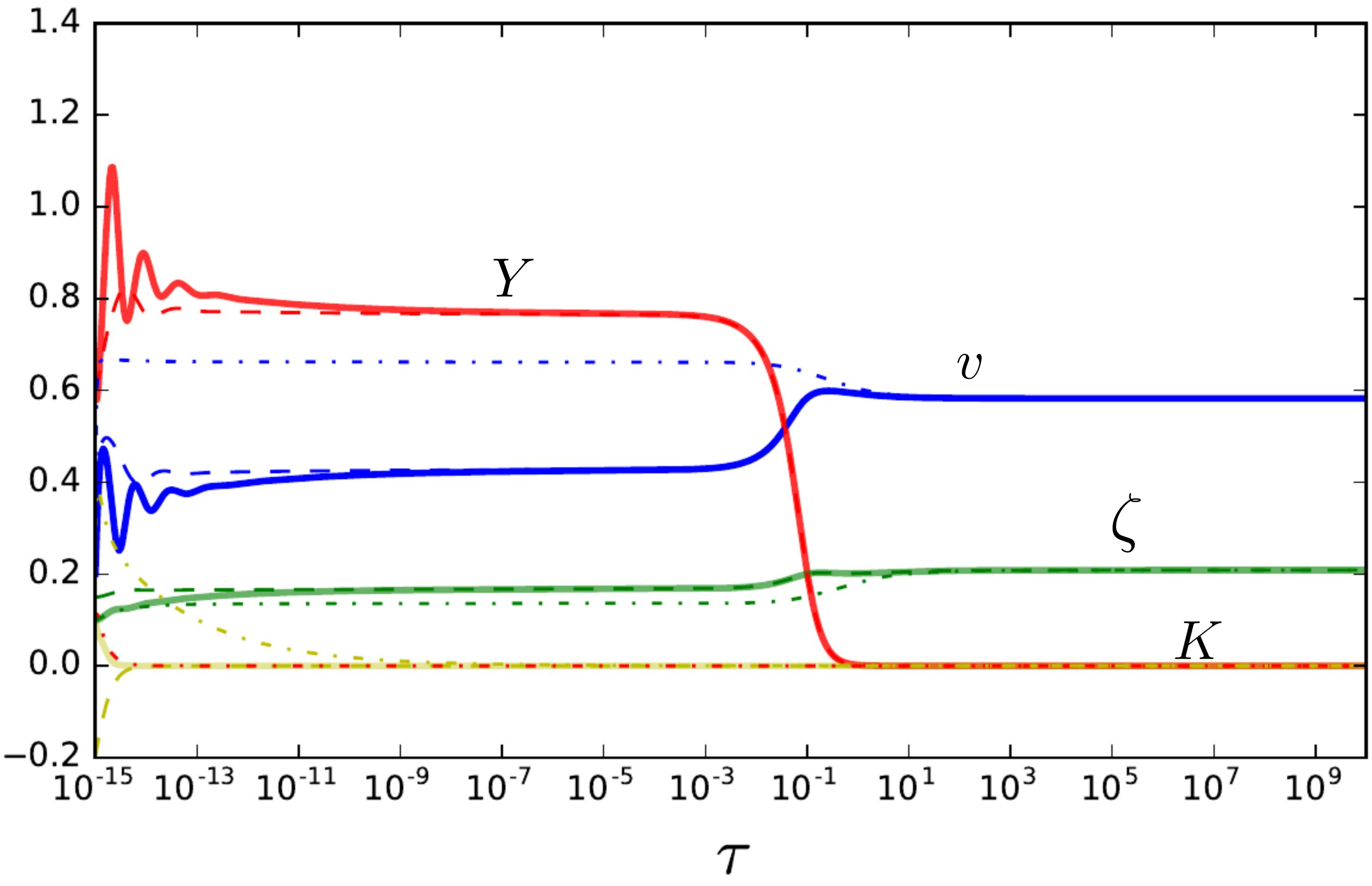
(A)

Charge/current scaling (with leakage)

Dynamical solutions

With leakage $g = 1 + 2bY$

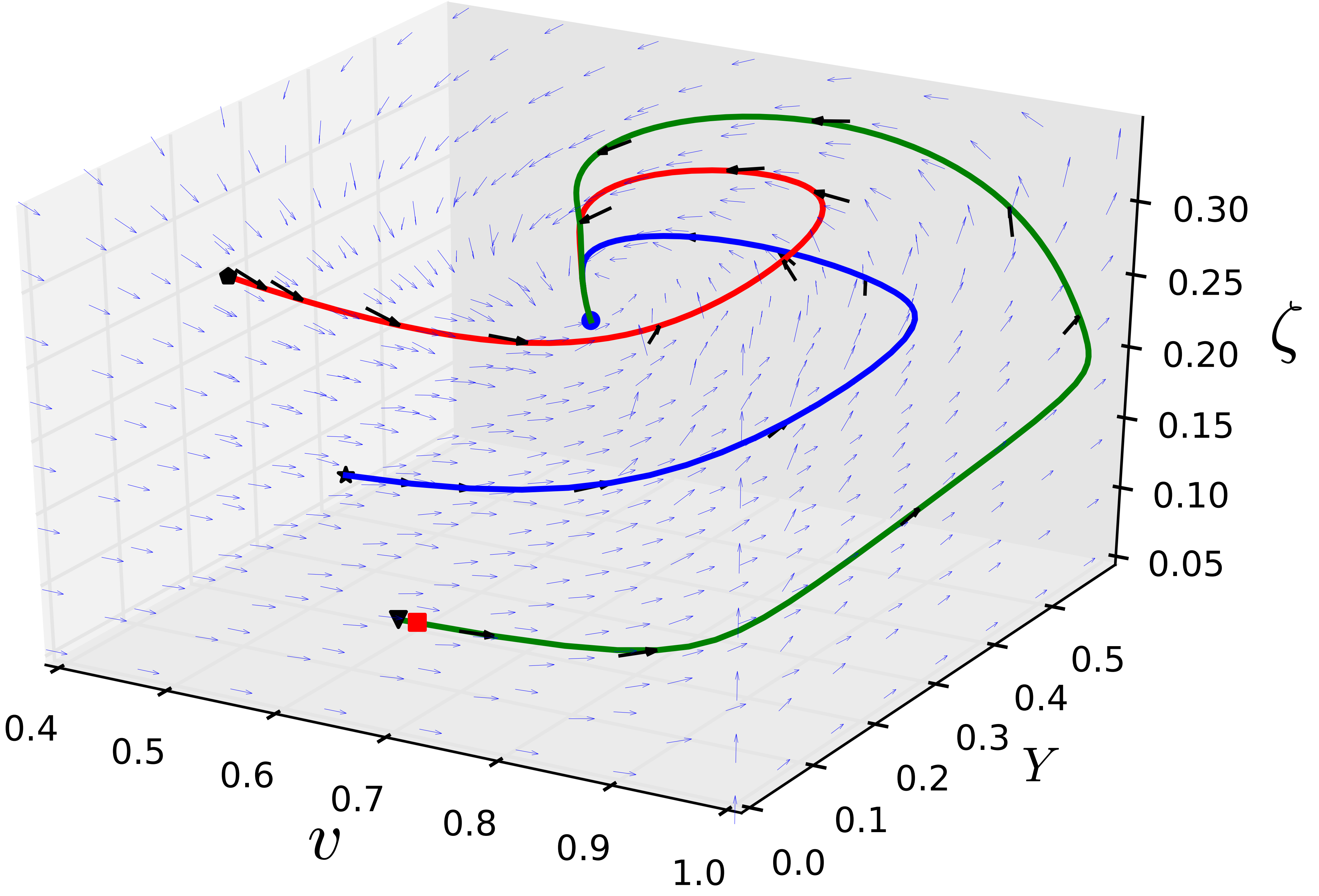
$c_o = 0.23 \quad b = 0 \quad A = 0.25$



Attractor phase diagram

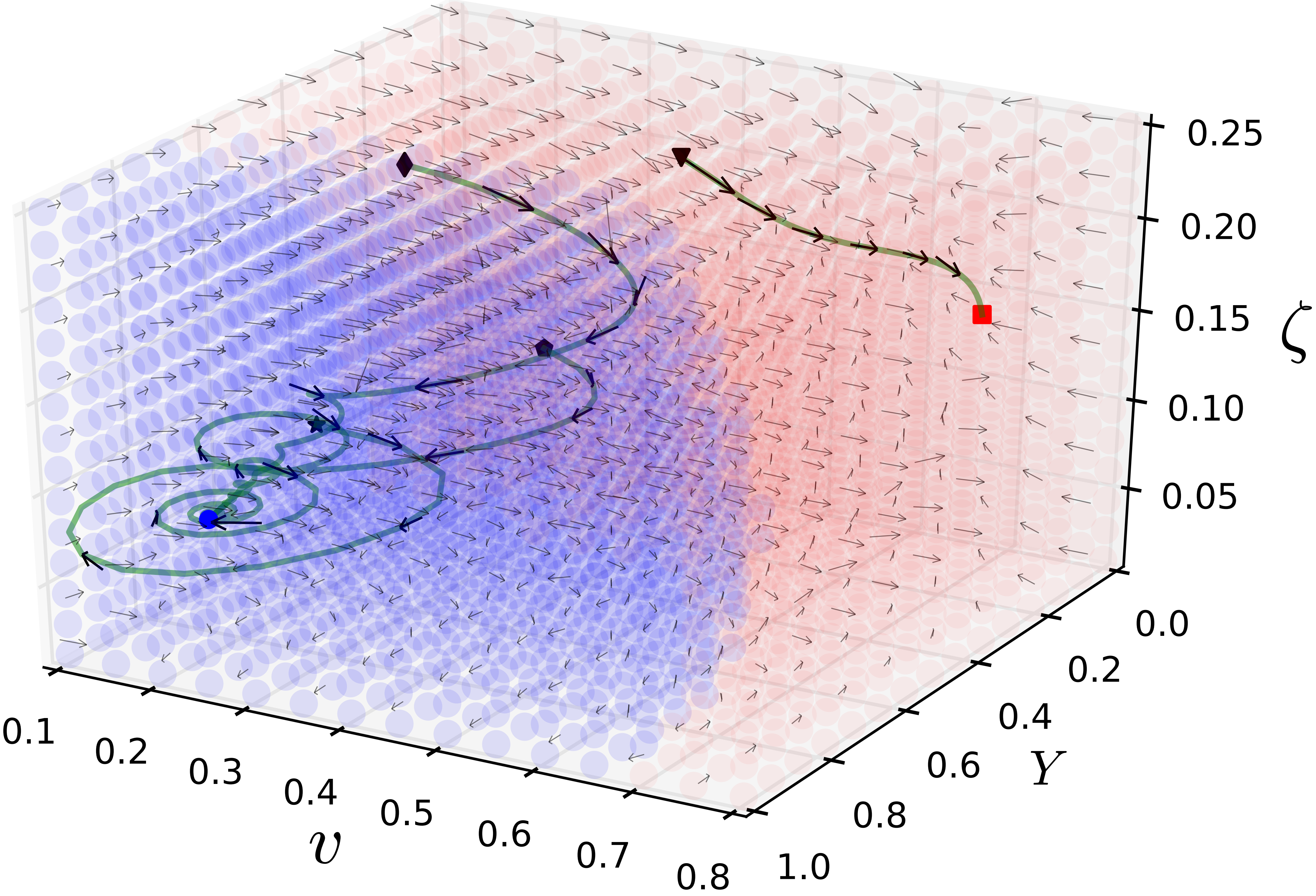
Martins, Peter, Rybak, EPS, arXiv:2108.03147

Attractor ($n = 1$, radiation)



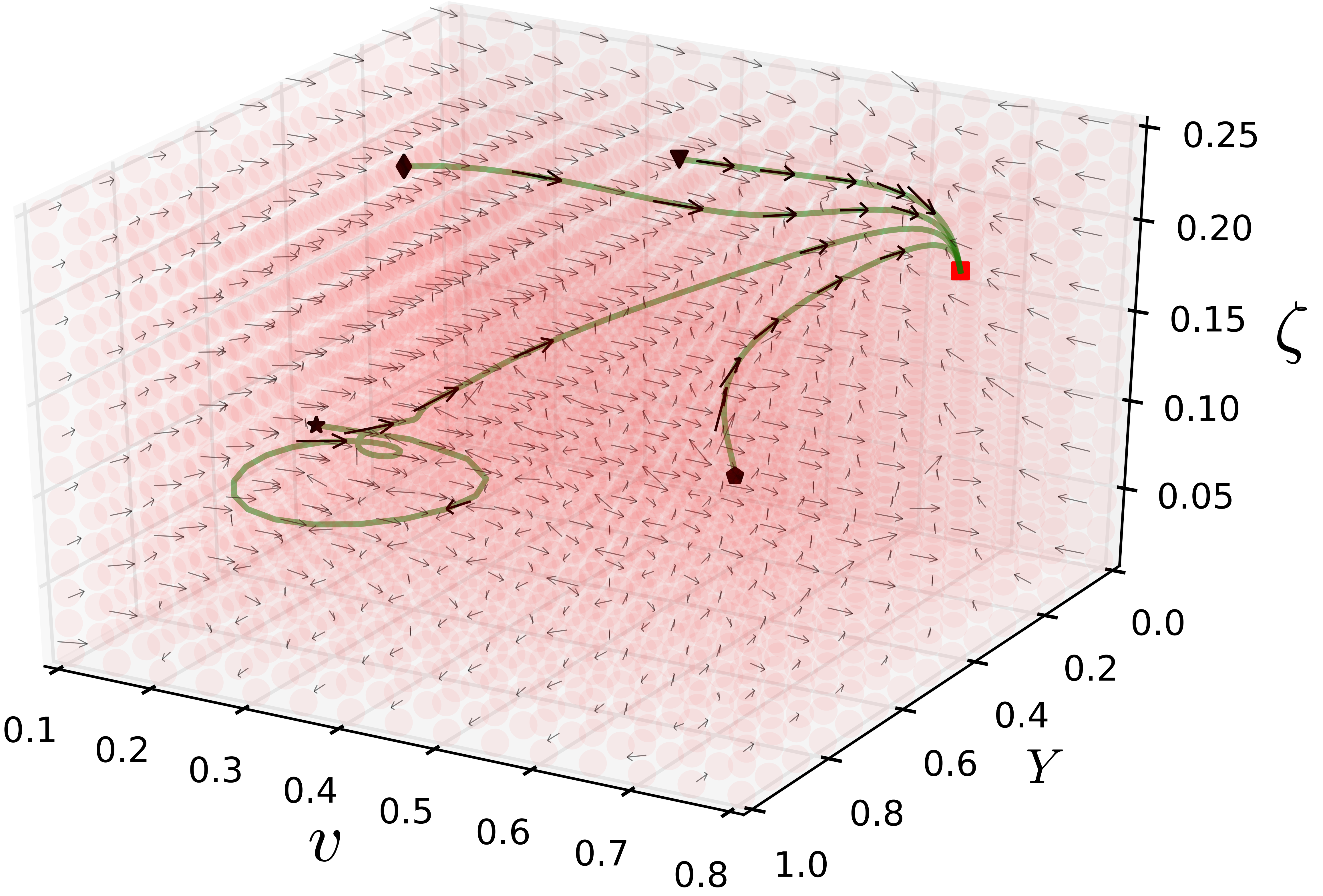
Dependence on initial conditions

Radiation era (charged scaling possible)



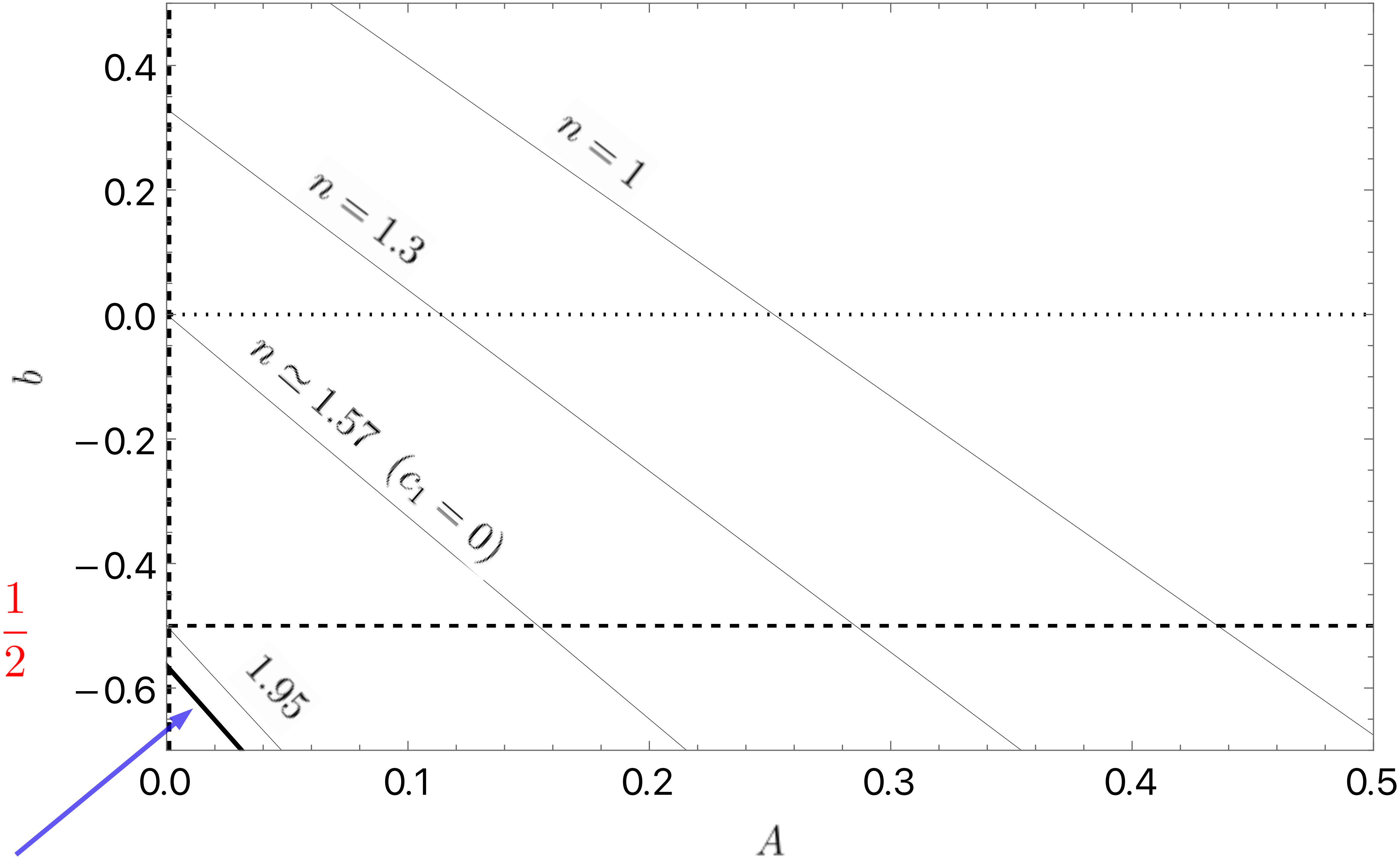
Dependence on initial conditions

Towards Matter era (only Nambu-Goto networks)



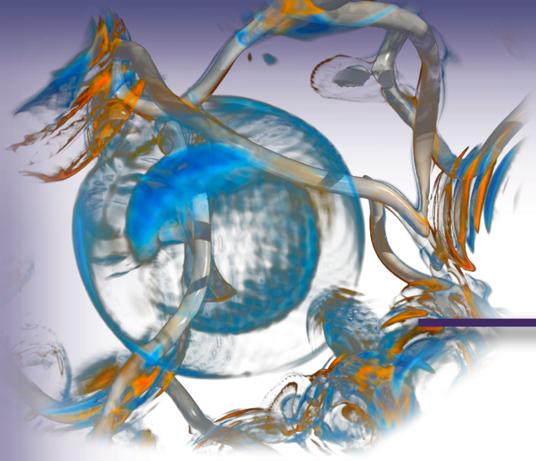
Scaling and cosmological epochs

Constraints



$b \leq -\frac{1}{2}$

$n = 2$



CVOS Model Conclusions

Martins, Peter, Rybak, EPS, arXiv:2011.09700 & arXiv:2108.03147

Current-carrying cosmic strings models with analytic EoS (arXiv:2011.09700)

A general formalism to describe integrated quantities

$$L_{\mathrm{C}} = \zeta\tau$$

$$v = \sqrt{\langle \dot{\boldsymbol{X}}^2 \rangle}$$

$$Q \text{ and } J \qquad \Longleftrightarrow \qquad K \text{ and } Y$$

Search for scaling solutions $\zeta \to \zeta_{\mathrm{SC}}\,,\;\; v \to v_{\mathrm{SC}}\,,\;\; K \to K_{\mathrm{SC}}$ and $Y \to Y_{\mathrm{SC}}$

Linear EoS model for first analysis (arXiv:2108.03147)

Charged configurations only possible for radiation era

Charge loss (leakage) needed to prevent charge domination

Sensitivity to initial conditions; Stability analysis

Matter domination leads to Nambu-Goto strings (no charge)

Future steps: Non-linear EoS models more realistic

Calibration of CVOS with numerical simulations (José Ricardo Correia)

Observational implications - CMB signatures (Andrei Lazanu) and GWs …

Happy Birthday, Brandon!