From vortons to gravitational wave constraints on cosmic strings

Danièle Steer

Carter Fest, July 2022

Symplectic structure for elastic and chiral conducting cosmic string models

B. Carter[♯] and D. A. Steer[♭]

[*Phys.Rev.D* 69 (2004) 125002]

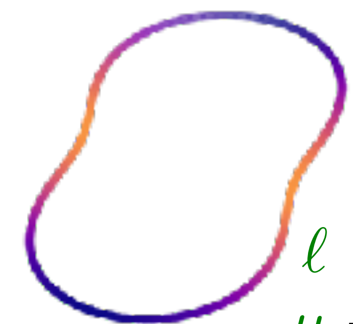
Since there are many different indices to keep under control in this analysis, we have tried to clarify the presentation by using a colour scheme. Black indices refer to spacetime quantites; green quantities are geometrical, describing the embedding of the brane (in this case a string) in the background spacetime; and blue indices on a vector run over both spacetime and internal indices. We have decided to write all physical quantities (invariant under gauge transformations and rescalings) in red; generalised momenta are written in brown; and quantities in purple are dynamical (generally gauge or normalisation dependent) variables.

$$\vartheta^{i} = p^{\,i}_{\mathrm{A}}\,\delta q^{\mathrm{A}}$$

current carrying strings with stabilized vortons: $\dot{\ell}(\ell, N)$

⇢ Constraints dark matter & from gravitational wave background (SGWB)

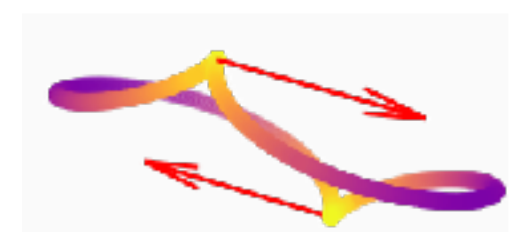
$\frac{\mathrm{d}\ell}{\mathrm{d}t} =$ rate at which a cosmic string loop of loses energy



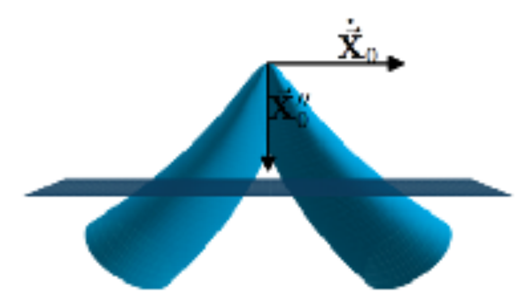
Vanilla NG string loops: GW emission is the dominant decay mode: $\dot{\ell} = -\Gamma G\mu$

⇢ Constraints from LIGO-Virgo O3 run: SGWB and search for individual GW bursts

⇢ LISA and PTA constraints



cusp



NG string loops with other decay channels: GWs and particle emission: $\dot{\ell}(\ell)$

⇢ Observable effects on both SGWB and diffuse gamma-ray background

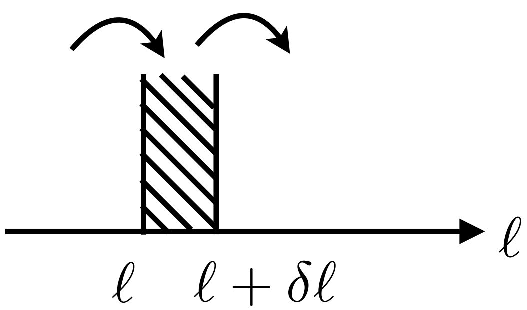
figures: Pierre Auclair

Loop distribution $n(t, \ell, N)$

– Satisfies a Boltzmann/continuity equation in an FRWL universe

[E.J.Copeland. T.W.B.Kibble, D.A.S., PRD 1998]

[P.Peter, C.Ringeval, 2013]



charge dependence for current carrying strings

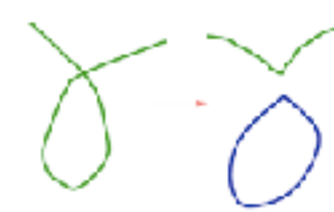
$$\boxed{\left.\frac{\partial}{\partial t}\right|_{\ell}\left[a^3 n(t,\ell,N)\right] + \left.\frac{\partial}{\partial \ell}\right|_{t}\left[\frac{\mathrm{d}\ell}{\mathrm{d}t} a^3 n(t,\ell,N)\right] = a^3 \mathcal{P}(t,\ell,N)}$$

scale factor
$a(t)$

rate at which
loops loose
energy

loop production function (LPF)

= Rate at which loops of length ℓ (assumed non-self-intersecting), are chopped off the infinite string network at time t, per unit volume



– given $\frac{\mathrm{d}\ell}{\mathrm{d}t}$, $\mathcal{P}(t,\ell,N)$, and some initial loop distribution at t_{ini},

integrate to find $n(\ell, t, N)$, and $n(t, \ell)$

→ observational signatures & constraints.

e.g. Energy density in loops $\int \mathrm{d}\ell \, n(t,\ell)(\mu\ell)$

Energy radiated…

I) Irreducible production of relic vortons, and dark matter

[P.Auclair, P.Peter, C.Ringeval, D.A.S., JCAP 03 (2021) 098]

standard NG strings — current carrying strings

$t_{\rm ini}$ $\longrightarrow$ $t_{\rm cur}$ $\longrightarrow$ t

$T_{\rm ini}$

$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = -\Gamma G\mu$$

$T_{\rm cur}$

string forming
phase transition

$$\mu = \mathcal{T} = m_\phi^2$$

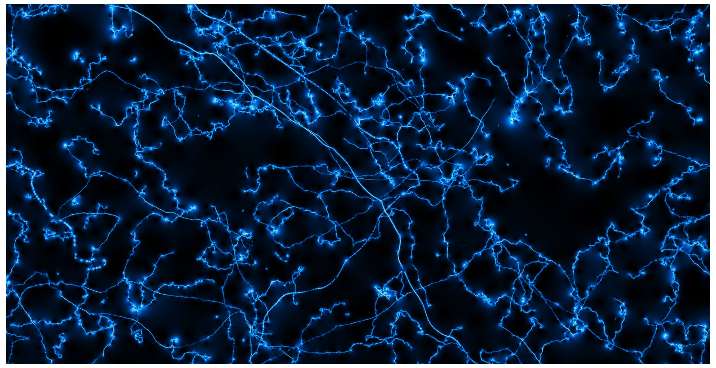
$$G\mu \sim 10^{-6}\left(\frac{T_{\rm ini}}{10^{16}\,\mathrm{GeV}}\right)^2$$

(weak) current

$$\lambda \sim m_\sigma^{-1}$$

$$\mu \simeq \mathcal{T}$$

Questions:
- Vorton distribution?
- *Vortons from initial conditions & those from continuous creation of loops?*
- Other loops, their distribution?
- Relic abundance of vortons today?



[C.Ringeval]

- take some initial (e.g. Vachaspati Vilenkin) loop distribution at $t_{\rm ini}$ amplitude $C_{\rm ini}$
- assume $\mathcal{R} = \lambda\sqrt{\mu} \simeq \dfrac{m_\phi}{m_\sigma} \gg 1$ So $T_{\rm cur} = \dfrac{T_{\rm ini}}{\mathcal{R}} \ll T_{\rm ini}$
- Saturation condition $0 < \dfrac{\mu - \mathcal{T}}{m_\phi^2} \leq \dfrac{1}{\mathcal{R}}$, so $\mu \simeq \mathcal{T}$

[B.Carter. P.Peter, Babul, Ringeval…]

standard NG strings — current carrying strings $\rightarrow t$

t_{ini} — t_{cur}

T_{ini} — T_{cur}

string forming phase transition
$\mu = \mathcal{T} = m_\phi^2$

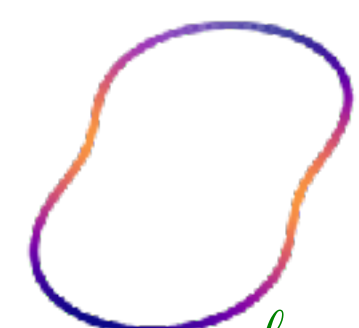
(weak) current
$\lambda \sim m_\sigma^{-1}$
$\mu \simeq \mathcal{T}$

$$\mathcal{R} = \lambda\sqrt{\mu} \simeq \frac{m_\phi}{m_\sigma} \gg 1$$

Assumptions:

- Approximately chiral loops: classically conserved quantum numbers $|Z| \sim N$ [B.Carter 1990]

- A loop formed with initial length ℓ_* , has conserved charge $N = \sqrt{\frac{\ell_*}{\lambda}}$



ℓ_*

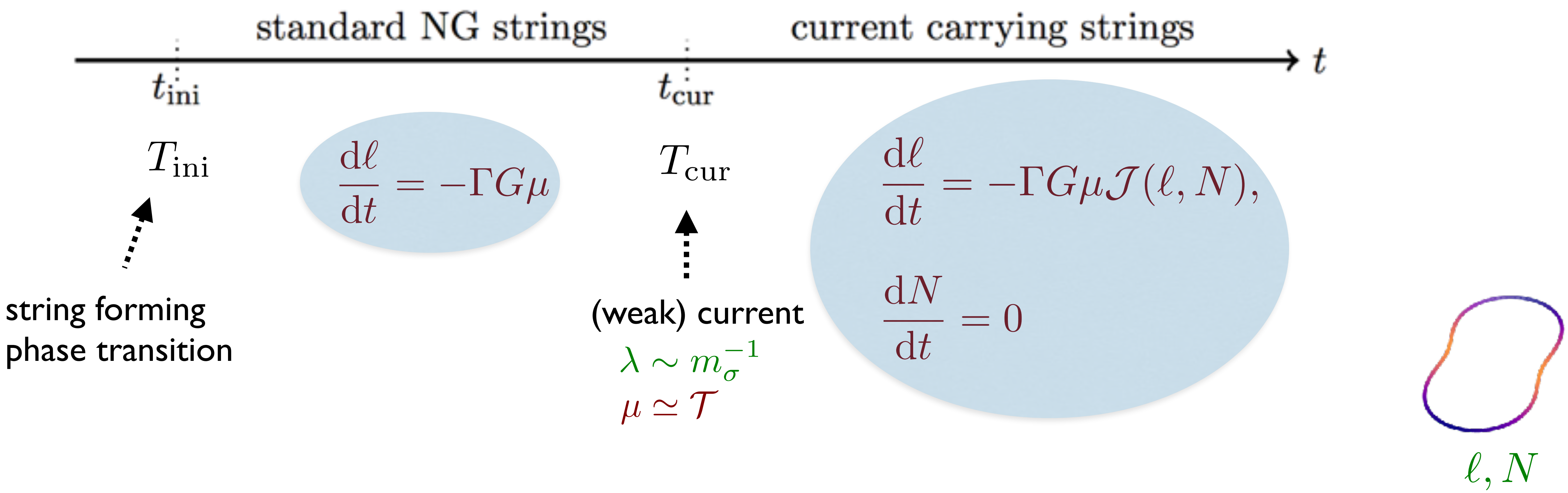
- Vortons are classically stable, with length

$$\ell_0 = \sqrt{\frac{2\pi}{\mu}} N \sim \sqrt{\frac{\ell_*}{\lambda\mu}}$$

provided $\ell_0 > \lambda$, or equivalently $N > \mathcal{R}$

[Carter, Davis & Shellard; Battye et al]

- gravitational wave emission is main damping mechanism by which larger loops will (or will not!) become vortons. Occurs on time scales much smaller that the Hubble time.



- If $N < \mathcal{R}$ then $\mathcal{J}(\ell, N) \approx 1$ "**doomed loops**" their initial size is too small to support a current and hence they decay through gravitational radiation never becoming vortons

- If $N > \mathcal{R}$ then
$$\begin{cases} \mathcal{J}(\ell \gg \ell_0, N) \approx 1 & \text{"Proto-vortons" loops which are initially large enough to be stabilised by a current, but have not yet reached the vorton size } \ell_0 \\ \mathcal{J}(\ell \ll \ell_0, N) \approx 0 & \text{"Vortons": all those proto-vortons which have decayed by gravitational radiation to become vortons.} \end{cases}$$

Example: $\mathcal{J}(\ell, N) = \Theta[\ell - \ell_0(N)]\,\Theta(N - \mathcal{R}) + \Theta(\mathcal{R} - N)$

vortons are loops which accumulate around $\ell_0(N)$

Solve the continuity equation

$$\boxed{\left.\frac{\partial}{\partial t}\right|_{\ell}\left[a^3 n(t,\ell,N)\right]-\Gamma G\mu\left.\frac{\partial}{\partial \ell}\right|_{t}\left[a^3\mathcal{J}(\ell,N)n(t,\ell,N)\right]=a^3\mathcal{P}(t,\ell,N)}$$

– assume scaling of the infinite string network described by a simple 1 scale model:

$$\mathcal{P}(\ell,t)=Ct^{-5}\delta\left(\frac{\ell}{t}-\alpha\right)$$

$$\mathcal{P}(\ell,t)=Ct^{-5}\delta\left(\frac{\ell}{t}-\alpha\right)\delta\left(N-\sqrt{\frac{\ell}{\lambda}}\right)$$

standard NG strings | current carrying strings

$t_{\rm ini}$ —— $t_{\rm cur}$ —— t

$$\frac{\mathrm{d}\ell}{\mathrm{d}t}=-\Gamma G\mu$$

$$\frac{\mathrm{d}\ell}{\mathrm{d}t}=-\Gamma G\mu\mathcal{J}(\ell,N),$$

– Exact solutions

$$n(\ell,t_{\rm cur},N)$$
$$=C\,t_{\rm cur}^{-3/2}\frac{(\alpha+\Gamma G\mu)^{3/2}}{(\ell+\Gamma G\mu t_{\rm cur})^{5/2}}\Theta(\alpha t_{\rm cur}-\ell)\Theta\left[\ell+\Gamma G\mu t_{\rm cur}-t_{\rm ini}(\alpha+\Gamma G\mu)\right]$$
$$+\,C_{\rm ini}\left(\frac{t_{\rm ini}}{\ell}\right)^{5/2}t_{\rm ini}^{-4}\Theta\left[(\alpha+\Gamma G\mu)t_{\rm ini}-\ell-\Gamma G\mu t_{\rm cur}\right]\delta\left(N-\sqrt{\frac{\ell}{\lambda}}\right)$$

Initial Vachaspati-Vilenkin distribution: random walk correlated over length scale $\ell_{\rm corr}$:

$$C_{\rm ini}\simeq 0.4\left(\frac{t_{\rm ini}}{\ell_{\rm corr}}\right)^{3/2}$$

Solve the continuity equation

$$\boxed{\left.\frac{\partial}{\partial t}\right|_{\ell}\left[a^3 n(t,\ell,N)\right] - \Gamma G\mu \left.\frac{\partial}{\partial \ell}\right|_{t}\left[a^3 \mathcal{J}(\ell,N) n(t,\ell,N)\right] = a^3 \mathcal{P}(t,\ell,N)}$$

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standard NG strings

current carrying strings

$t_{\rm ini}$ $\qquad$ $t_{\rm cur}$ $\qquad$ t

$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = -\Gamma G\mu$$

$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = -\Gamma G\mu \mathcal{J}(\ell,N),$$

– Exact solutions

$$n(\ell, t_{\rm cur}, N)$$
$$= C\, t_{\rm cur}^{-3/2}\frac{(\alpha+\Gamma G\mu)^{3/2}}{(\ell+\Gamma G\mu t_{\rm cur})^{5/2}}\Theta(\alpha t_{\rm cur}-\ell)\Theta\left[\ell+\Gamma G\mu t_{\rm cur} - t_{\rm ini}(\alpha+\Gamma G\mu)\right]$$
$$+\, C_{\rm ini}\left(\frac{t_{\rm ini}}{\ell}\right)^{5/2} t_{\rm ini}^{-4}\Theta\left[(\alpha+\Gamma G\mu)t_{\rm ini} - \ell - \Gamma G\mu t_{\rm cur}\right]\delta\left(N-\sqrt{\frac{\ell}{\lambda}}\right)$$

Initial Vachaspati-Vilenkin distribution: random walk correlated over length scale $\ell_{\rm corr}$:

$$C_{\rm ini} \simeq 0.4\left(\frac{t_{\rm ini}}{\ell_{\rm corr}}\right)^{3/2}$$

"relaxed"

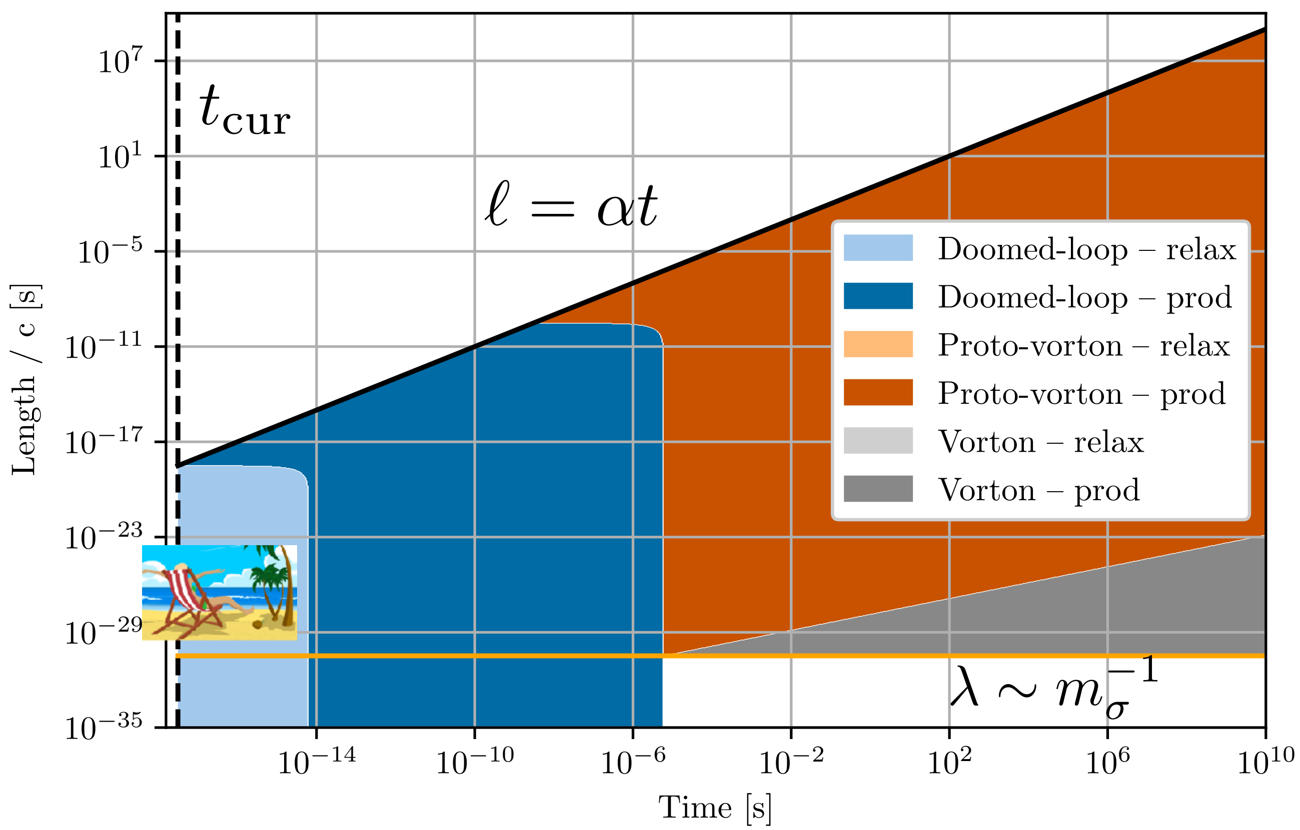
"produced"

Solutions: (ℓ, t) plane

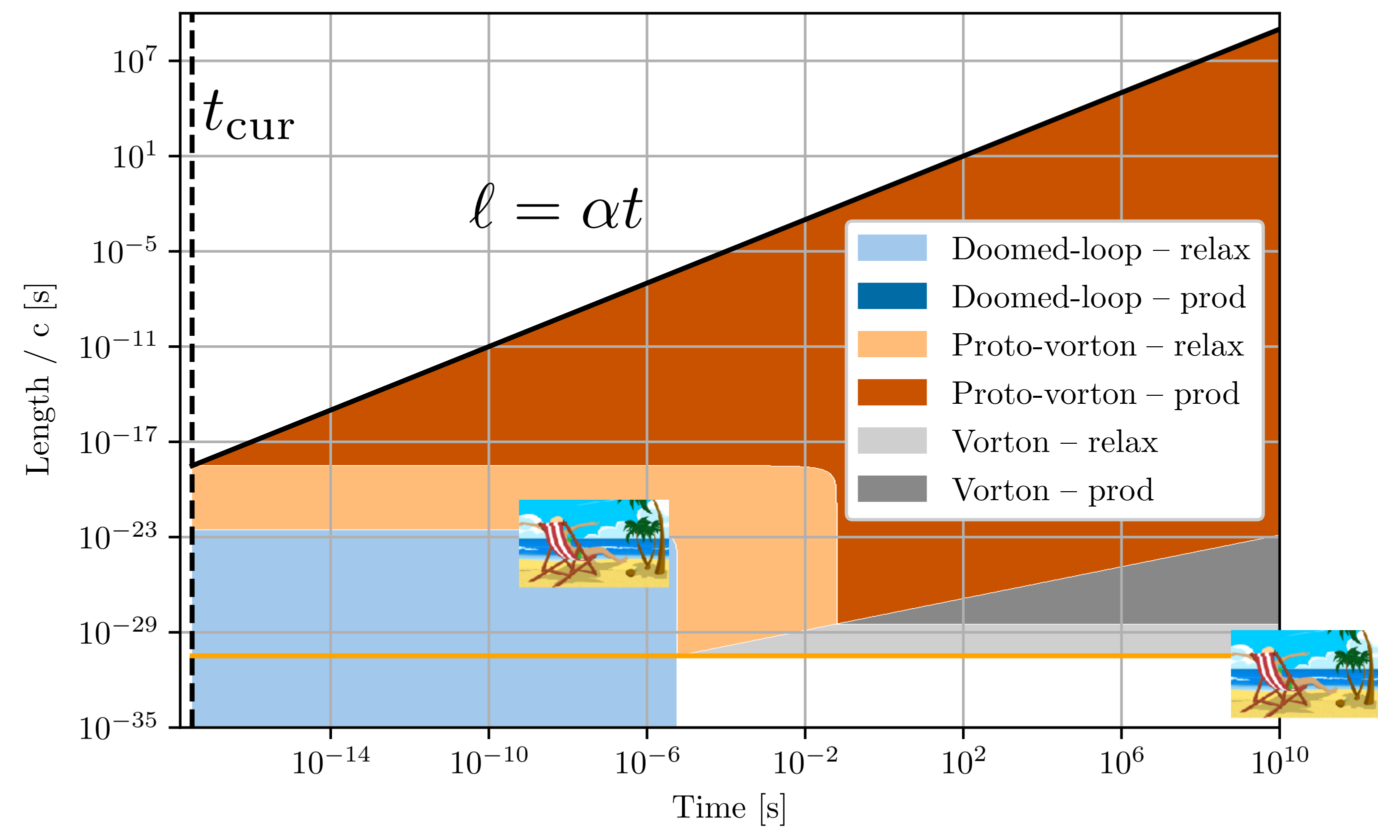
$$C = 1, \ell_{\rm corr} = \mu^{-1/2}, \alpha = 0.1, \Gamma = 50$$

$$T_{\rm cur} = 10^6 \mathrm{GeV}$$

$$G\mu = 10^{-6}$$
$$T_{\rm ini} \sim 10^{16} \mathrm{GeV}$$



$$G\mu = 10^{-19}$$
$$T_{\rm ini} \sim 10^{9} \mathrm{GeV}$$



- No "relaxed" vortons for $G\mu > \dfrac{\alpha G t_{\rm cur}}{\lambda^3}$
- For example, "produced" vorton distribution, scales like matter and is given by

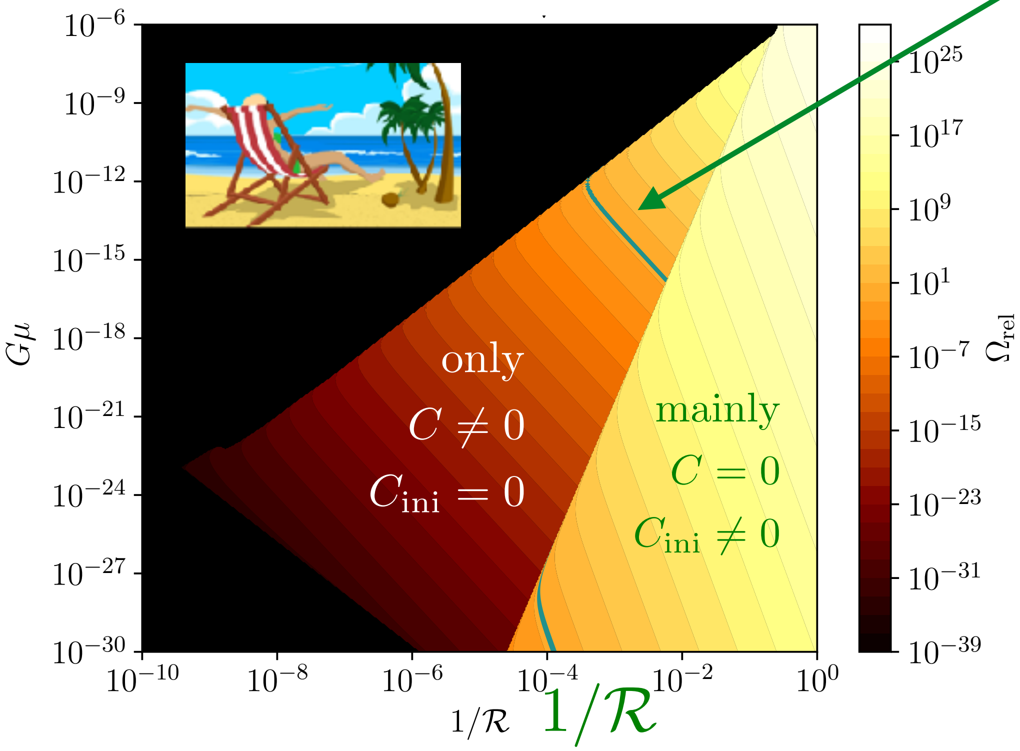
$$n(\ell, t)|_{\rm vort,prod} = \frac{2\lambda\mu\ell}{\alpha} C \left[\frac{a\left(\lambda\mu\ell^2/\alpha\right)}{a(t)}\right]^3 \left(\frac{\lambda\mu\ell^2}{\alpha}\right)^{-4} \Theta(\lambda\mu\ell^2 - \alpha t_{\rm cur}) \Theta\left(\frac{\Gamma G\mu t + \ell}{\alpha + \Gamma G\mu} - \frac{\lambda\mu\ell^2}{\alpha}\right) \Theta\left(\ell - \lambda\right)$$

Relic abundance

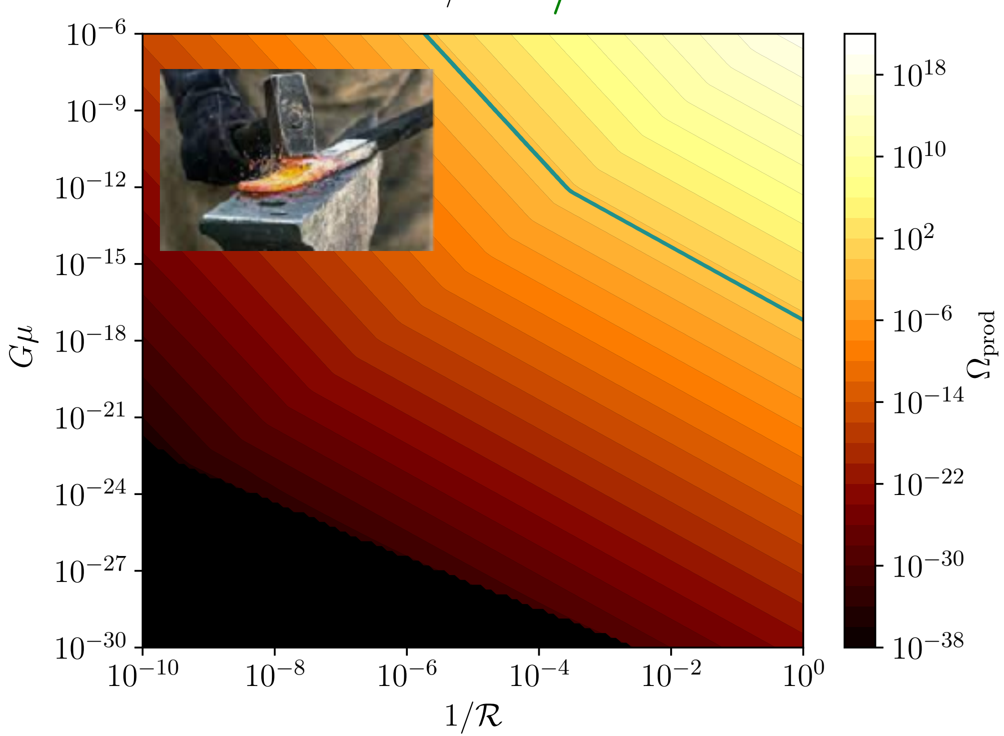
- Fraction of energy contained in vortons today $\Omega \equiv \dfrac{8\pi G}{3H_0^2}\displaystyle\int_0^\infty (\mu\ell)n(\ell,t_0)\,\mathrm{d}\ell.$

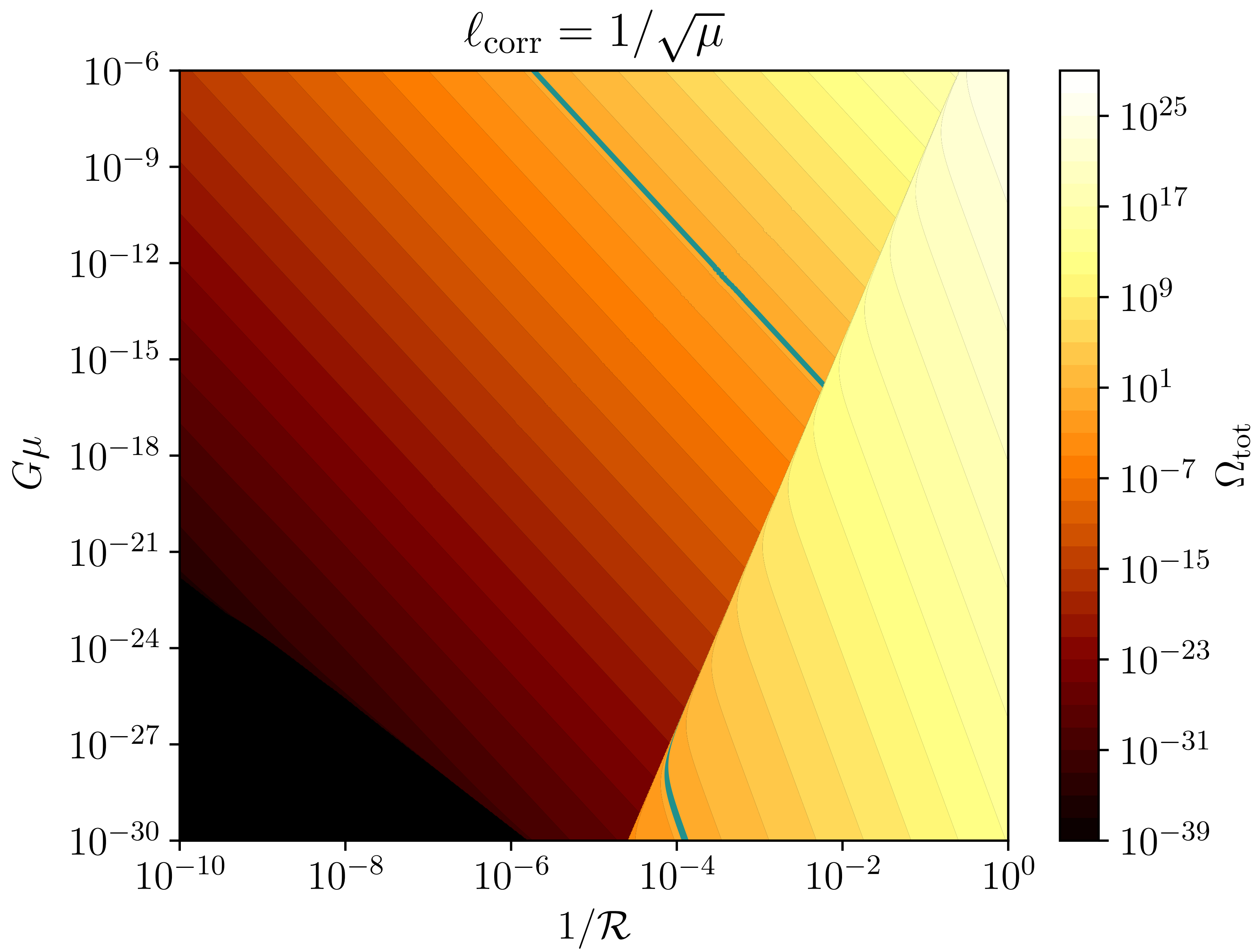
$G\mu$

$\Omega = 0.3$



$1/\mathcal{R}$





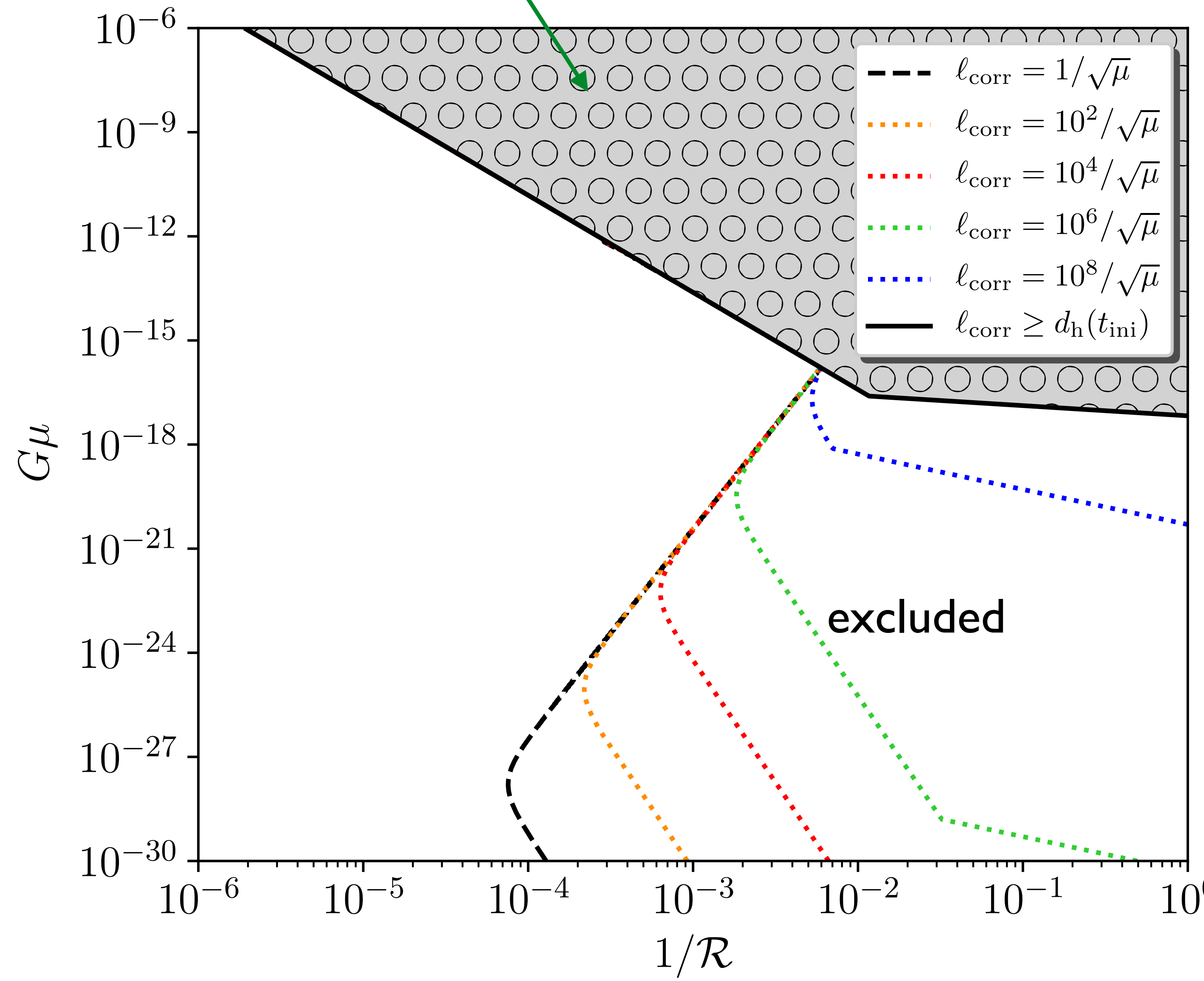
no acceptable regions for $\mathcal{R} \lesssim 10^2$

over production of vortons: **excluded**,
independently of the initial conditions at $t_{\rm ini}$
not considered previously

Dependence on Initial conditions

–thermal initial conditions $\ell_{\rm corr} = 1/\sqrt{\mu}$

–Kibble causality argument $\ell_{\rm corr} = d_h(t_{\rm ini})$



Summary:

– Even if no loops are created at the time of string formation, loop production means vortons are massively present today

– Enables us rule out new domains of the parameter space, independently of initial conditions.

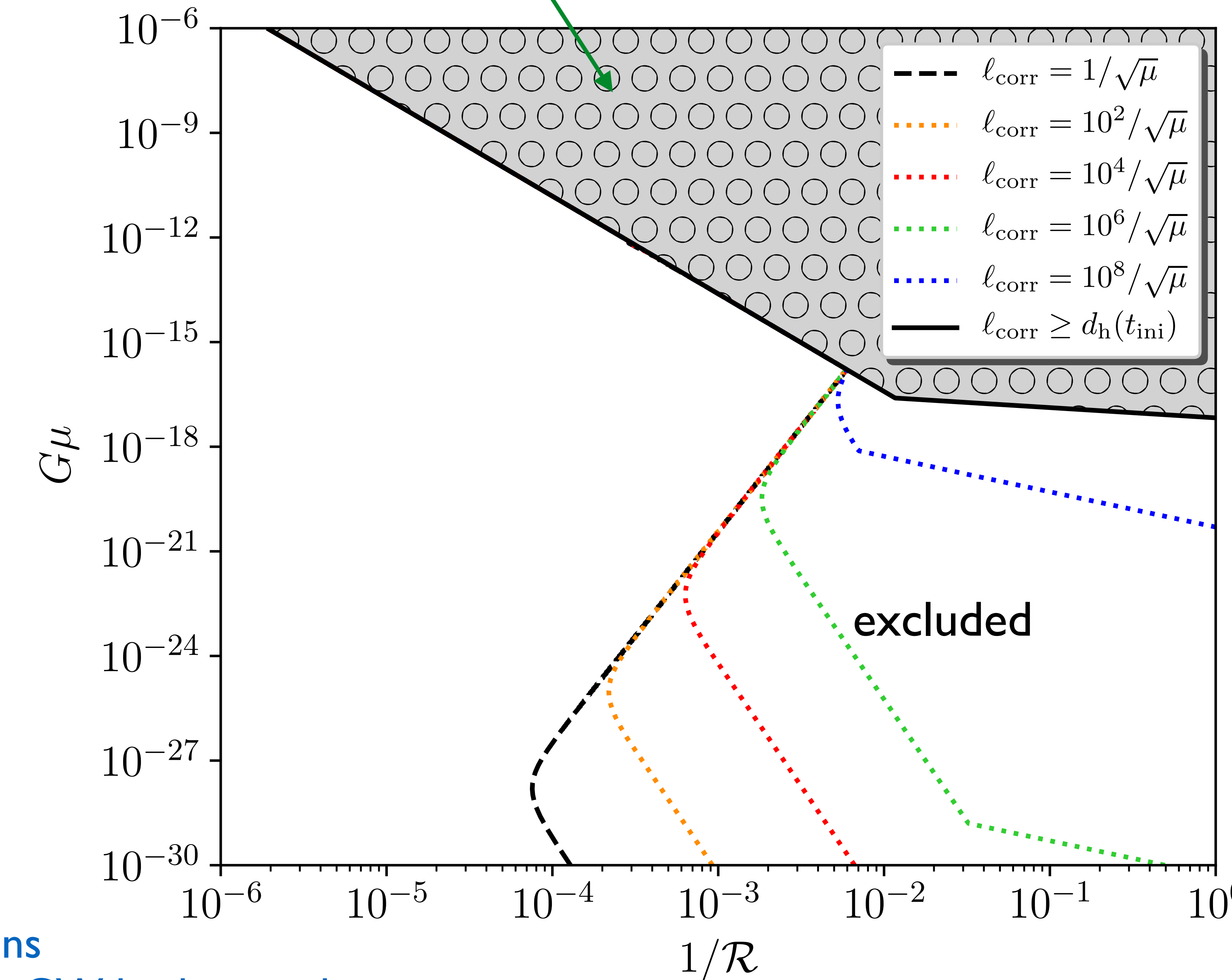
– In some regimes of parameter space vortons can provide a viable and original dark matter candidate

over production of vortons: **excluded**,
independently of the initial conditions at $t_{\rm ini}$
not considered previously

Effect of Initial conditions

–thermal initial conditions $\ell_{\rm corr} = 1/\sqrt{\mu}$

–Kibble causality argument $\ell_{\rm corr} = d_h(t_{\rm ini})$



Other observables?

- Since GWs are emitted from proto-vortons and doomed-loops, contribute to stochastic GW background

- Stabilisation of vortons expected to present a part of the energy being converted to GWs

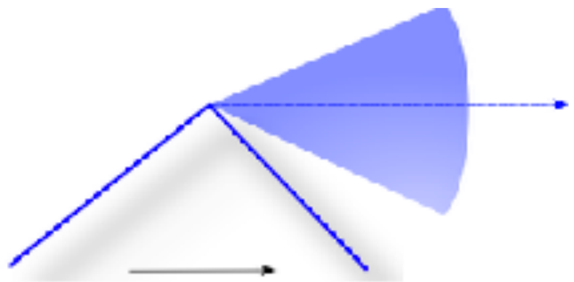
- Shown that due to the very small size of vortons, the lack of energy in GWs is negligible, and the predictions for the stochastic GW background are unchanged relative to "standard" (non-current carrying, Nambu-Goto) strings with a 1-scale loop production function

GW constraints on “standard” cosmic strings: existing and future.

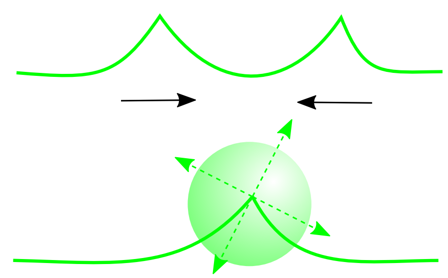
$G\mu, \Gamma$

• Occasional sharp **individual bursts** (resolved GW signals)

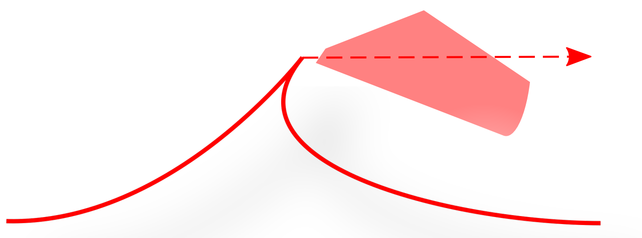
– kinks



– kink-kink collisions

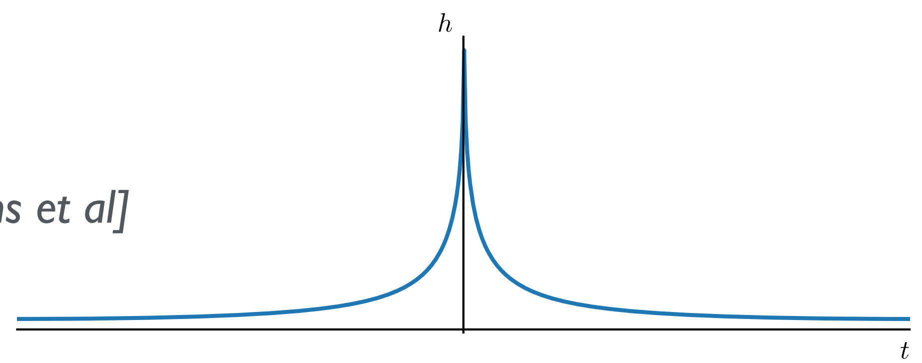


– cusps



GW-form	$h_i(\ell, z, f) = A_i(\ell, z) f^{-q_i}$
Amplitude	$A_i(\ell, z) = g_{1,i} \dfrac{G\mu \ell^{2-q_i}}{(1+z)^{q_i-1} r(z)}$

[Vachaspati+Vilenkin, Damour+Vilenkin; Siemens et al]



$i = \{c, k, kk\} \qquad q_c = 4/3, \quad q_k = 5/3 \quad q_{kk} = 2$

$\theta_m(\ell, z, f) = (g_2 f (1+z) \ell)^{-1/3}$
$\theta_m < 1$

• If an observer/experiment lies in one of those directions: stronger signal.

• **Stochastic GW background**

$$\Omega_{\rm gw}(t_0, f) = \frac{8\pi G}{3H_0^2} f \frac{d\rho_{\rm gw}}{df}(t_0, f)$$

At a given frequency, add up GW emission from all the loops throughout entire history of the Universe that contribute to that frequency (removing infrequent bursts)

GW constraints on "standard" cosmic strings: existing and future.

$G\mu, \Gamma$

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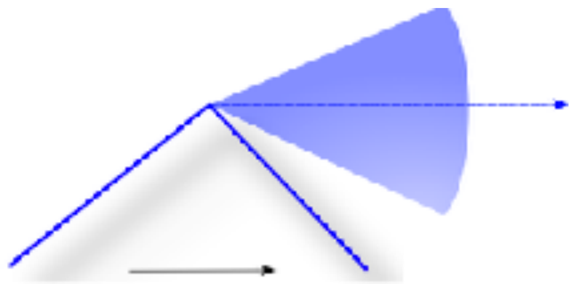

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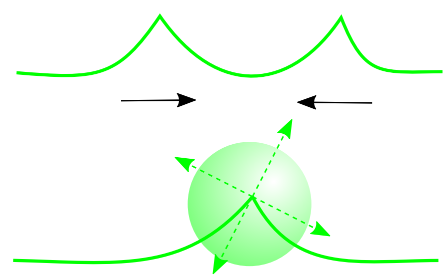
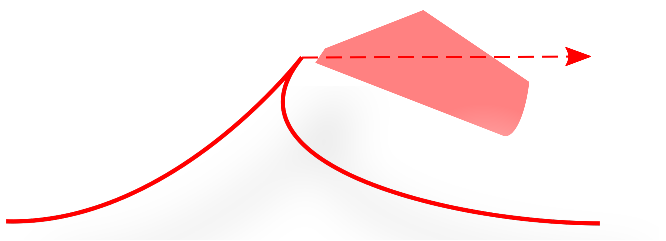


– cusps



GW-form $\quad h_i(\ell, z, f) = A_i(\ell, z) f^{-q_i}$

Amplitude $\quad A_i(\ell, z) = g_{1,i} \dfrac{G\mu \ell^{2-q_i}}{(1+z)^{q_i-1} r(z)}$

$i = \{c, k, kk\} \qquad q_c = 4/3, \quad q_k = 5/3 \quad q_{kk} = 2$

$\theta_m(\ell, z, f) = (g_2 f (1+z)\ell)^{-1/3}$

$\theta_m < 1$

$$\Gamma \geq \Gamma_c + \Gamma_k + \Gamma_{kk}$$

$\downarrow \qquad \downarrow \qquad \downarrow$

$\propto N_c \qquad N_k \qquad N_{kk} \sim N_k^2/4$

average number of burst events/loop oscillation period $\ell/2$

- **Stochastic GW background**

$$\boxed{\Omega_{\rm gw}(t_0, f) = \frac{8\pi G}{3H_0^2} f \frac{d\rho_{\rm gw}}{df}(t_0, f)}$$

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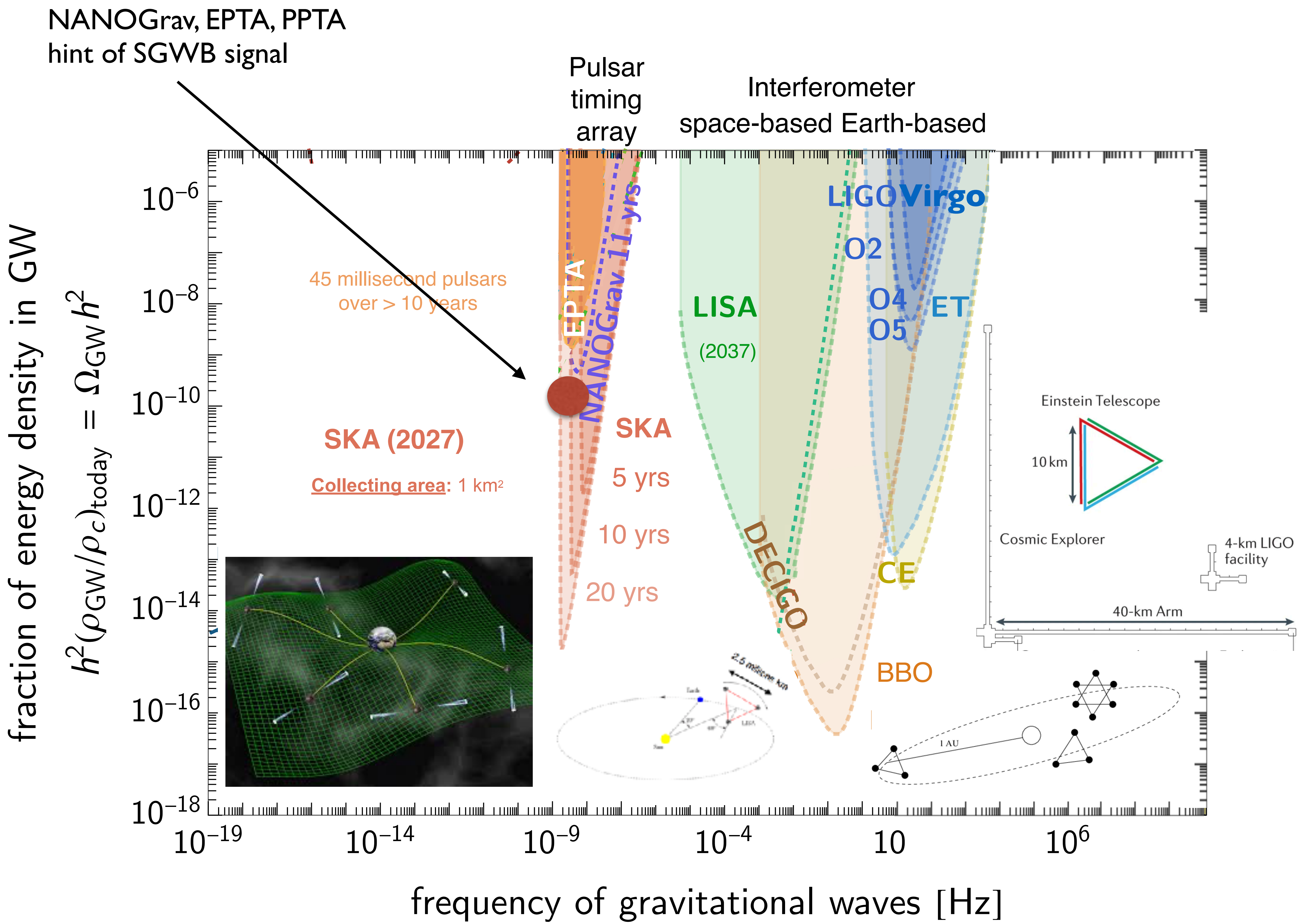


Courtesy of Yann Gouttenoire

LIGO-Virgo-Kagra O3 constraints $G\mu, \Gamma$

[Constraints on Cosmic Strings Using Data from the Third Advanced LIGO–Virgo Observing Run, by LIGO, Virgo+Kagra collaborations, Phys.Rev.Lett. 126 (2021) 24, 241102, arXiV: 2101.12248]

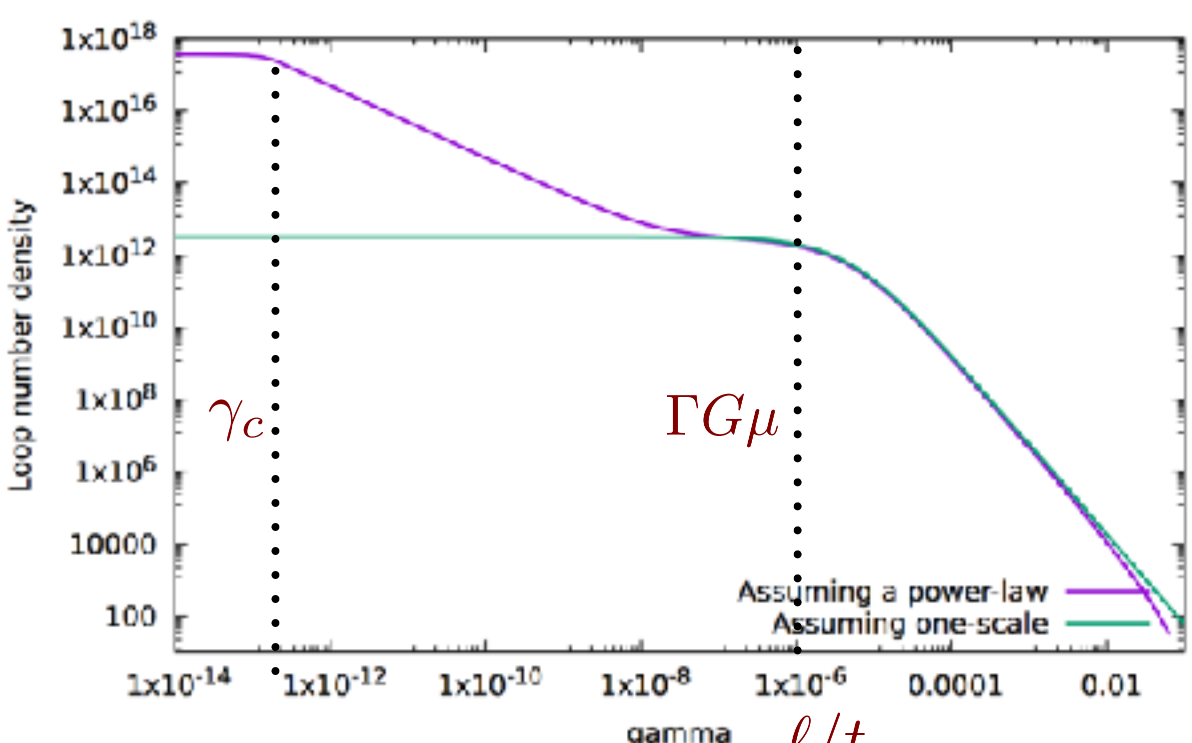
• Considered 4 different Nambu-Goto cosmic string models. Amongst these:

Model A

[Blanco-Pillado, Olum and Shlaer, 2014]

$$\mathcal{P}(\ell,t) = Ct^{-5}\delta\left(\frac{\ell}{t} - \alpha\right)$$

$n(\ell, t)$



ℓ/t

Model B

[Lorentz, Ringeval + Sakellariadou, 2010] *[Polchinski et al]*

$$\mathcal{P}(t,\ell) = \tilde{C}t^{-5}\left(\frac{\ell}{t}\right)^{2\chi-3}\Theta\left(\alpha - \frac{\ell}{t}\right)\Theta\left(\frac{\ell}{t} - \gamma_c\right)$$

• loops produced up to a "gravitational backreaction scale"

$$\gamma_c = \ell_c/t \simeq 10(G\mu)^{1+2\chi} \ll \Gamma G\mu$$

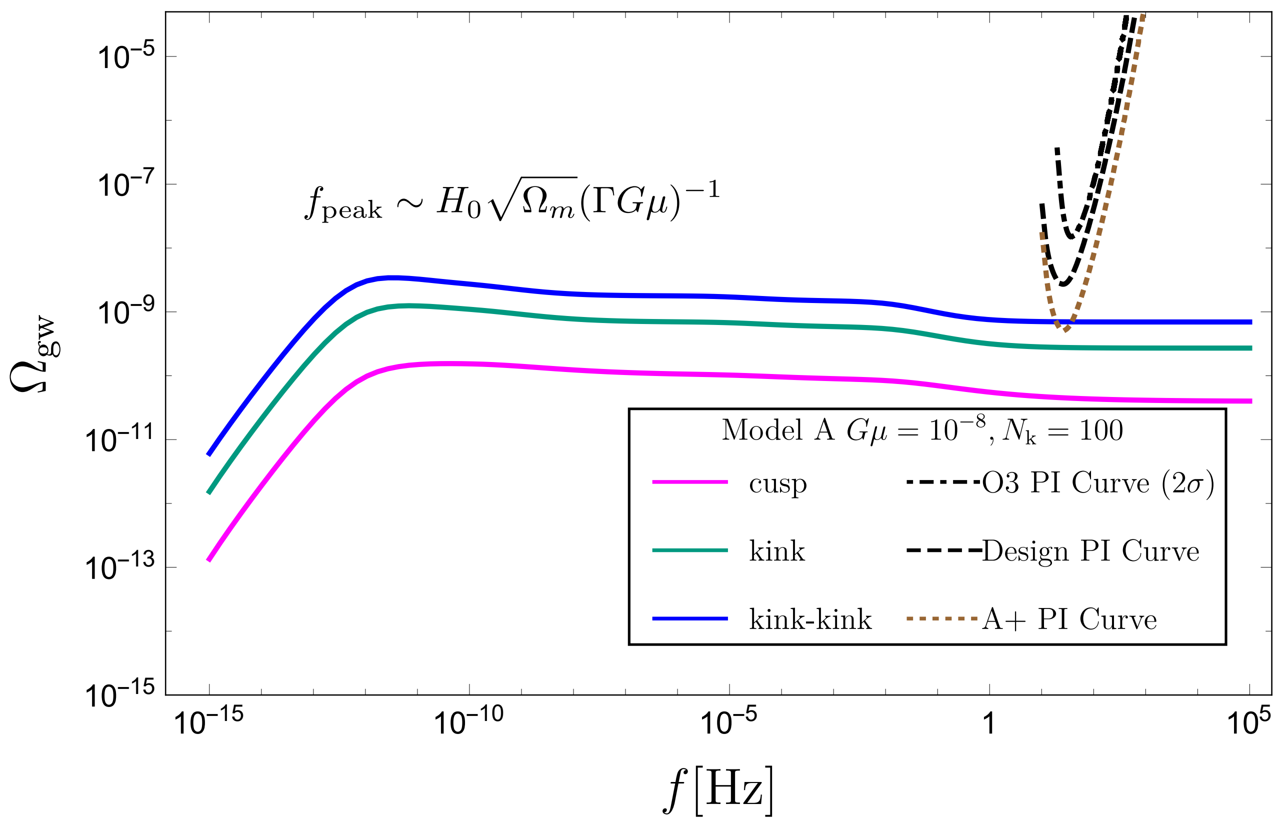
• Solution of Boltzmann equation calibrated to simulations of Ringeval et al on large scales

2 more [P.Auclair, 2020]

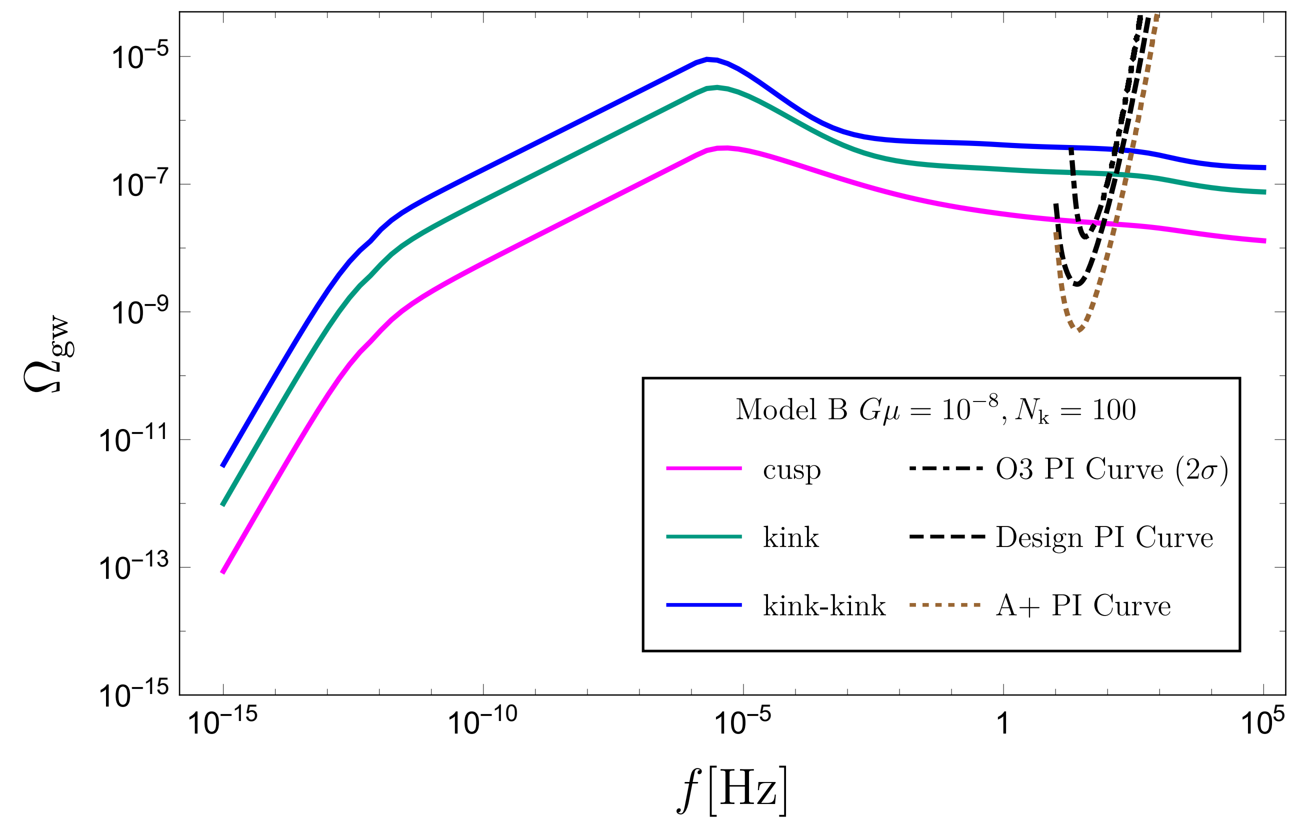
LIGO-Virgo-Kagra O3 constraints $G\mu, \Gamma$

$$G\mu = 10^{-8}$$
$$\Gamma \sim 100$$

Model A



Model B



[LIGO-Virgo 2101.12130]

$$\Omega_{\rm GW} \leq 5.8 \times 10^{-9}$$

Generic shape:

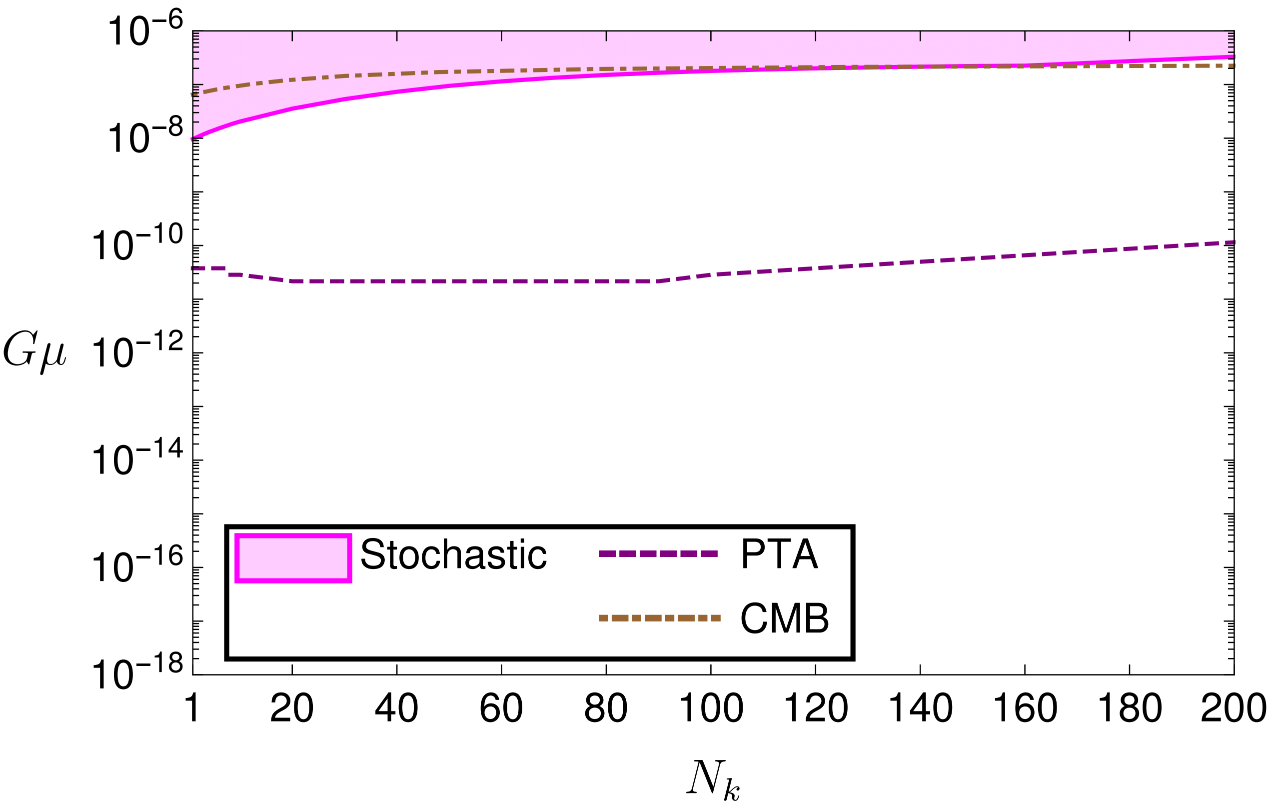
– emission in *radiation era -> flat spectrum* (exact compensation between redshifting of GW energy density, and loop production required for network to scale)

– emission in *matter era* (less loop production, redshifting of GW energy density "wins")

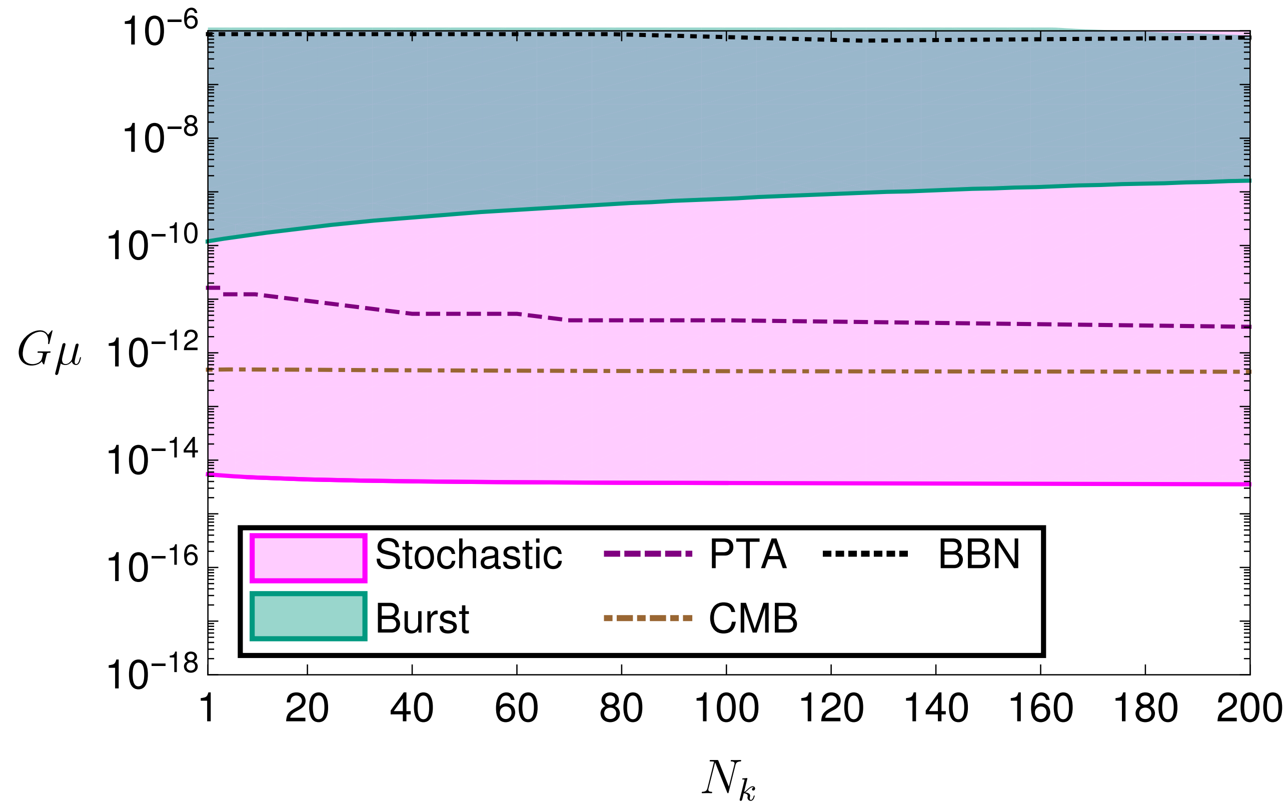
Exclusion plots $G\mu, \Gamma$

Bounds on integrated GW energy density generated before BBN, and before photon decoupling

Model A



Model B



Excluded:

$$G\mu \gtrsim (9.6 \times 10^{-9} - 10^{-6})$$

Excluded:

$$G\mu \gtrsim (4.0 - 6.3) \times 10^{-15}$$

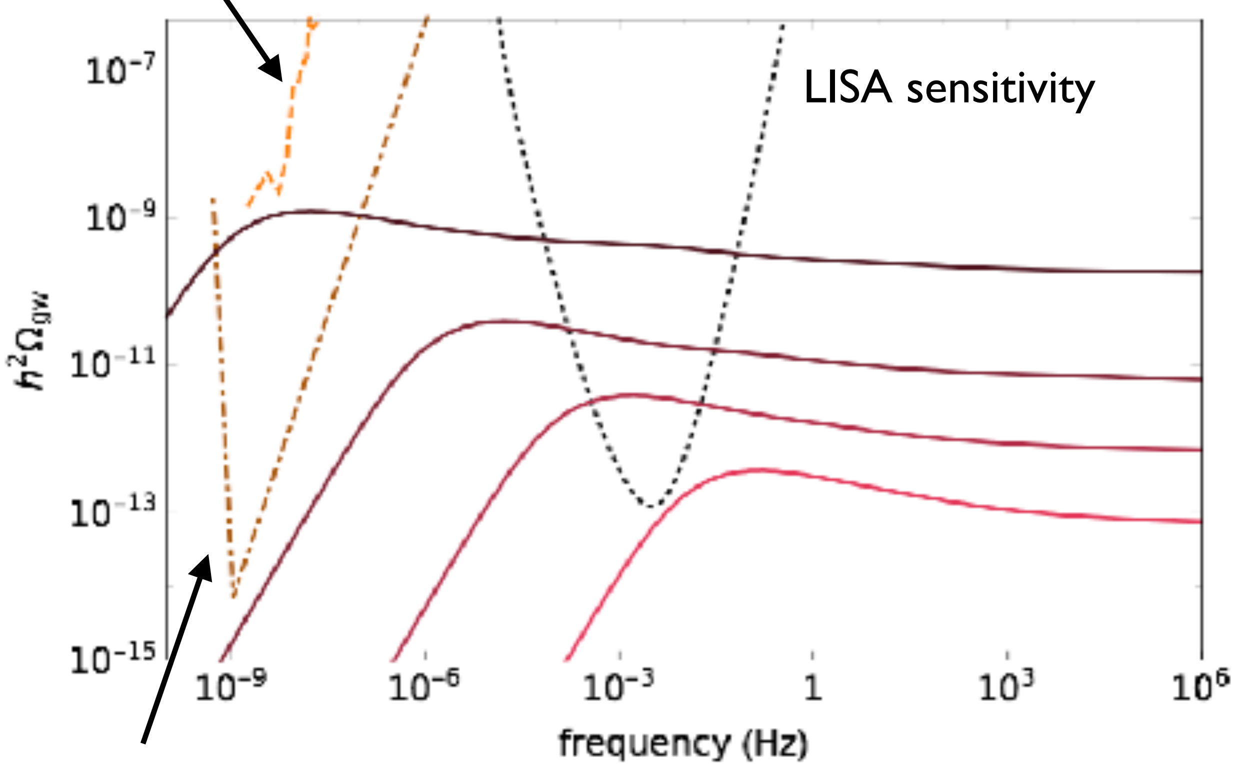
- Relative to O1&O2 analysis (Nk=1), constraints on Gmu stronger by ~2 orders of magnitude for model A, and by ~1 for model B

LISA band $G\mu, \Gamma = 50$

[P.Auclair, DAS, +LISA cosmology group JCAP 04 (2020) 034]

EPTA sensitivity

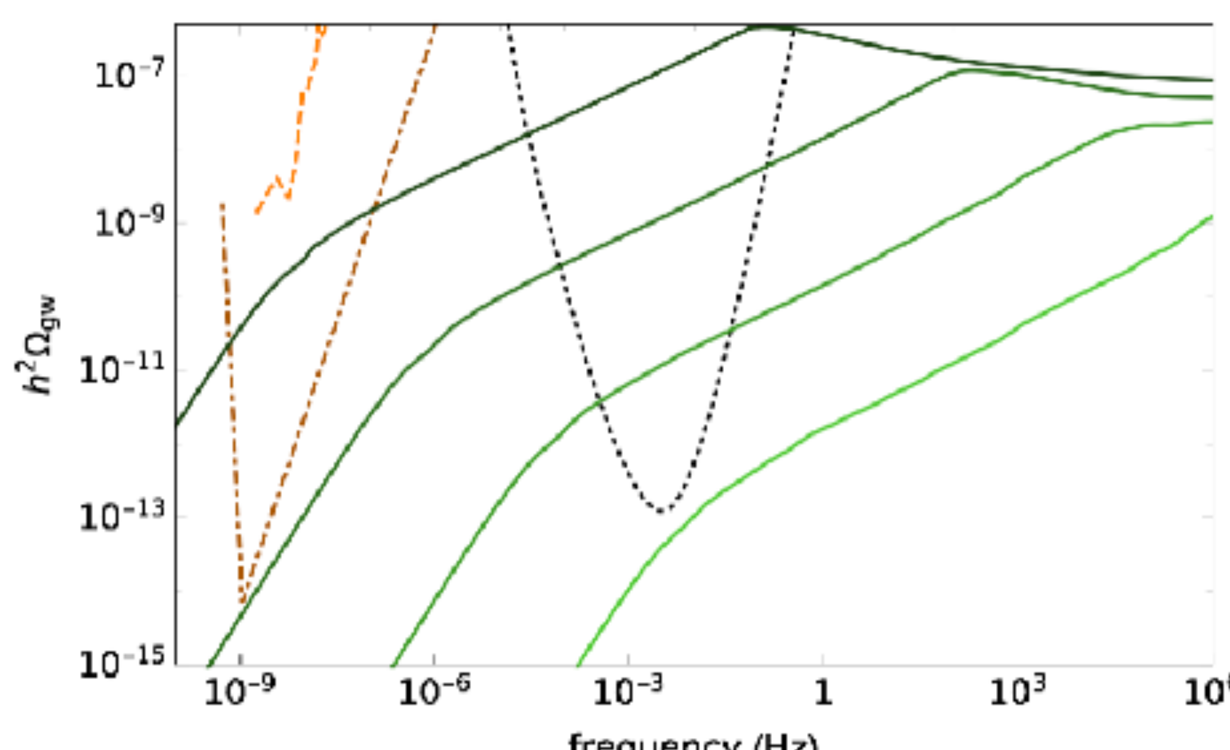
Model A



(projected) SKA sensitivity

LISA will be able to probe cosmic strings with tensions
$G\mu \gtrsim \mathcal{O}(10^{-17})$

Model B



Spectral shape => tightening of constraints not expected from low frequency experiments

- Parkes PTA constraint, for both models

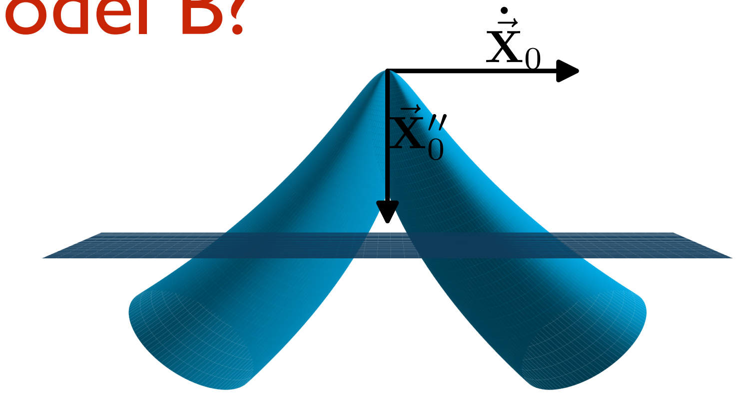
$$G\mu \sim 10^{-10} - 10^{-11}$$

[L.Bian et al 2205.07293]

[S.Babak, H.Quelquejay-Leclere, DAS, in progress, with EPTA Data release 2, 24 years observation of 6 pulsars]

Particle production… A window onto Model B?

[P.Auclair, D.A.S, T.Vachaspati, 2020,
P.Auclair, K.Leyde and DAS, 2022]



• At kinks and cusps, a realistic string can "overlap" leading to other forms of energy loss: emission of particles
[Matsunami et al, PRL 122 , 201301 (2019)]

$$\boxed{\frac{d\ell}{dt} = \begin{cases} -\Gamma G\mu, & \ell \gg \ell_c \\ -\Gamma G\mu\sqrt{\frac{\ell_c}{\ell}}, & \ell \ll \ell_c \end{cases}}$$

$$\ell_c \sim w(\Gamma G\mu)^{-2}$$

• again, can solve the Boltzmann equation exactly and, from the resulting loop distribution, calculated the energy emitted into particles.

• Emitted particles decay into standard model Higgs particles, of which a fraction cascade down into gamma-rays -> contribute to the **diffuse gamma-ray background:**

$$\boxed{\omega_{\rm DGRB}^{\rm obs} \lesssim 5.8 \times 10^{-7}\ {\rm eV cm^{-3}}}$$

A. A. Abdo et al. (Fermi-LAT),

total EM energy injected since universe became transparent to GeV gamma-rays $t_\gamma \simeq 10^{15}{\rm s}$

• combined with GW constraints -> possibly new constraint

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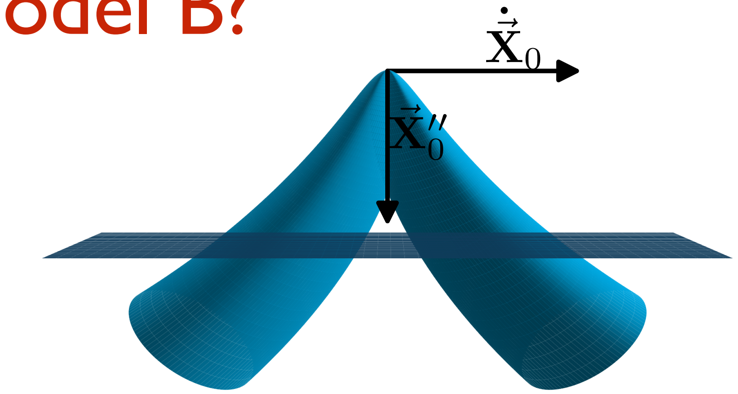

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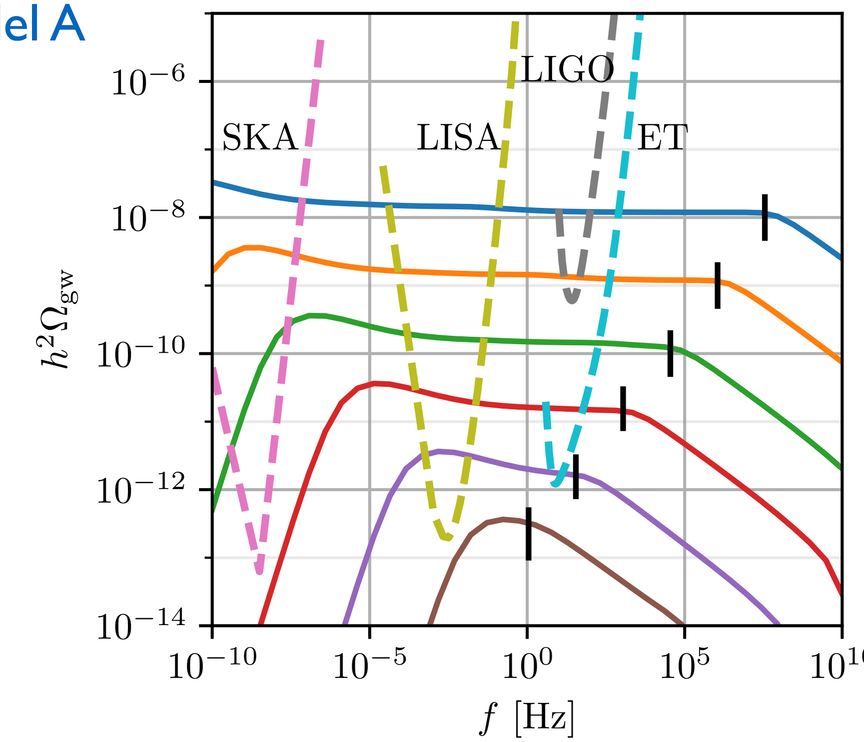
Model A



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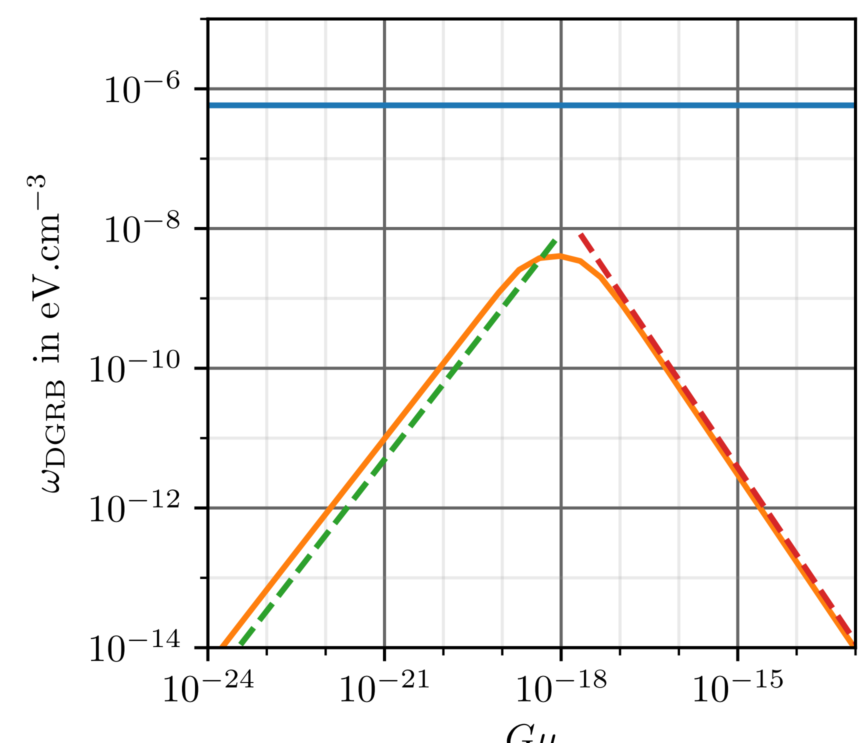

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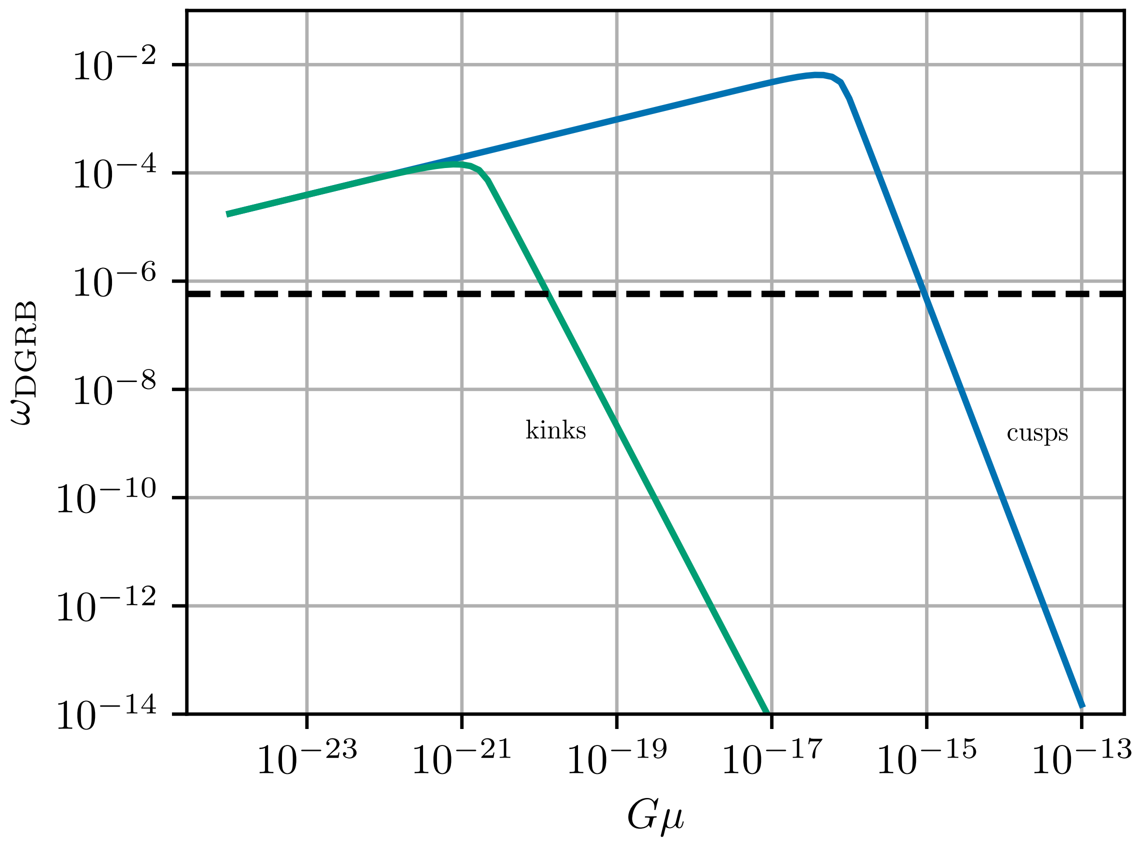
For model B

[Auclair, Leyde, Steer, 2021]



$$10^{-15} \lesssim (G\mu)_{\rm cusps} \lesssim 4.0 \times 10^{-15}$$

gamma rays   GWs

- Future LIGO-Virgo O4/O5 observations will either rule out model B…or discover cosmic strings?!

Conclusions

- Vortons may be there, and be an original form of dark matter!

– framework to calculate their as a function of the string tension and the current carrier energy scale
– Sizable "irreducible" population, independent of initial conditions
– rule out new areas of parameter space.

- GW constraints on NG strings for different models.

– LIGO-Virgo excludes $G\mu \gtrsim (9.6 \times 10^{-9} - 10^{-6})$, model A $\quad 10\,\mathrm{Hz} < f < 5\,\mathrm{kHz}$
$G\mu \gtrsim (4.0 - 6.3) \times 10^{-15}$ model B

– LISA will probe strings with $G\mu \gtrsim \mathcal{O}(10^{-17})$ independently of the model $\quad 10^{-4}\,\mathrm{Hz} < f < 1\,\mathrm{Hz}$

– PTA are starting to exclude strings with $G\mu \sim 10^{-10} - 10^{-11}$ for model A $\quad 10^{-9}\mathrm{Hz} < f < 10^{-7}\mathrm{Hz}$

- If loops formed with a power-law loop production function (Model B), and if particles are emitted from cusps, then the model will get hot under the collar quite soon!