COALESCING BINARY BLACK HOLES
and
GRAVITY BEYOND GENERAL RELATIVITY

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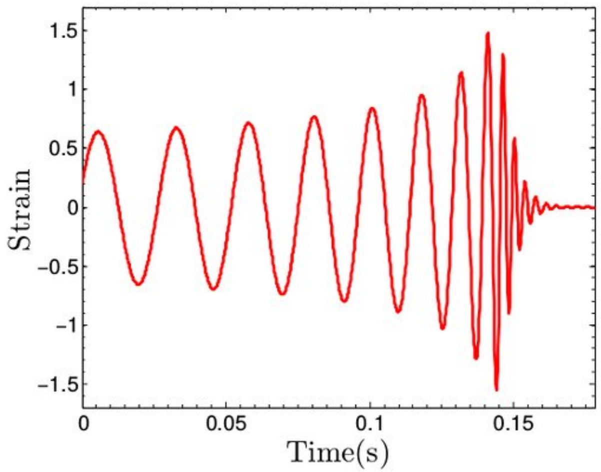
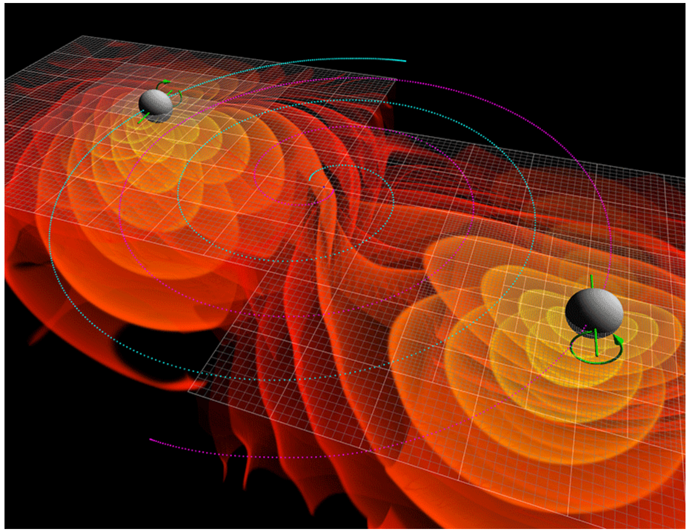
Institut des Hautes Etudes Scientifiques



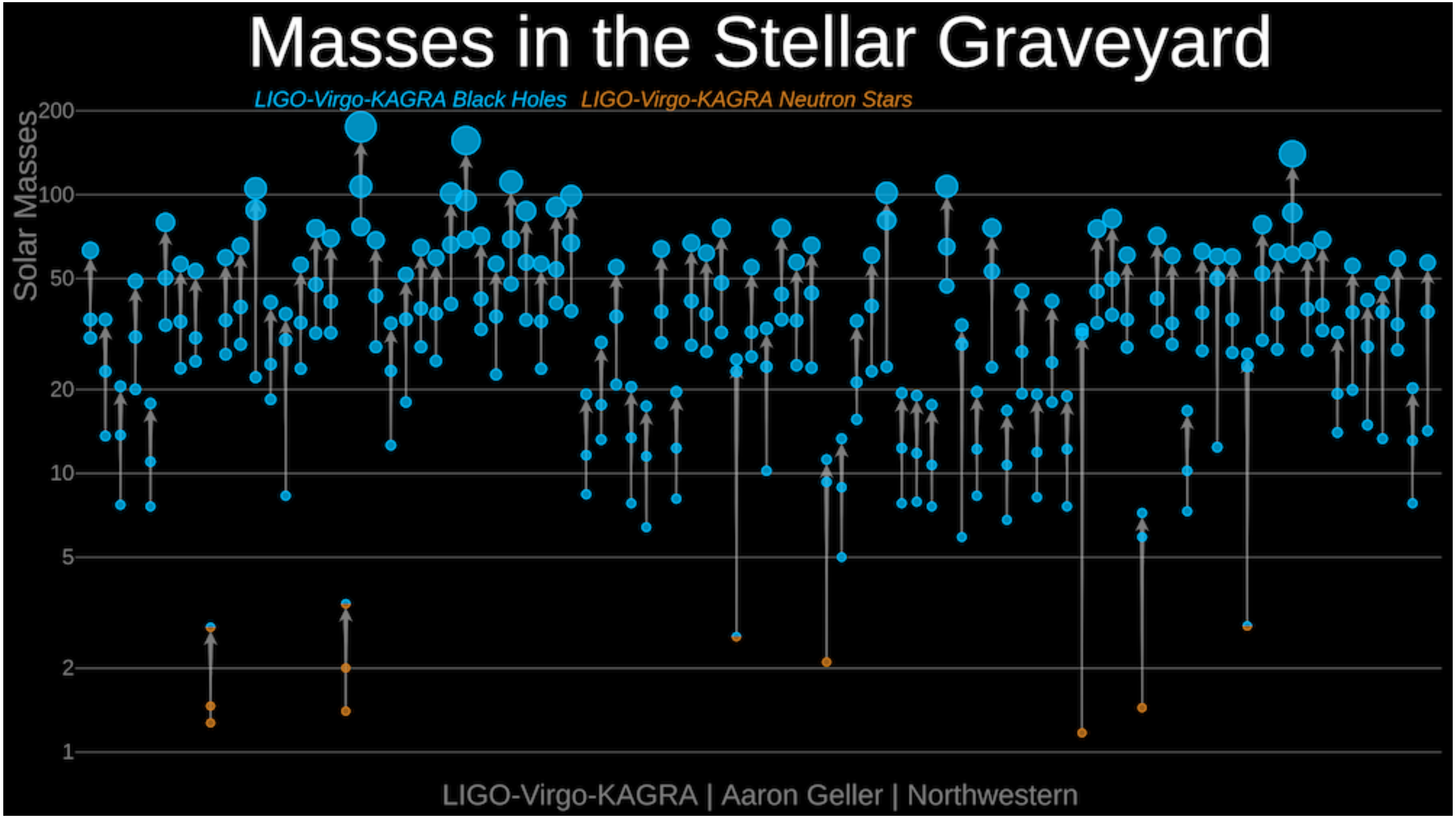


Carter Fest: Black Holes and Other Cosmic Systems
A conference in celebration of Brandon Carter's 80th birthday
4-6 July 2022, IAP and Obs. Paris-Meudon, France

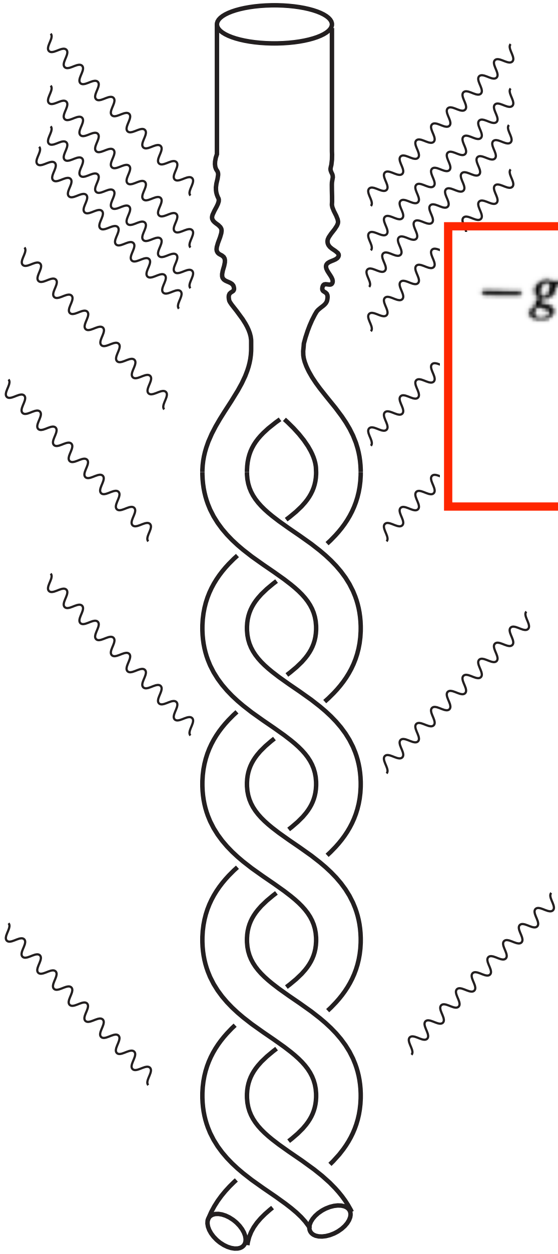
LIGO-Virgo p>0.5 Events

(O1-O2-O3a-O3b; nov 2021)

90 events, incl.: 2 NS-NS; 3 NS-BH; 85 BH-BH

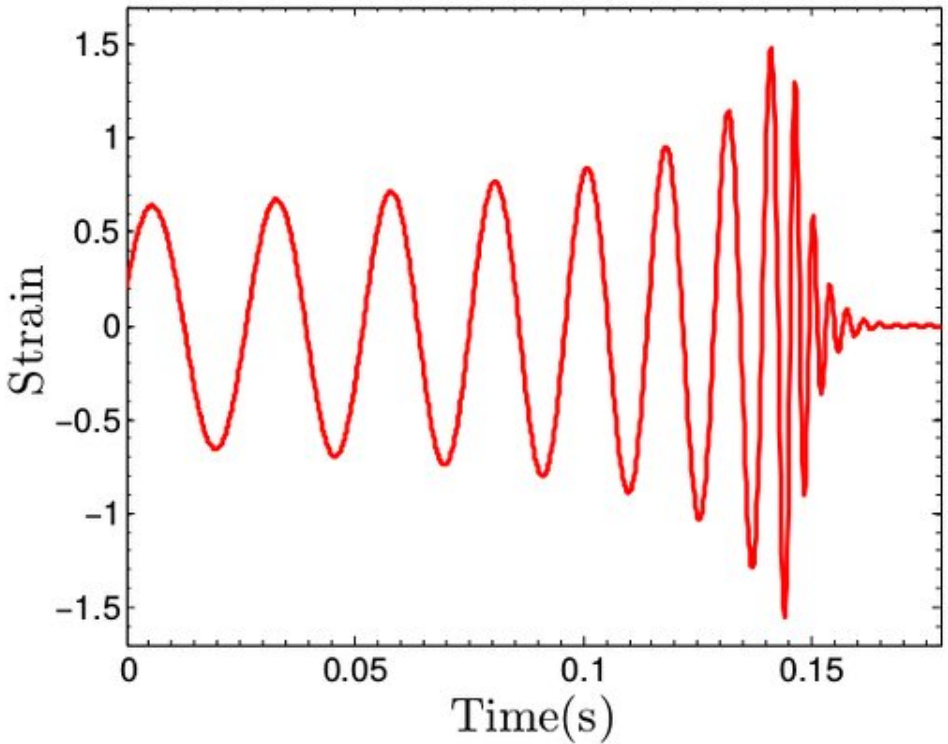


$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad\qquad R_{\mu\nu} = 0$$



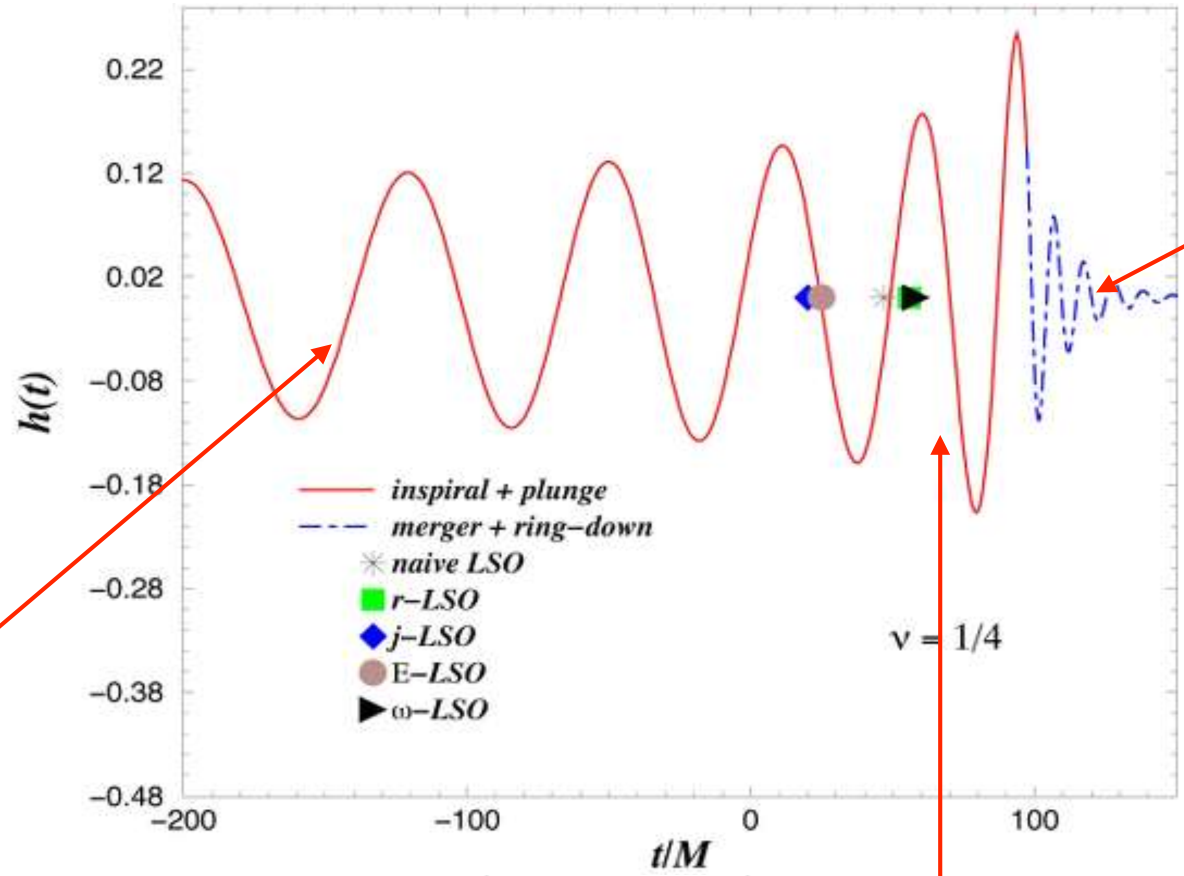
$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$

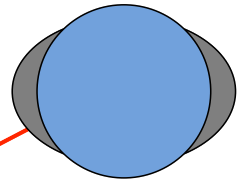
$$-g^{\mu\nu} g_{\alpha\beta,\,\mu\nu} + g^{\mu\nu} g^{\rho\sigma} \big(g_{\alpha\mu,\,\rho} g_{\beta\nu,\,\sigma} - g_{\alpha\mu,\,\rho} g_{\beta\sigma,\,\nu} + g_{\alpha\mu,\,\rho} g_{\nu\sigma,\,\beta} + g_{\beta\mu,\,\rho} g_{\nu\sigma,\,\alpha} - \frac{1}{2} g_{\mu\rho,\,\alpha} g_{\nu\sigma,\,\beta} \big) = 0$$



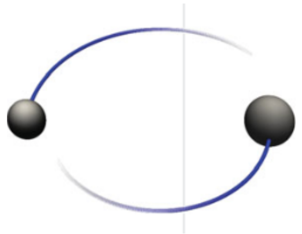
Gravitational Waves emitted by the Merger of two Black Holes

Buonanno-TD'2000

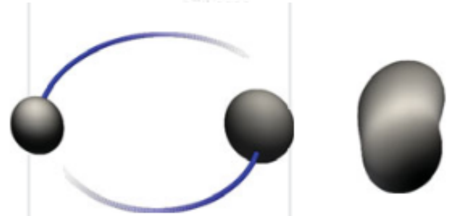




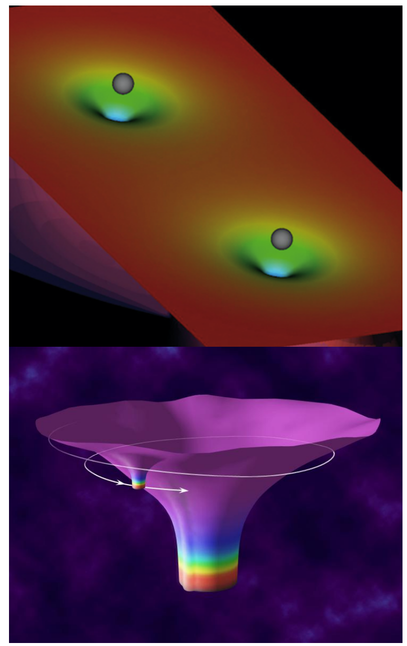
Ringdown (BBH):
« vibration modes »
of final BH (QNM);
perturbation
of BHs à la
Regge-Wheeler-Zerilli-
Teukolsky
+Vishveshwara



Inspiral:
perturbative
computation
of higher-order
contributions
to E=H and F
(expansion in v^2/c^2
tidal polarizability
of NS)



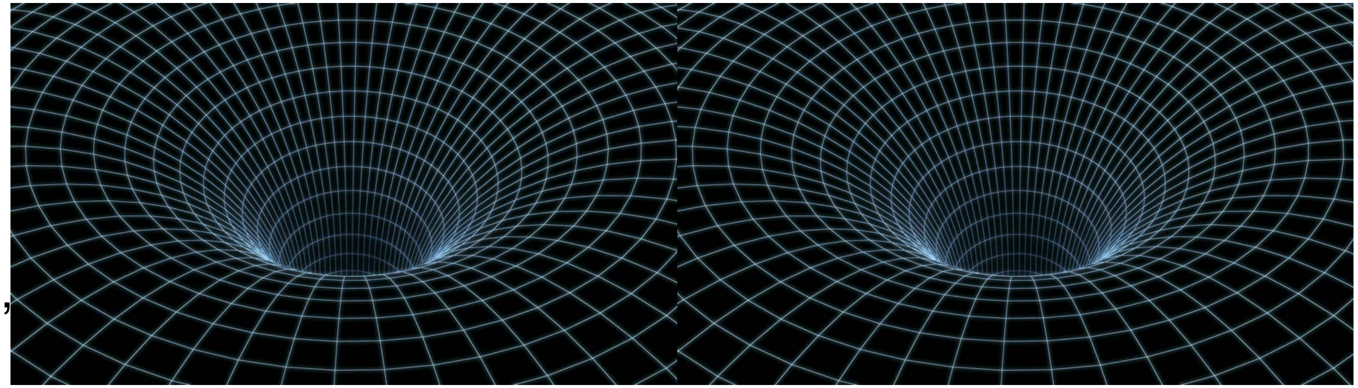
Late inspiral, « plunge » and merger:
first estimated by the Effective One-Body method (AB-TD 2000)
later confirmed and improved by using
numerical simulations (Pretorius…2005)



Importance of the No-Hair Theorem for BBH

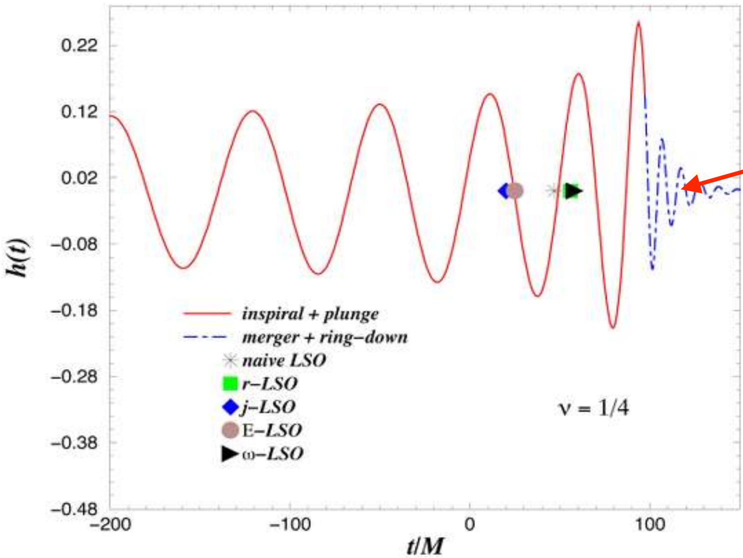
Allows matched asymptotic expansions

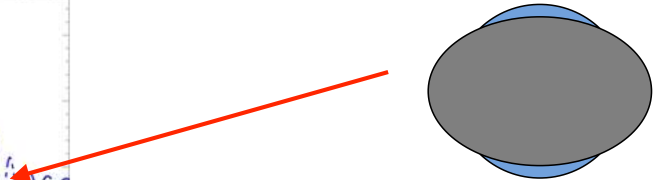
[EIH '38], Manasse '63, Demianski-Grishchuk '74, D'Eath'75, Kates '80, Damour '82



—> **« Effacing Principle » (vanishing Love numbers)** TD 83 up to G^6

Determines the QNM ringing

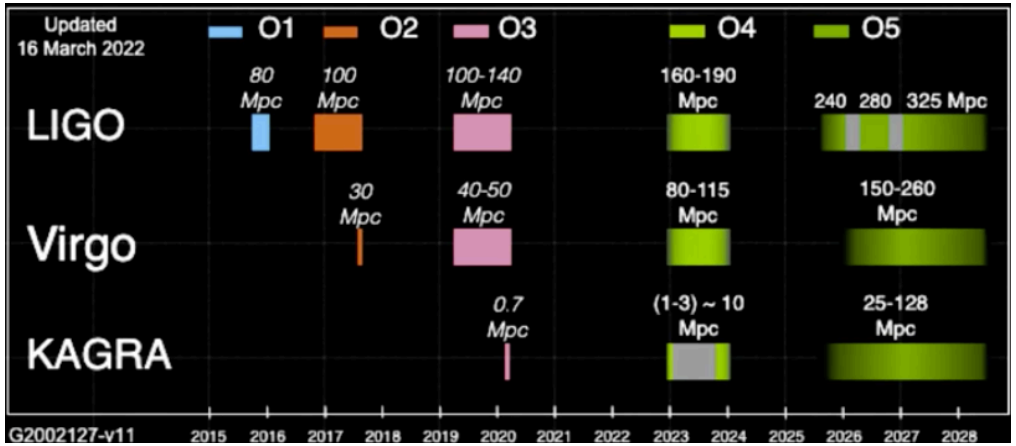


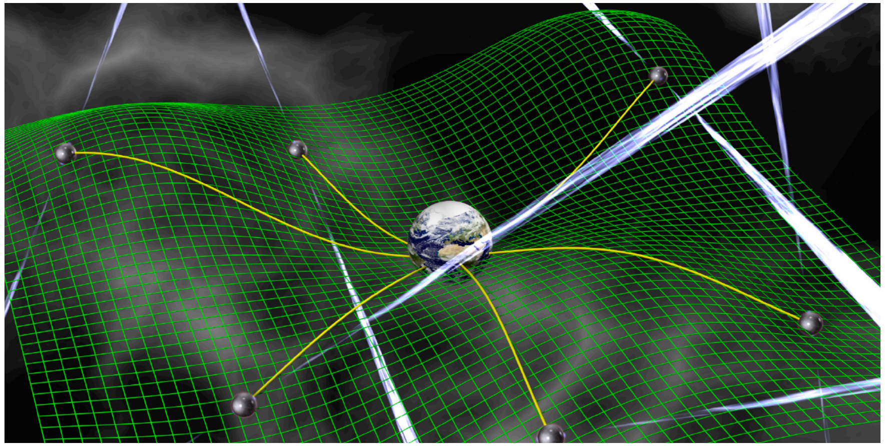


Ringdown (BBH):
« vibration modes » of final BH (QNM); perturbation of BHs à la Regge-Wheeler-Zerilli-Teukolsky +Vishveshwara

No-Hair —> BBH dynamics and waveform fully determined by (initial) masses and spins

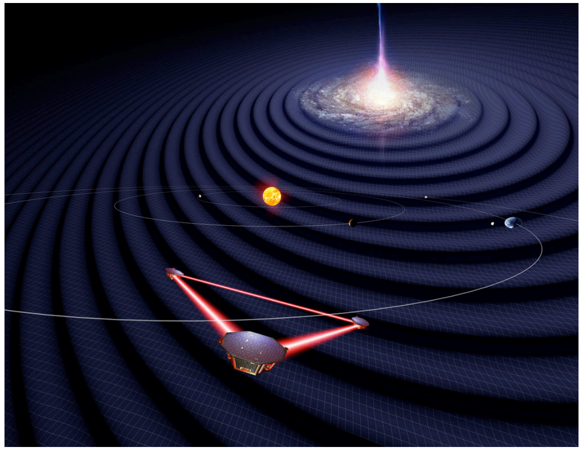
Towards the Future



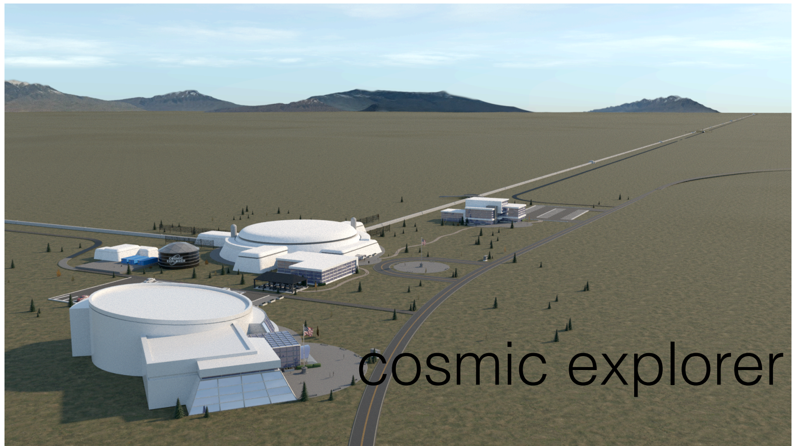


pulsar timing array

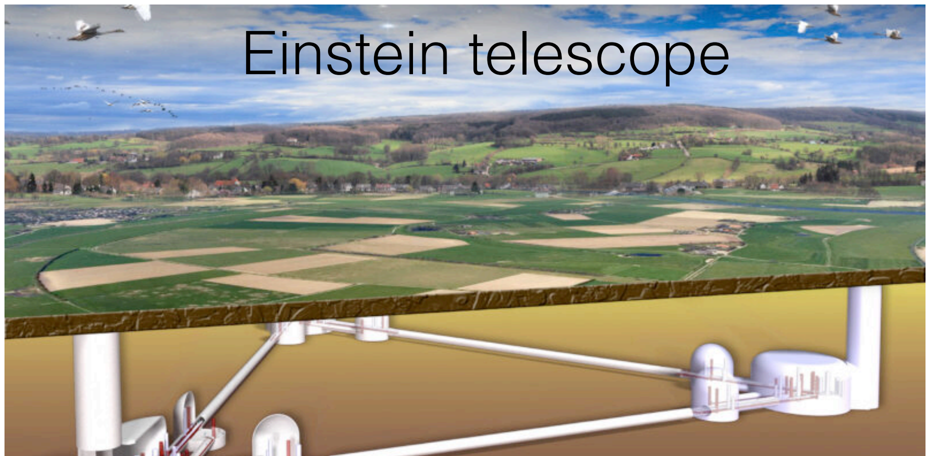
ligo india



lisa



cosmic explorer

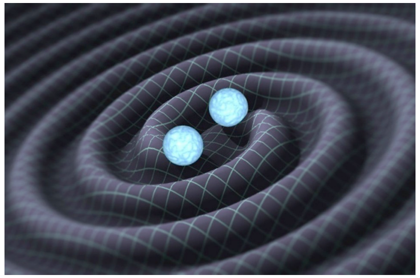


Einstein telescope

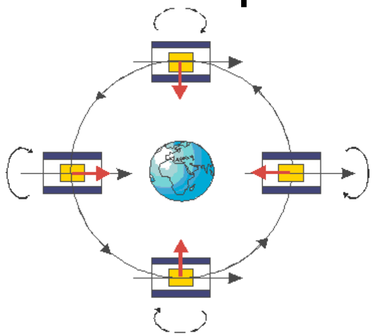
Probing Fundamental Physics with Gravitational Data (GR and Beyond)

(review: particle data group)

grav. waves

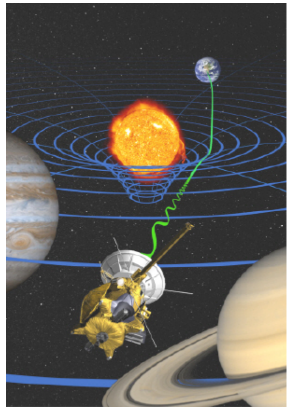


Microscope

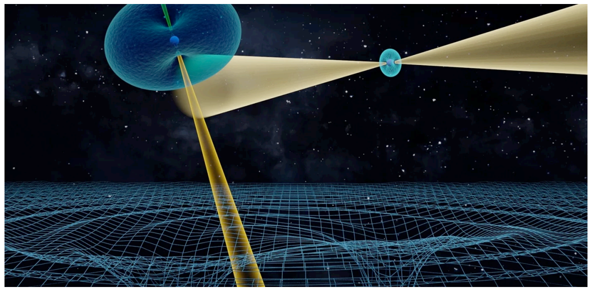


Lunar laser ranging

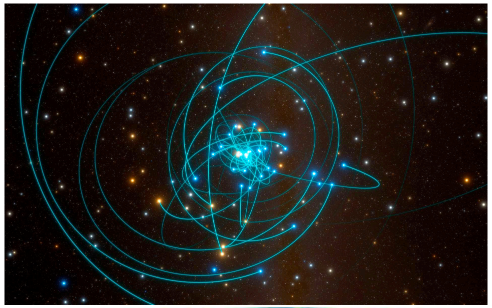
Cassini



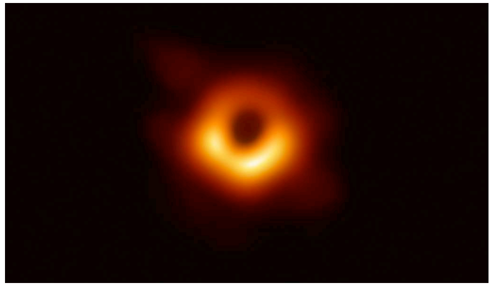
Binary pulsars



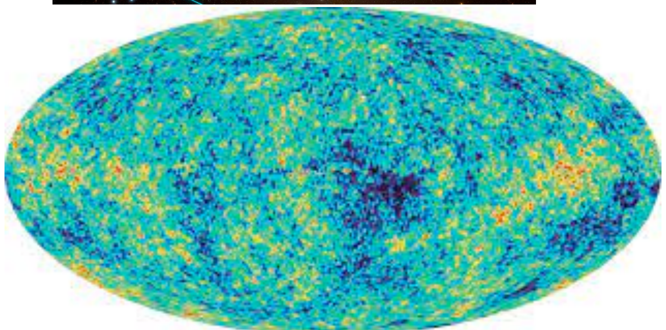
Galactic center



EHT



Cosmology



Phenomenological approaches to GR deviations

Equivalence Principle:

$$\frac{a_A - a_B}{\bar{a}} = 0\ ? \quad \frac{d\alpha/dt}{\alpha} = 0\ ?$$

Post-Einsteinian

Weak-field Gravity (PPN):

$$g_{00} = -1 + \frac{2}{c^2}V - \frac{2\beta}{c^4}V^2 + O\left(\frac{1}{c^6}\right)$$

$$g_{0i} = -\frac{2(\gamma+1)}{c^3}V_i + O\left(\frac{1}{c^5}\right),$$

$$g_{ij} = \delta_{ij}\left[1 + \frac{2\gamma}{c^2}V\right] + O\left(\frac{1}{c^4}\right),$$

$$\beta = 1\ ?\ \gamma = 1$$

Binary-pulsar timing (PPK formalism based on DD model) strong-field +radiative gravity:

Keplerian

Post-Keplerian

$$t_N - t_0 = F[T_N(\nu_p, \dot{\nu}_p, \ddot{\nu}_p)\,;\ \{p^K\}\,;\ \{p^{PK}\}]$$

$$k^{GR}(m_1, m_2) = 3(1-e^2)^{-1}(GMn/c^3)^{2/3},$$

$$\gamma^{GR}_{\rm timing}(m_1, m_2) = en^{-1}(GMn/c^3)^{2/3}m_2(m_1 + 2m_2)/M^2,$$

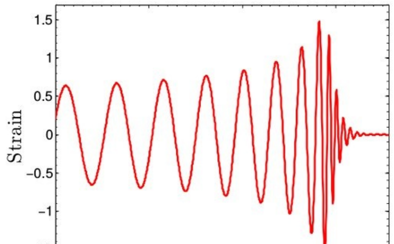
$$\dot{P}^{GR}_b(m_1, m_2) = -(192\pi/5)(1-e^2)^{-7/2}\left(1 + \tfrac{73}{24}e^2 + \tfrac{37}{96}e^4\right) \times (GMn/c^3)^{5/3}m_1 m_2/M^2,$$

$$r^{GR}(m_1, m_2) = Gm_2/c^3,$$

$$s^{GR}(m_1, m_2) = nx(GMn/c^3)^{-1/3}M/m_2.$$

Phenomenological tests of GW data

$$h_{obs}(t) = h_{GR}(t; p_i)\ ?$$



$$\psi(f) = \sum_i \left[p_i^{\rm GR,NS}(m_1, m_2)(1 + \delta\hat{p}_i) + p_i^{\rm GR,S}(m_1, m_2, S_1, S_2)\right] u_i(f).$$

$$\omega_a = (c^3/GM_f)[2\pi \hat{f}_a^{\rm QNM}(a_f) - i/\hat{\tau}_a^{\rm QNM}(a_f)]$$

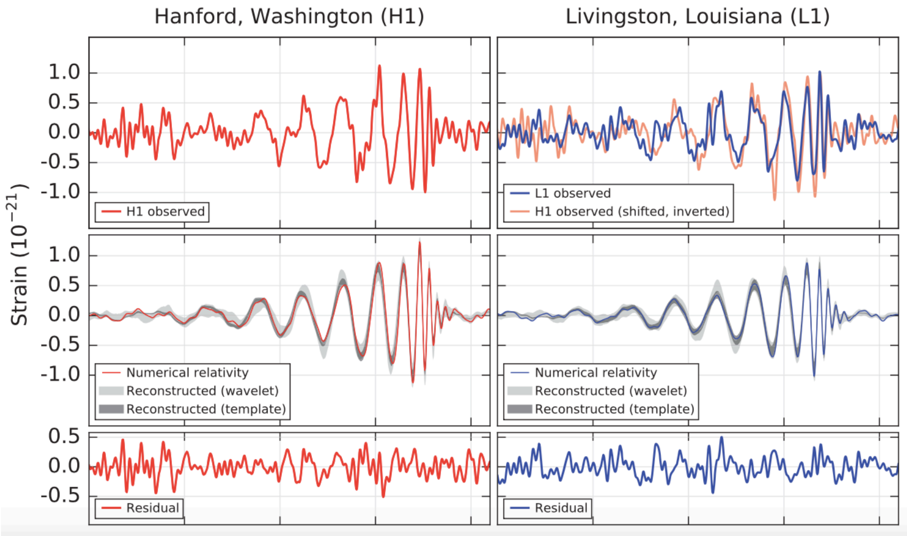
Phenomenological GR tests from LIGO-Virgo

The most direct evidence that the BHs predicted by GR exist and have the expected structure

GW150914

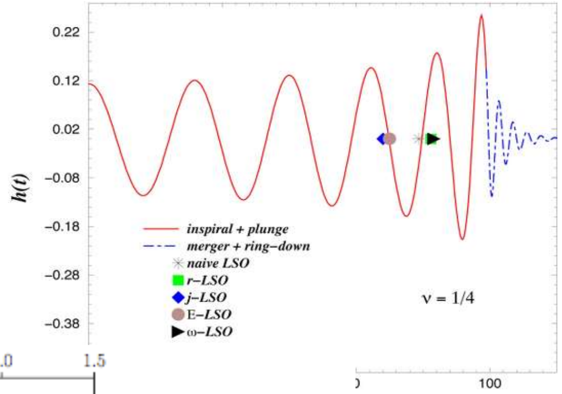
$$\frac{SNR_{GR}}{\sqrt{SNR^2_{GR} + SNR^2_{res,90}}} = 0.97$$

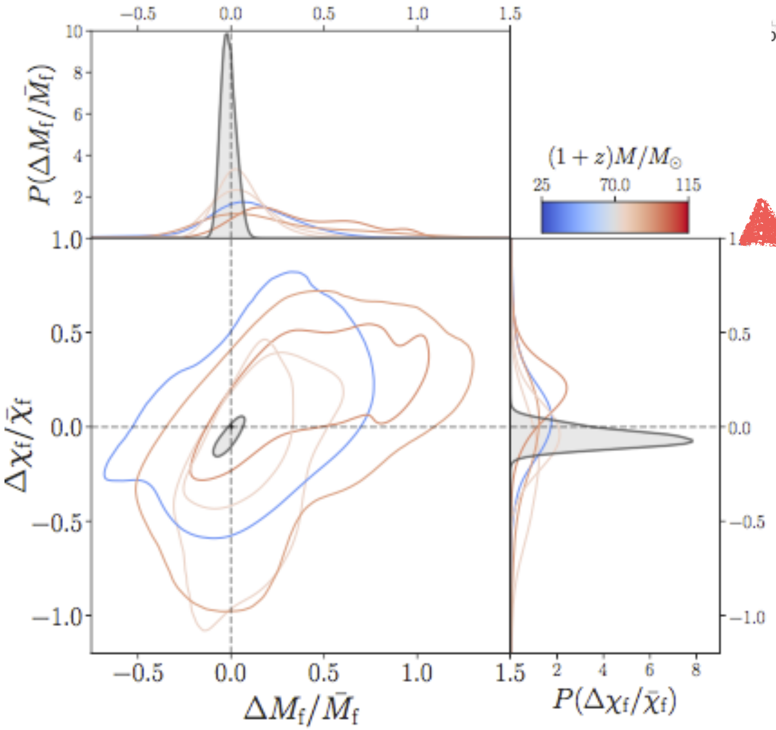
Global Fitting factor observed/predicted signal



Dividing in inspiral and post-inspiral

Confirmation of final damped vibration modes





GR tests with GWTC3 (dec 21)

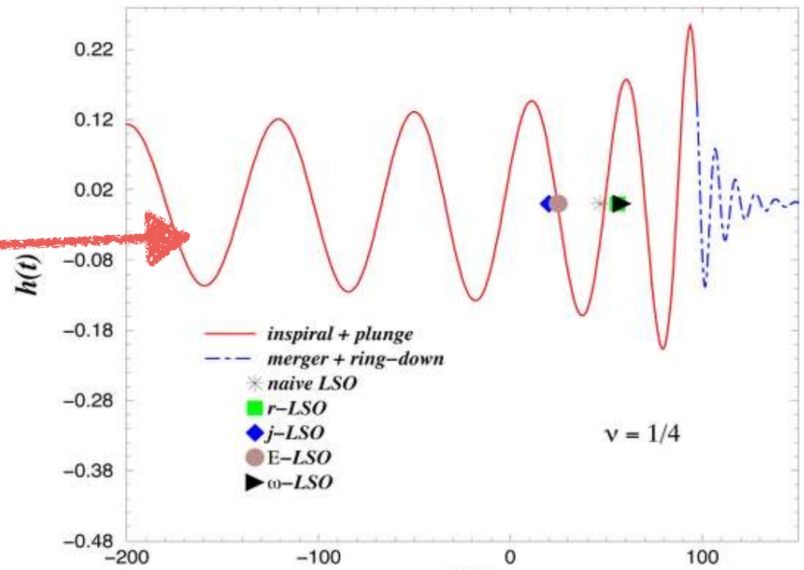
recent pSEOBNRv4HM analysis of GWTC3

$$\delta\hat{f}_{220} = 0.02^{+0.03}_{-0.03}; \delta\hat{\tau}_{220} = 0.13^{+0.11}_{-0.11}$$

Phenomenological GR tests LIGO-Virgo (2)

Phenom. tests of inspiral signal

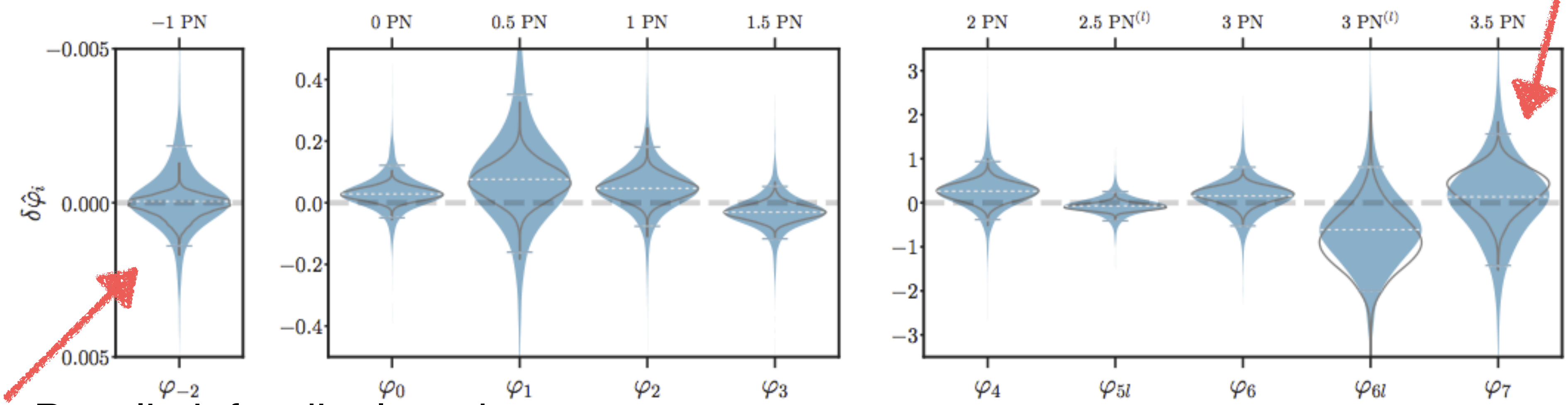
$$h(f) = A(f)e^{i\Psi(f)}$$



GR tests with GWTC3 (dec 21)

$$\psi(f) = \sum \left[p_i^{\mathrm{GR,NS}}(m_1, m_2)(1 + \delta\hat{p}_i) + p_i^{\mathrm{GR,S}}(m_1, m_2, S_1, S_2)\right] u_i(f).$$

O(1) limit



Best limit for dipole rad:
10^-3 level (pulsars -> 10^-9)

Speed of GWs vs light (GW170817)

$$-3 \times 10^{-15} < \frac{c_{\mathrm{GW}} - c}{c} < +7 \times 10^{-16}.$$

Tests of the Equivalence Principle

Variation of « constants »

$$\alpha \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137.036}$$

$$\omega_a = (c^3/GM_f)[2\pi \hat{f}_a^{\rm QNM}(a_f) - i/\hat{\tau}_a^{\rm QNM}(a_f)]$$

$$d\ln(\alpha_{\rm em})/dt = (-2.5 \pm 2.6)\times 10^{-17}{\rm yr}^{-1},$$
$$d\ln(\mu)/dt = (-1.5 \pm 3.0)\times 10^{-16}{\rm yr}^{-1},$$
$$d\ln(m_q/\Lambda_{\rm QCD})/dt = (7.1 \pm 4.4)\times 10^{-15}{\rm yr}^{-1}.$$

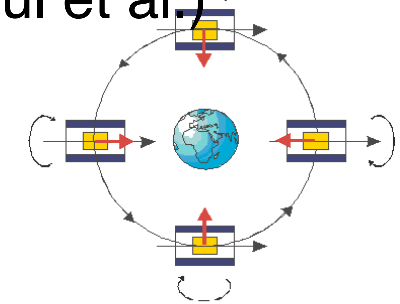
cosmological
Oklo
Atomic-clocks

Eotvos (Adelberger et al)

$$(\Delta a/a)_{\rm BeTi} = (0.3 \pm 1.8)\times 10^{-13};$$
$$(\Delta a/a)_{\rm BeAl} = (-0.7 \pm 1.3)\times 10^{-13};$$

Microscope (Touboul et al.)

$$(\Delta a/a)_{\rm TiPt} = (-1 \pm 9({\rm stat}) \pm 9({\rm syst}))\times 10^{-15}$$



$$(\Delta a/a)_{\rm EarthMoon} = (-3 \pm 5)\times 10^{-14}.$$

Lunar-Laser ranging

Tests of the 1/r^2 law

$$V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha\exp(-r/\lambda)],$$
$$|\alpha| < 1 \text{ down to } 56\mu$$

(Kapner et al)

Phenomenological tests of post-Newtonian gravity (solar system)

Two main post-Newtonian parameters

$$g_{00} = -1 + \frac{2}{c^2}V - \frac{2\beta}{c^4}V^2 + O\left(\frac{1}{c^6}\right)$$

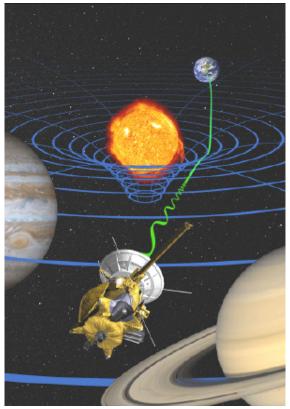
$$g_{0i} = -\frac{2(\gamma+1)}{c^3}V_i + O\left(\frac{1}{c^5}\right),$$

$$g_{ij} = \delta_{ij}\left[1 + \frac{2\gamma}{c^2}V\right] + O\left(\frac{1}{c^4}\right),$$

$$\bar{\gamma} = \gamma - 1$$

$$\bar{\beta} = \beta - 1$$

Cassini Mission



$$\overline{\gamma} = (2.1 \pm 2.3) \times 10^{-5}$$

$$|\overline{\beta}| < 7 \times 10^{-5}$$

Phenomenological Binary Pulsar Tests: strong and radiative fields

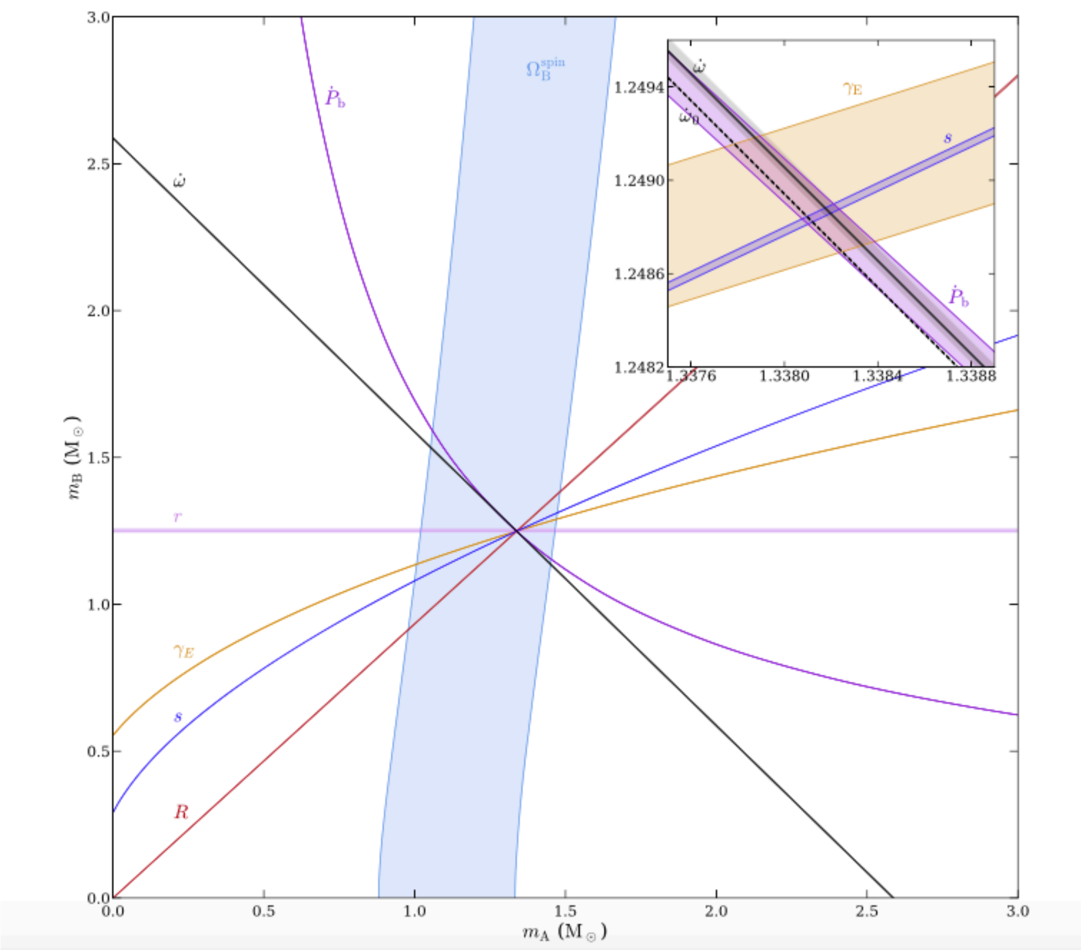
PSR1913+16 (Hulse-Taylor)

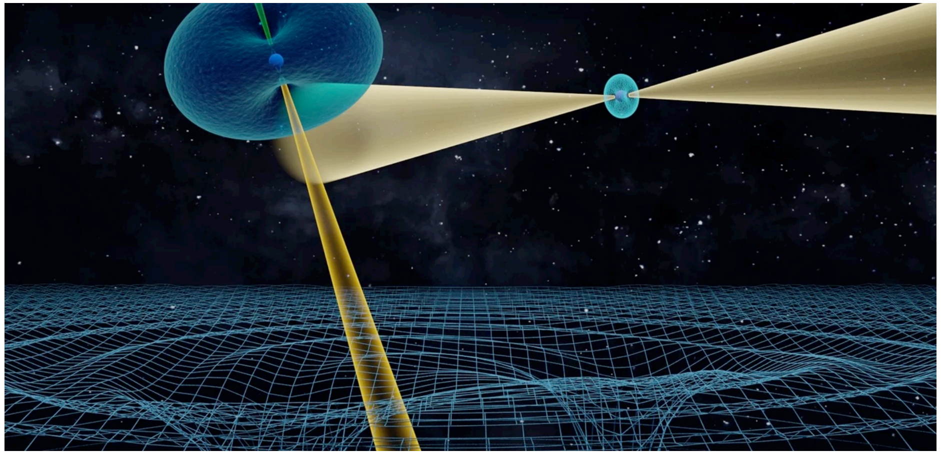
$$\left[\frac{\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal}}{\dot{P}_b^{\rm GR}[k^{\rm obs}, \gamma_{\rm timing}^{\rm obs}]}\right]_{1913+16} = 0.9983 \pm 0.0016$$

Double Pulsar(Kramer et al)

5 precision tests of GR

M. KRAMER *et al.* PHYS. REV. X **11,** 041050 (2021)





$$\dot{P}_{\rm b}^{\rm GW} / \dot{P}_{\rm b}^{\rm GW,GR} = 0.999963(63)$$

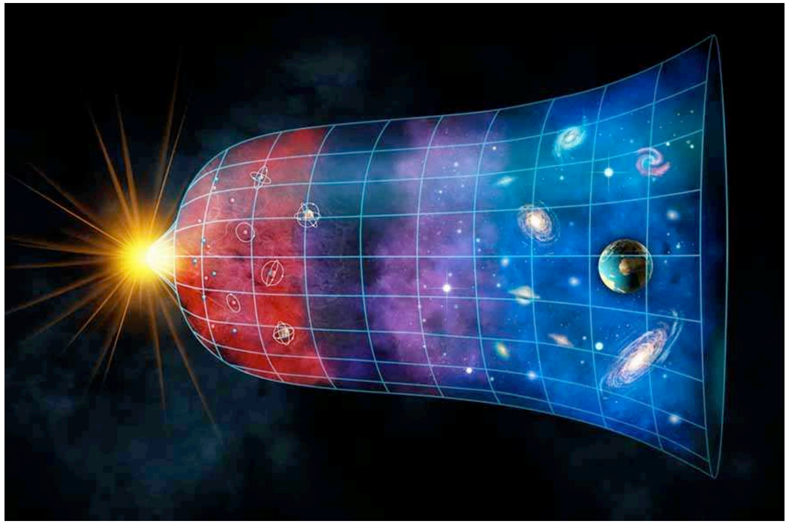
$$s^{\rm obs} / s^{\rm GR} = 1.000\,09(18)$$

Triple Pulsar (SEP)

$$|\Delta a / a| < 2.05 \times 10^{-6} \text{ (95\% C.L.)}$$

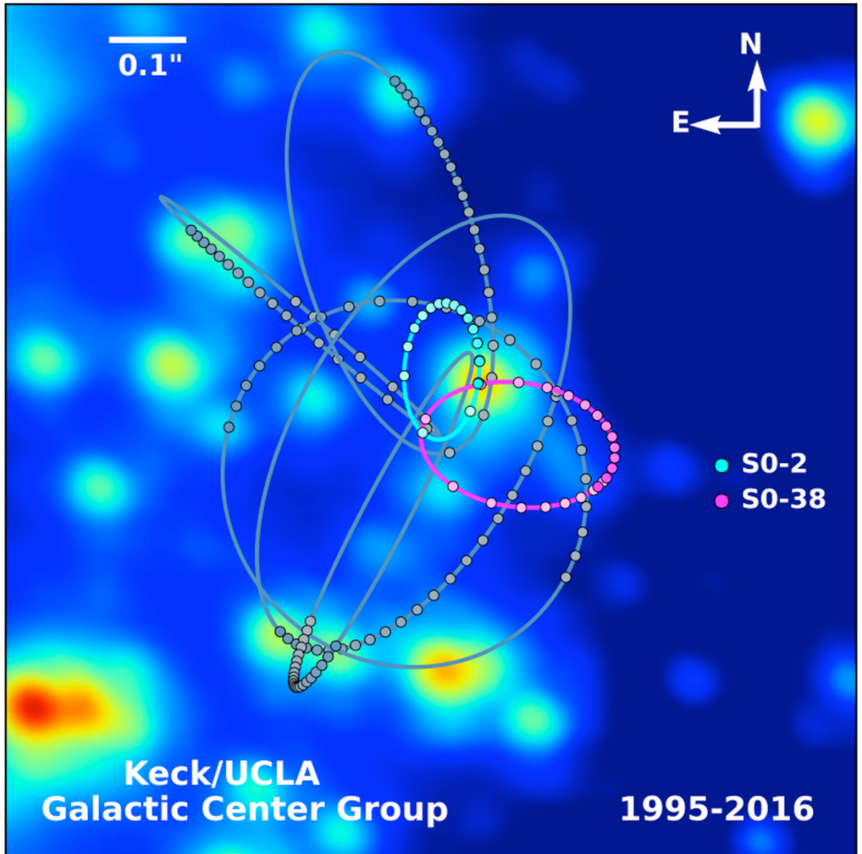
Other tests of GR

cosmology

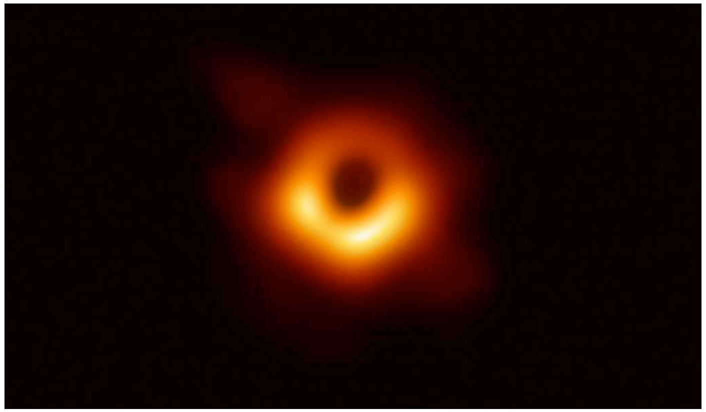


centre of our Galaxy SgrA*: notably S2

periastron precession
$f\_SP = 1.10 \pm 0.19$
(GRAVITY'20)



EHT



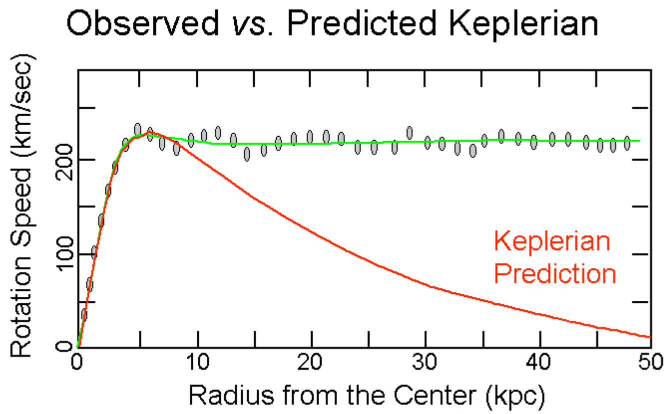
Project: Shape of photon ring (Gralla-Lupsasca-…'20)

Theory-based approaches to GR deviations ?

Puzzles posed by GR

short-distance incompleteness

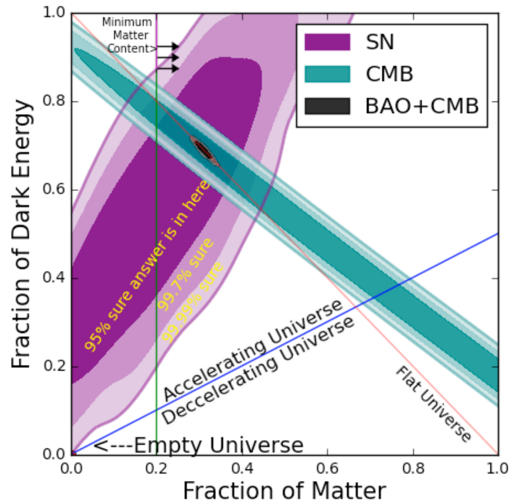
UV completion at L > L_Planck ?



dark matter

long-distance « black clouds »

dark energy



Theory-based approaches to GR deviations

Completions/extensions of GR

« Historical » extensions:

Kaluza-Klein, Jordan-Fierz: dilaton-like scalar field

Cartan: torsion

Einstein: $G_{\mu\nu} = g_{\mu\nu} + B_{\mu\nu}$

String theory: tree-level massless sector

$$G_{\mu\nu}; \Phi; B_{\mu\nu}$$

$$+ \text{moduli from compactified dim}$$

$$+ \ell_s - \text{corrections}$$

\+ Kaluza-Klein tower of massive states depending on **KK compactification lenth-scale** L_KK

Recent suggestion: « Dark dimension » (Montero-Vafa-Valenzuela'22)

$$\ell_{KK} \sim 10^{-2} \Lambda^{-\frac{1}{4}} \sim 1\,\mu \;\; \text{with} \;\; M_* \sim 10^{10}\,\text{GeV}$$

Consequences of adding a scalar dof

Generic EP violations
from dilaton-like coupling

$$\mathcal{L}_{\text{int}\phi} = \kappa\phi\Big[+ \frac{d_e}{4e^2} F_{\mu\nu}F^{\mu\nu} - \frac{d_g \beta_3}{2g_3} F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=e,u,d} (d_{m_i} + \gamma_{m_i} d_g) m_i \bar{\psi}_i \psi_i \Big].$$

Weak-field deviations from
composition-independent
coupling to T=T^mu_mu

$$\mathcal{L}_{\text{tot}}[g_{\mu\nu}, \varphi, \psi, A_\mu, H] = \frac{c^4}{16\pi G_*}\sqrt{g}(R(g_{\mu\nu}) - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) - \sqrt{g}V(\varphi) + \mathcal{L}_{\text{SM}}[\psi, A_\mu, H, \tilde{g}_{\mu\nu}].$$

two functions: potential V(phi), coupling a(phi)

$$\tilde{g}_{\mu\nu} = \exp(2a(\varphi))g_{\mu\nu}$$

field-dependent coupling

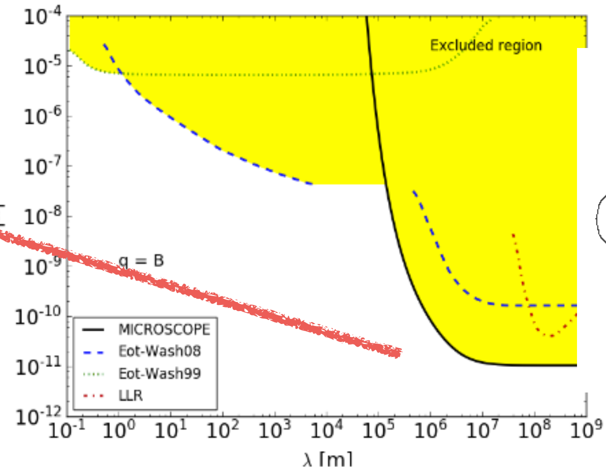
$$\alpha(\varphi) \equiv \partial a(\varphi)/\partial\varphi$$

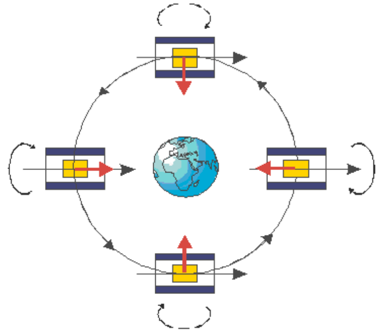
$$\overline{\gamma} = -2\frac{\alpha_0^2}{1+\alpha_0^2}\,; \qquad \overline{\beta} = +\tfrac{1}{2}\frac{\beta_0\alpha_0^2}{(1+\alpha_0^2)^2}$$

$$\alpha_0 \equiv \alpha(\varphi_0), \text{ and } \beta_0 \equiv \partial\alpha(\varphi_0)/\partial\varphi_0$$

Stringent constraints on light scalar dof

From EP tests (Bergé et al'18)



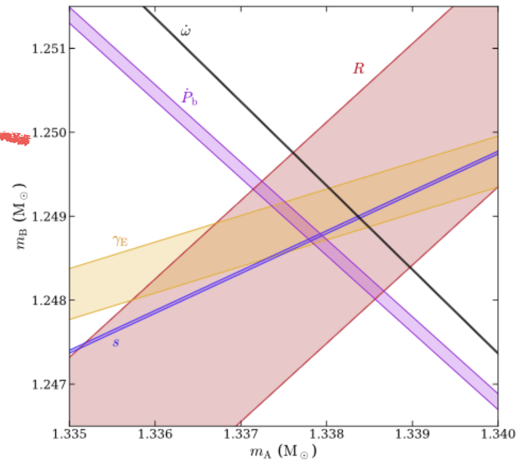


$$\alpha_0^2 = \alpha < 10^{-11}$$

From solar-system tests

$$\alpha_0^2 < 10^{-5}$$

More stringent constraints from binary-pulsar strong-field+radiative tests



More general 2-derivative scalar-tensor (Horndeski) $X \equiv -\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$

$$\begin{aligned}L_{\rm tot}[g_{\mu\nu},\varphi,\psi] = & G_2(\varphi,X) - G_3(\varphi,X)\Box_g\varphi + G_4(\varphi,X)R \\ & + G_{4X}(\varphi,X)[(\Box_g\varphi)^2 - \varphi^{\mu\nu}\varphi_{\mu\nu}] \\ & + G_5(\varphi,X)G^{\mu\nu}\varphi_{\mu\nu} - \frac{1}{6}G_{5X}(\varphi,X)[(\Box_g\varphi)^3 \\ & - 3\Box_g\varphi\varphi^{\mu\nu} + 2\varphi_{\mu\nu}\varphi^{\mu\lambda}\varphi^{\nu}_{\lambda}] + L_{\rm matter}[g_{\mu\nu},\psi]\end{aligned}$$

But $\dfrac{c_{\rm GW}^2}{c^2} = \dfrac{G_4 - X(\ddot{\varphi}G_{5X} + G_{5\varphi})}{G_4 - 2XG_{4X} - X(H\dot{\varphi}G_{5X} - G_{5\varphi})}$

$$-3\times 10^{-15} < \frac{c_{\rm GW} - c}{c} < +7\times 10^{-16}$$

Naturalness of phenom-relevant GR deviations ???

Cosmological attractor
(TD-Polyakov, TD-Piazza-Veneziano)

$$\alpha_0^2 \lll 1$$

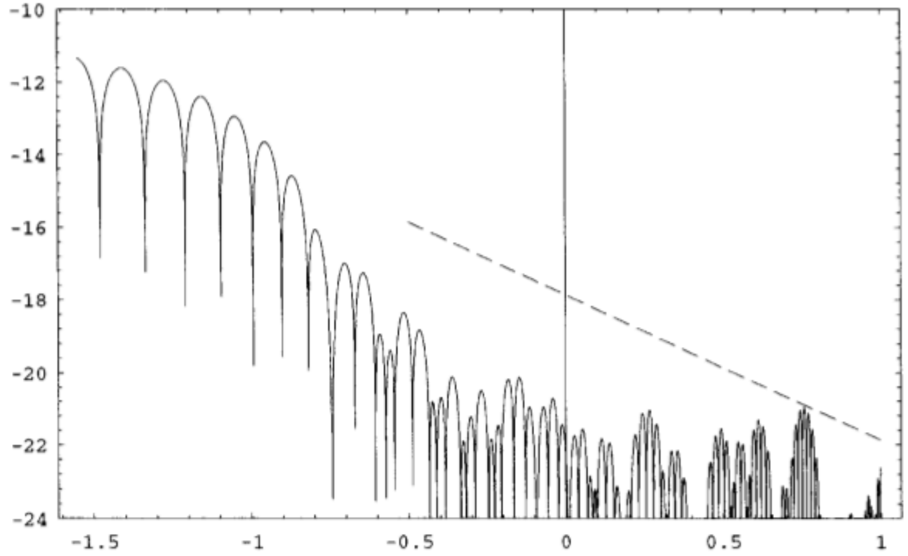


Fig. 3. The solid line represents $\log_{10}(\Delta a / a)_{\max}$ as a function of $\log_{10} \kappa$, i.e. the expected present level of violation of the equivalence principle (when comparing uranium with a light element) as a function of the curvature κ of the (string-loop induced) function $\ln B^{-1}(\phi)$ near a minimum ϕ_m. The dashed

Chameleon effect of V(phi)
(Khoury-Weltman)

O(10^-18) EP tests would be the best probes

Can one expect to see GR deviations in GW observations of BH coalescences ???

No-hair theorems in D=4 very much restrict possibilities

Few theories can predict hairy BHs. Interesting exceptions:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} (\nabla_\mu \phi)^2 + M_{\rm Pl} \alpha \phi \mathcal{R}_{\rm GB}^2 + M_{\rm Pl} \tilde{\alpha} \phi R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \right)$$

Length-squared coupling
$\alpha = \ell^2$

Gauss-Bonnet
$\mathcal{R}^2_{\rm GB} \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2$

Pontryagin

Need $\ell \sim 10 \text{ km}$

$(|\alpha| < 1 \text{ down to } 56\mu)$

for LIGO-observable deviations:
both through \alpha_eff=O(1)
and through QNM modifications
(Sotiriou…,Yagi…,Yunes…, Julié-Berti,…)

However:
classical causality PDE problems in strong fields (Pretorius…)
quantum causality constraints (Serra^2-Trincherini-Trombetta'22
à la Camanho-Edelstein-Maldacena-Zhiboedov + Caron-Huot, Arkani-Hamed…,Bern…,Bellazini,…)

$\ell \lesssim \ell_{EFT}$

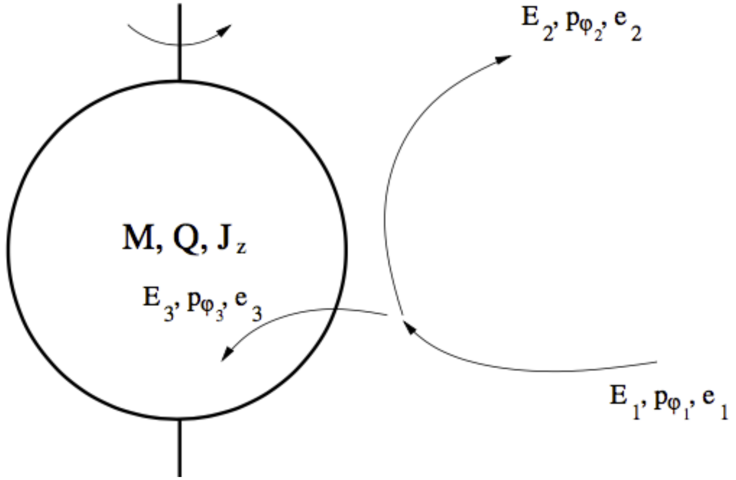
Phenom. tests still interesting notably BH Love #

Other possible new signals

Ultra-light bosonic fields
(e.g. Axion-like particles)
and BH superradiance

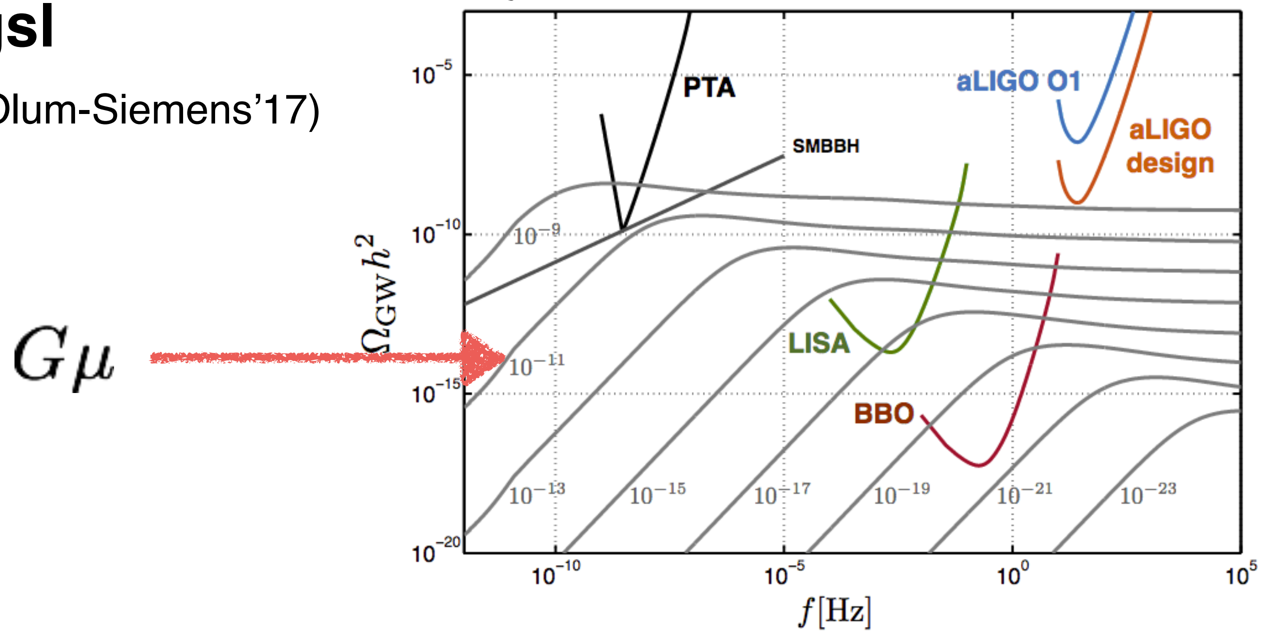
Penrose process

(review Berti-Cardoso-Pani'21)



Cosmic (super-)stringsl

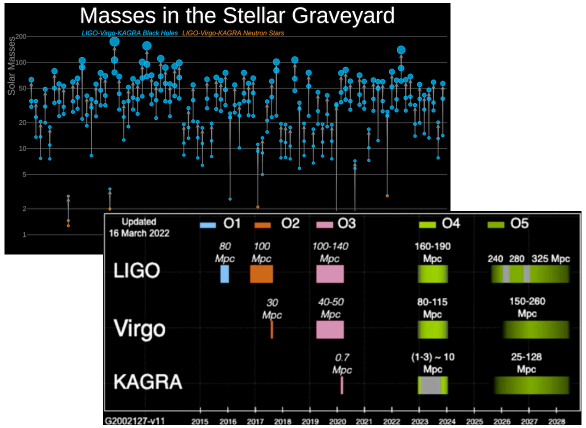
(TD-Vilenkin'00,…BlancoPillado-Olum-Siemens'17)

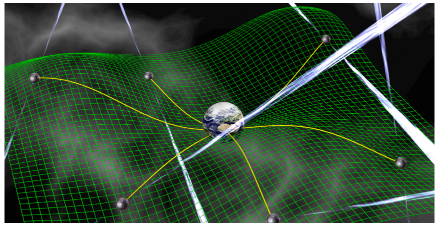


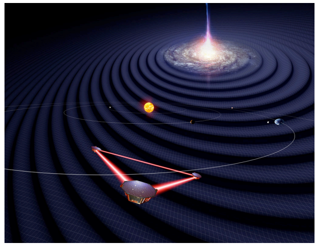
Quantum-generated **GWs from inflation in CMB** (B modes)

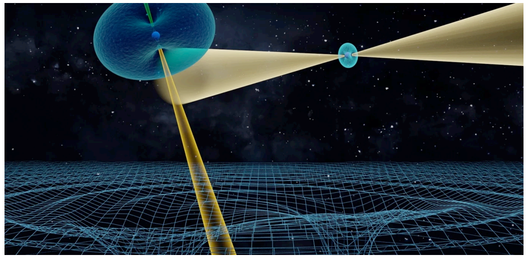
GWs from phase transitions

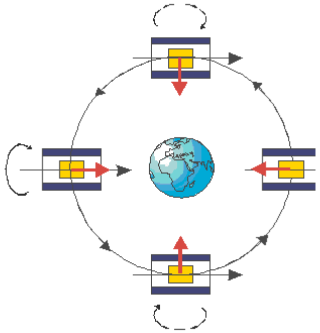
TAKE HOME IMAGES

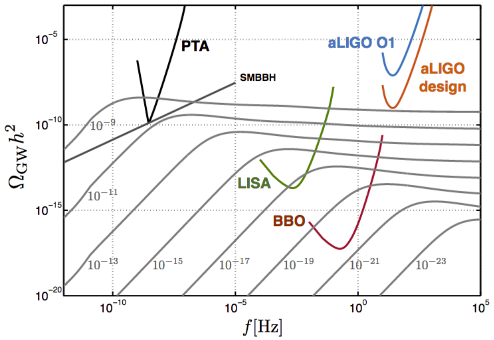


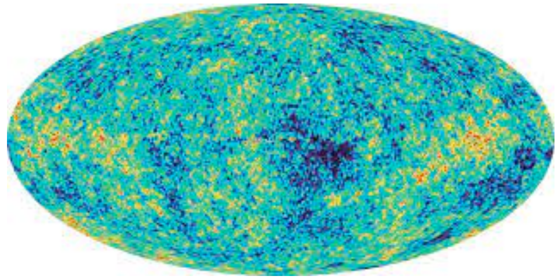


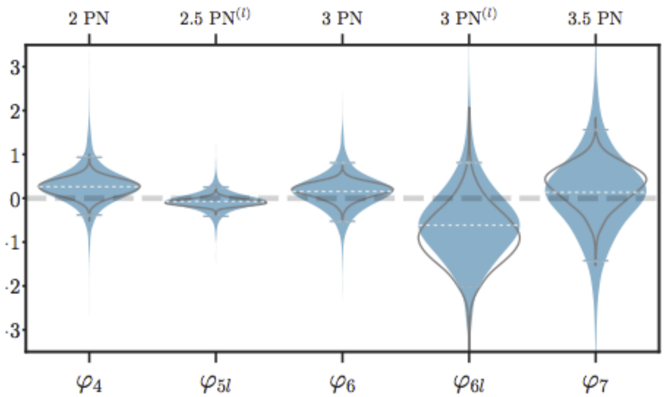


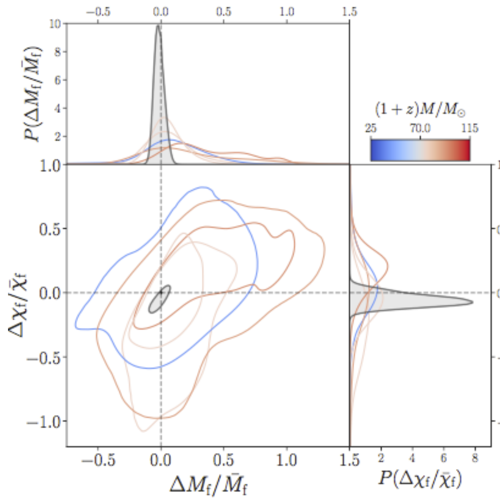












$$-3\times10^{-15} < \frac{c_{\rm GW}-c}{c} < +7\times10^{-16}\,.$$

$$\mathcal{L}_{\rm tot}[g_{\mu\nu},\varphi,\psi,A_\mu,H] = \frac{c^4}{16\pi G_*}\sqrt{g}\left(R(g_{\mu\nu})-2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) - \sqrt{g}V(\varphi)+\mathcal{L}_{\rm SM}[\psi,A_\mu,H,\tilde{g}_{\mu\nu}]\,.$$

$$\mathcal{L}_{{\rm int}\phi} = \kappa\phi\left[+\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu}-\frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=e,u,d}(d_{m_i}+\gamma_{m_i}d_g)m_i\bar{\psi}_i\psi_i\right].$$

$$S=\int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R-\frac{1}{2}(\nabla_\mu\phi)^2+M_{\rm Pl}\alpha\phi\mathcal{R}^2_{\rm GB}+M_{\rm Pl}\tilde{\alpha}\phi R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\right)$$