

**An assessment of the failures and optimization in the
computation of asteroid synthetic proper elements**

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Tides and motions in planetary systems

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Synthetic proper elements of asteroids (> 1M) available from:

AstDyS2:

<https://newton.spacedys.com/astdys2/index.php?pc=5>

AFP:

<http://asteroids.matf.bg.ac.rs/fam/properelements.php>

PDS A&D subnode:

https://sbnarchive.psi.edu/pds4/non_mission/ast.nesvorny.families_V2_0/data/

Using basically the same software: Orbitfit (orb507) – different integration time spans and dynamical models

Knežević and Milani, Synthetic proper elements for outer main belt asteroids, CMDA, 78, 17–46, 2000

Synthetic method to compute proper elements:

- Numerical integration of orbits;
 - digital filtering of short-periodic perturbations: mean elements
proper semimajor axis
- Fourier analysis:
 - removal of forced perturbations (Ferraz-Mello, 1981);
 - computation of frequencies from time series of slow angles;
 - extraction of proper modes from time series of equinoctial elements:
proper eccentricity and (sine of) proper inclination

Output:

Numerical integration:

vast.dat/fil – osculating/filtered elements for asteroids

vpla.dat/fil – osculating/filtered elements for planets

Fourier analysis:

vprs.pro - proper elements

vprs.sig – standard deviations

vprs.del – maximum excursions

vprs.ang – proper angles

vprs.err - error in elements (NaN in LCE)

vprs.out – auxiliary data

Failures in computation of proper elements

Errors in numerical integration – model and time span dependent:

- error in elements - e.g. hyperbolic orbits ($a > 100$ au): discarded, no proper elements produced;
- too short integration step – asteroid removed from integration;

Errors manifested in proper elements computation:

- unreasonable semimajor axis and large errors of proper elements
- Good proper elements but format problems (****) in the output due to too large errors; if in vprs.sig & vprs.del: asteroid removed

Partial failures:

- failure of the computation of variational equations – NaN in LCE
- proper elements and LCE acceptable but **** in vprs.del
asteroid accepted

Statistics: numbered asteroids

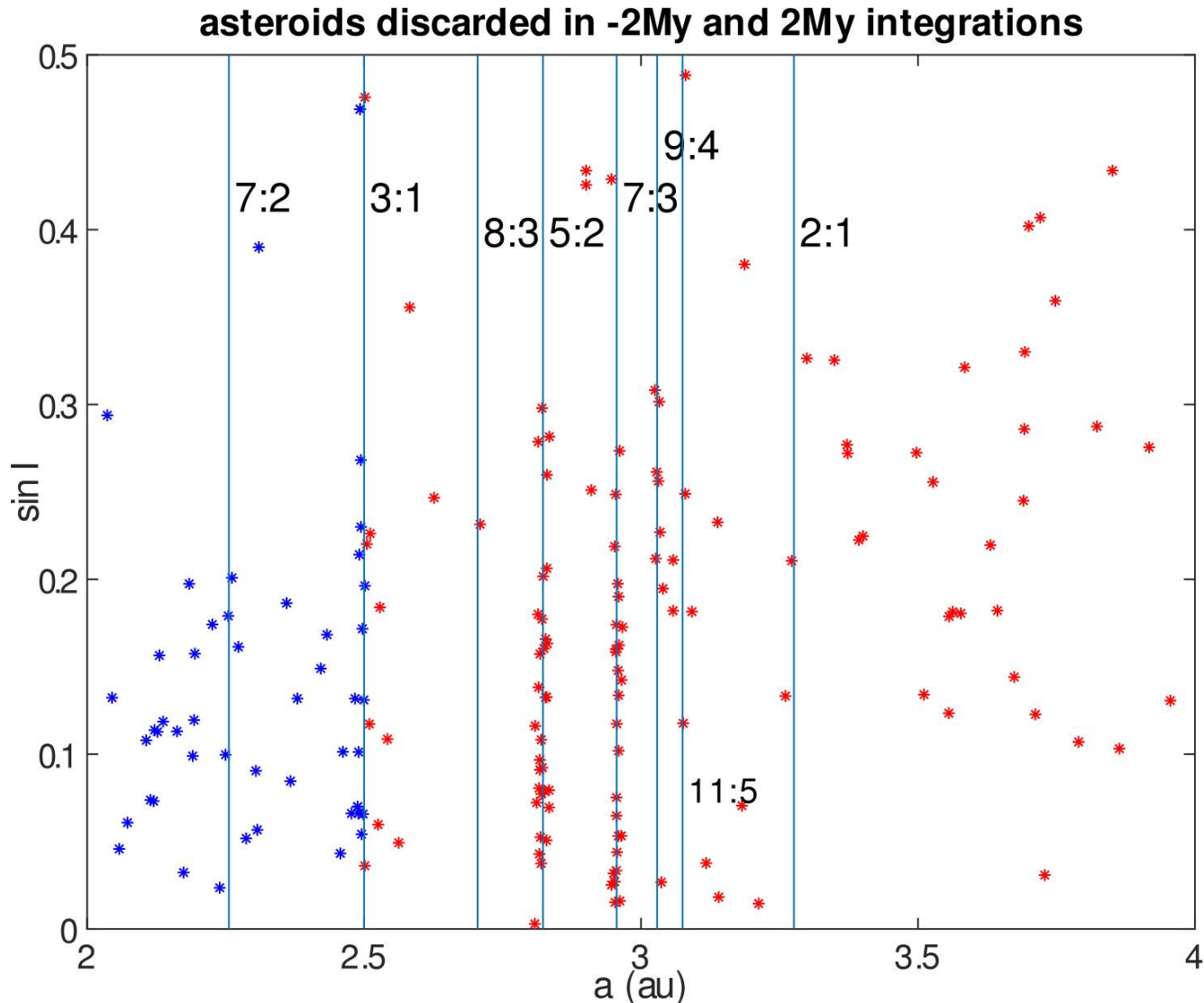
Total of 390992 asteroids (185655 till April 2008 + 295347 after June 2017)

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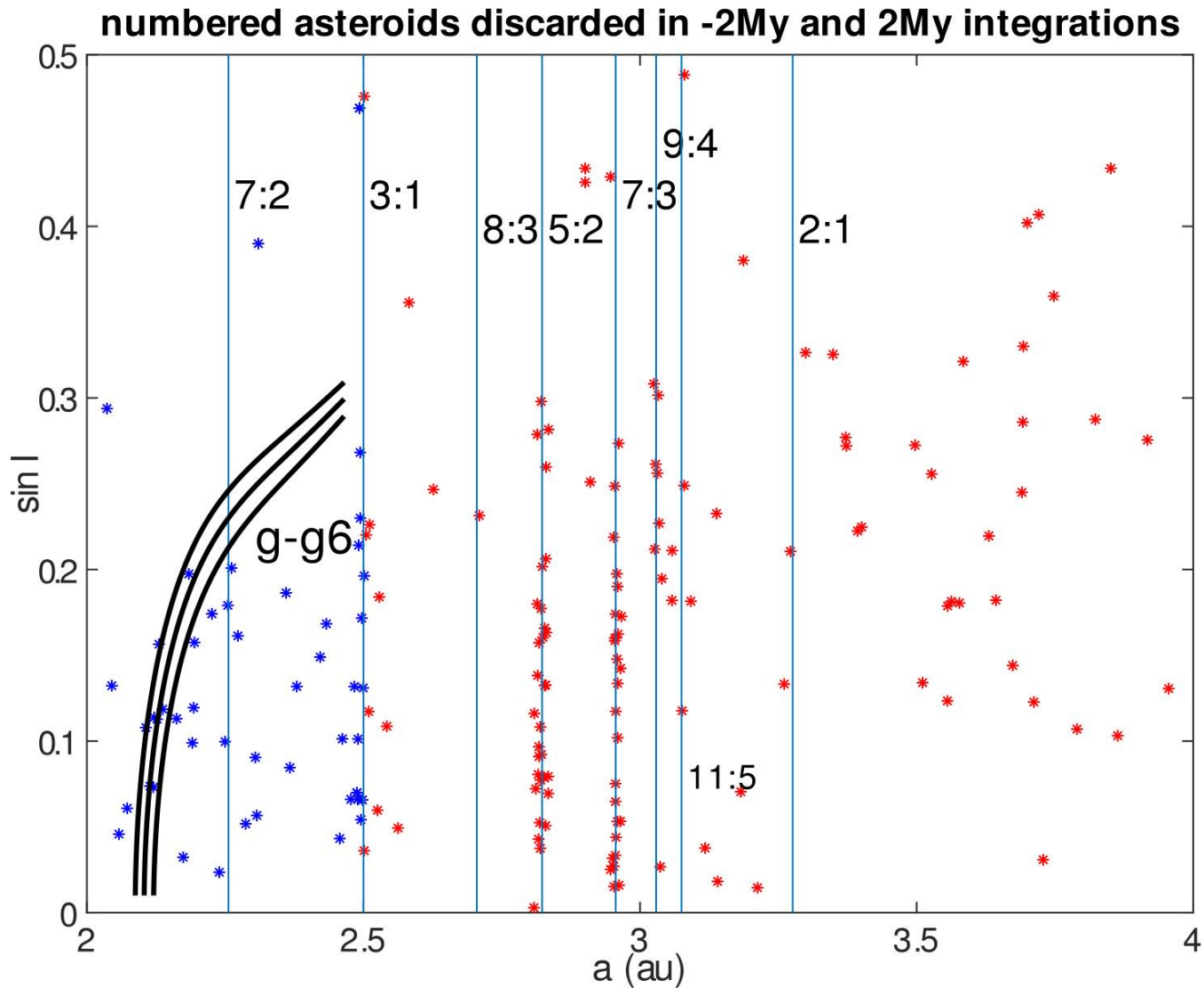
130 outer belt discarded in -2 My integration (4 for format problems)
499 outer belt discarded in 10 My integration (15 for format problems)
54 inner belt discarded in +-1My (2) integration (5 for format problems)

91 outer belt with LCE set to 999.99 in -2 My integration
384 outer belt with LCE set to 999.99 in 10 My integration
49 inner belt with LCE set to 999.99 in +-1 My integration

Overview of the Main Belt (numbered asteroids)

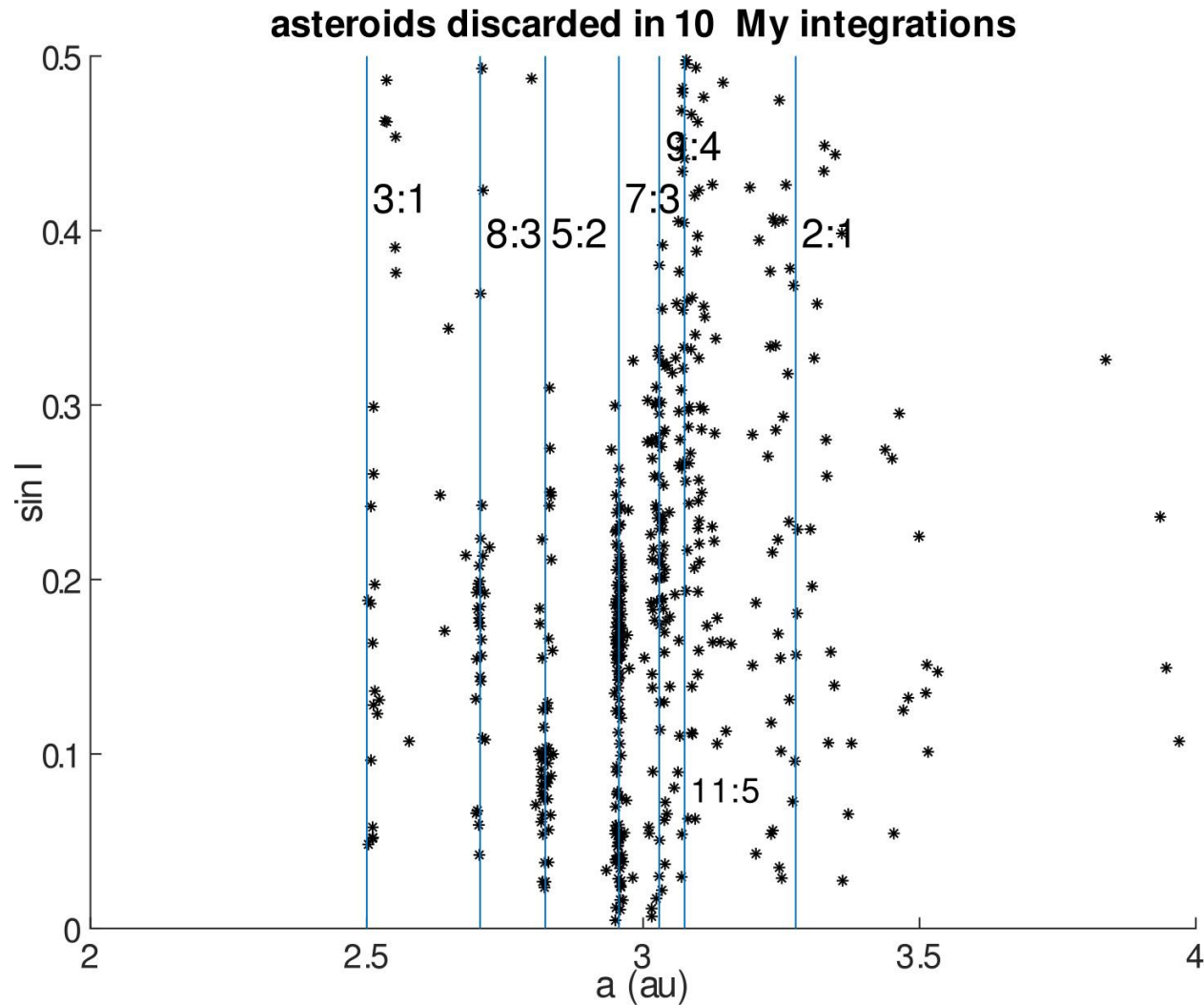


Overview of the Main Belt (numbered asteroids)



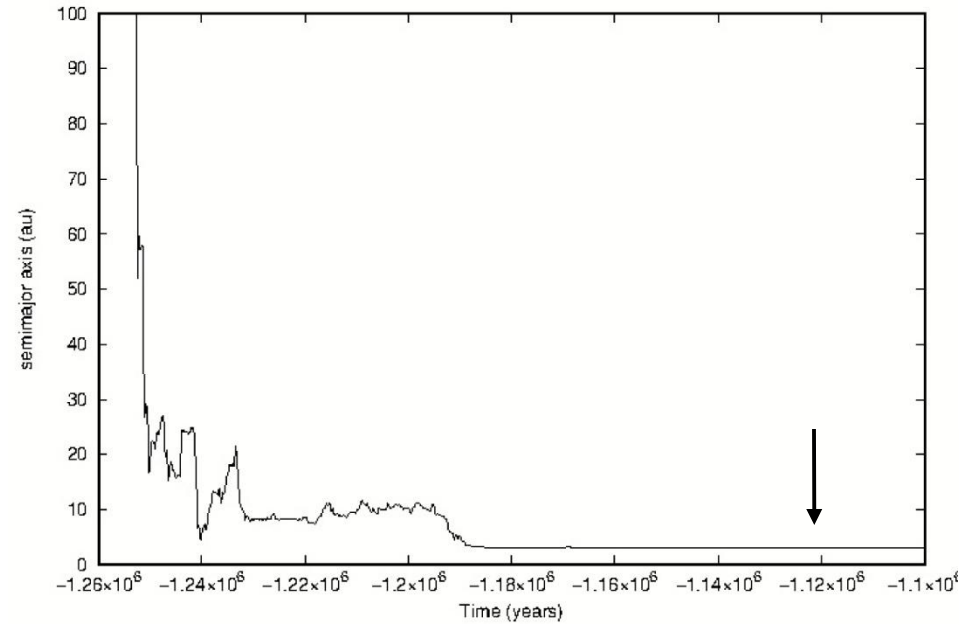
SFM et al,
1998

Overview of the Main Belt (numbered asteroids)



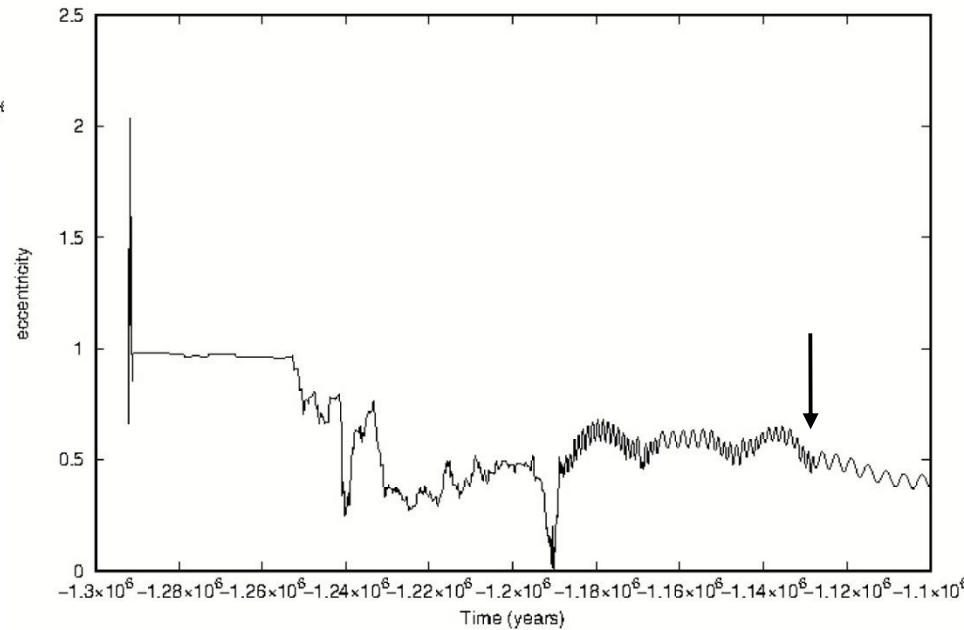
Examples: (494895) 2008UA53

494895

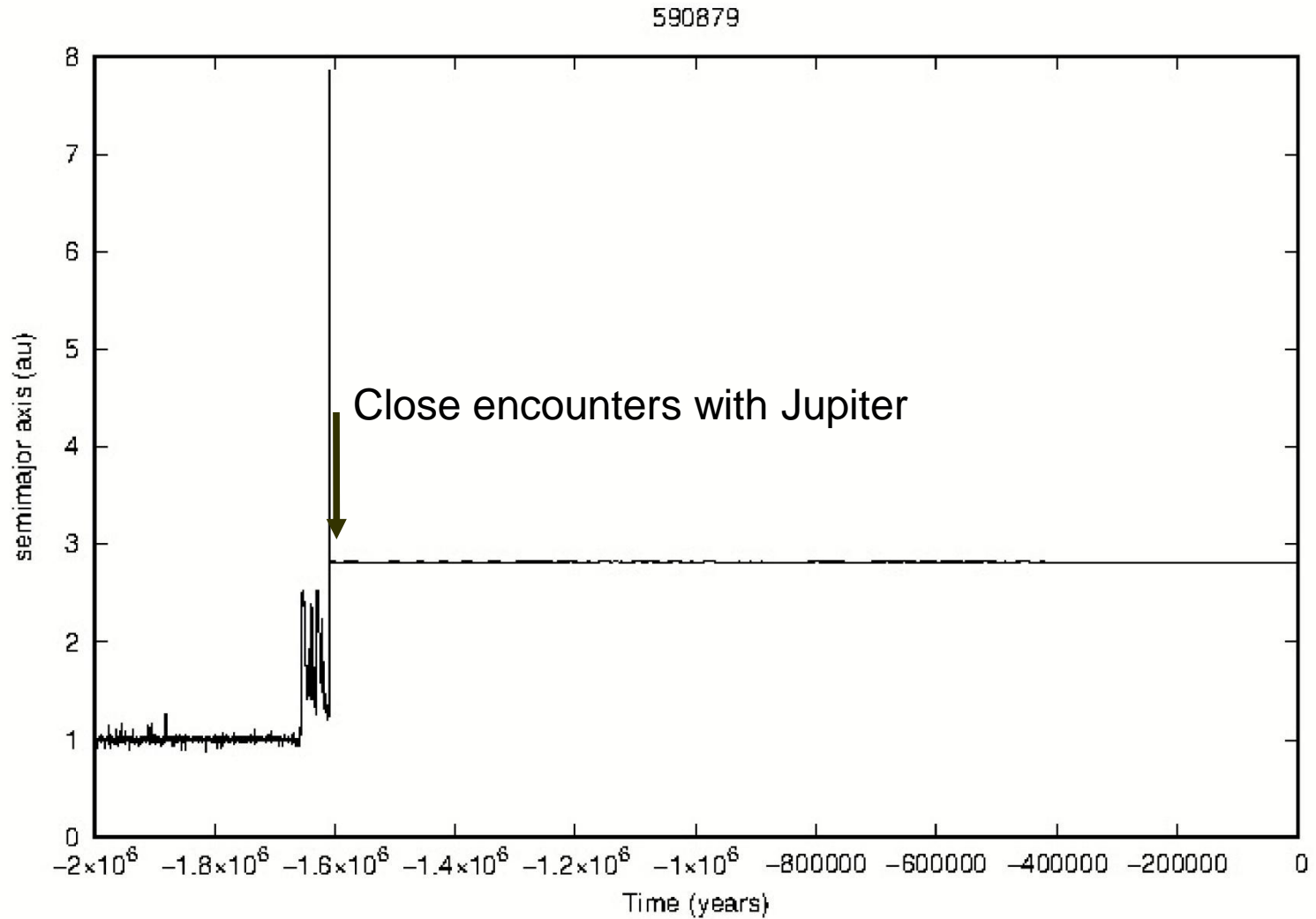


Onset of close encounters with
Jupiter at -1.127 My
(shown by arrow)

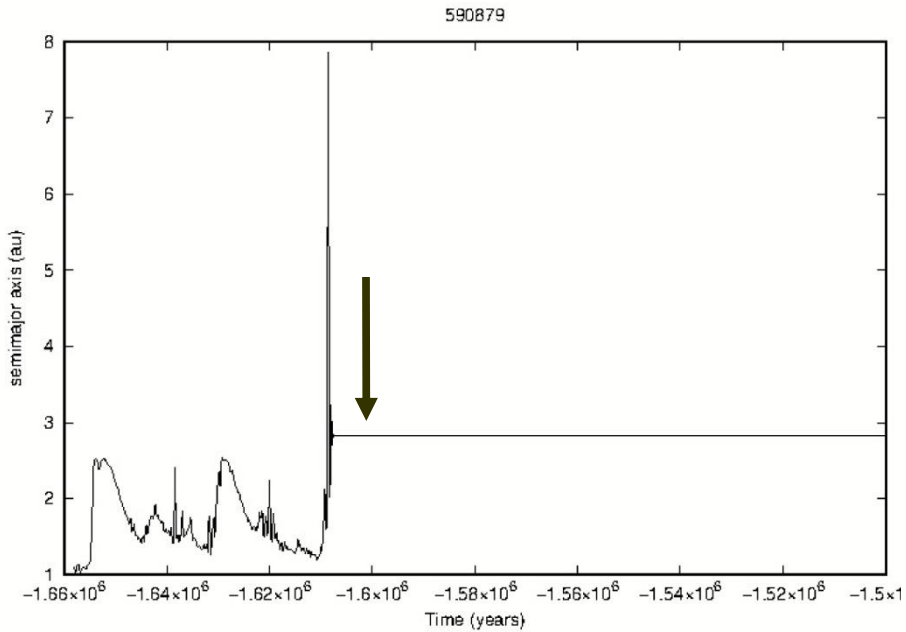
494895



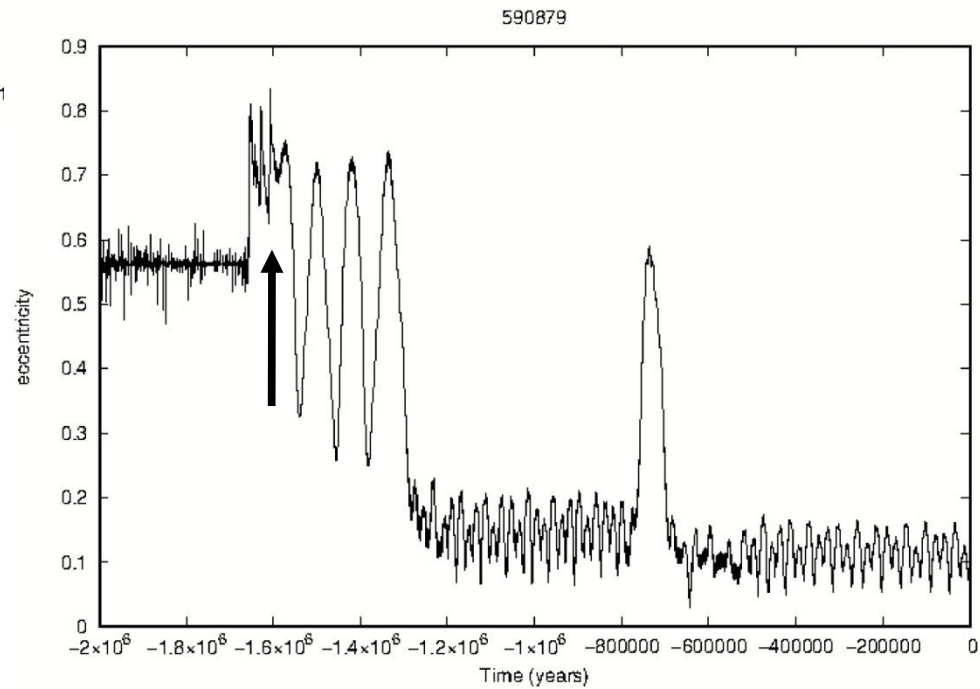
Examples: (590879) 2012VK14

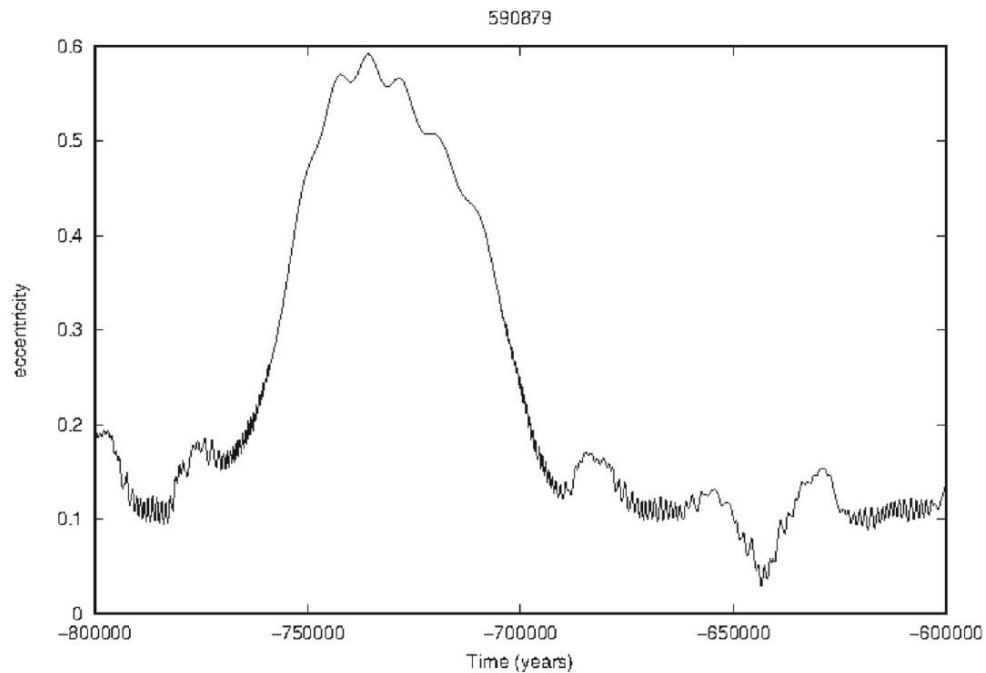
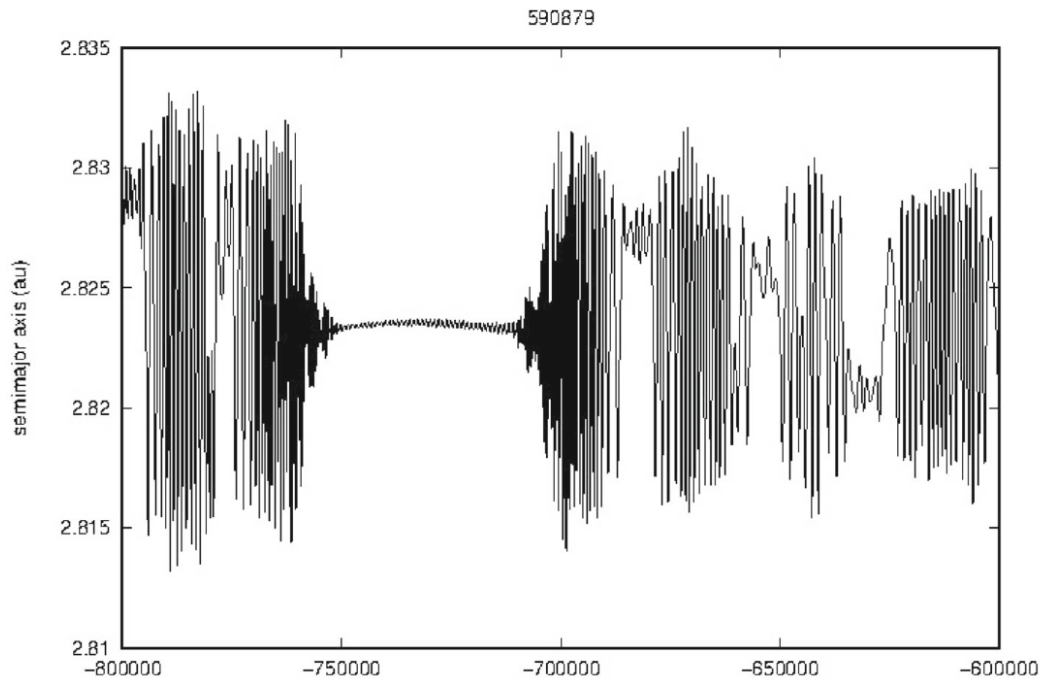


Examples: (590879) 2012VK14



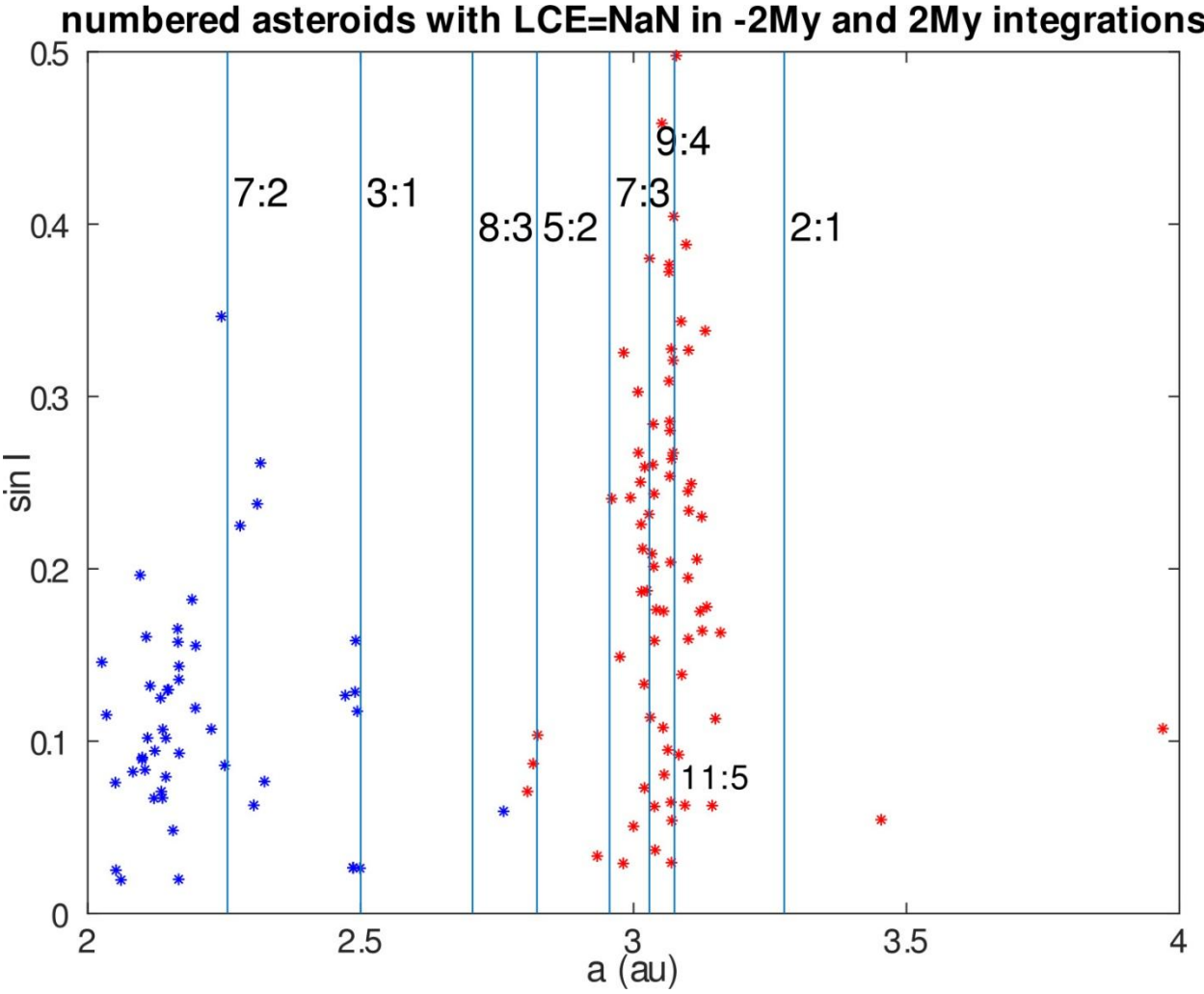
Onset of close encounters with
Jupiter at -1.608 My
(shown by arrow)



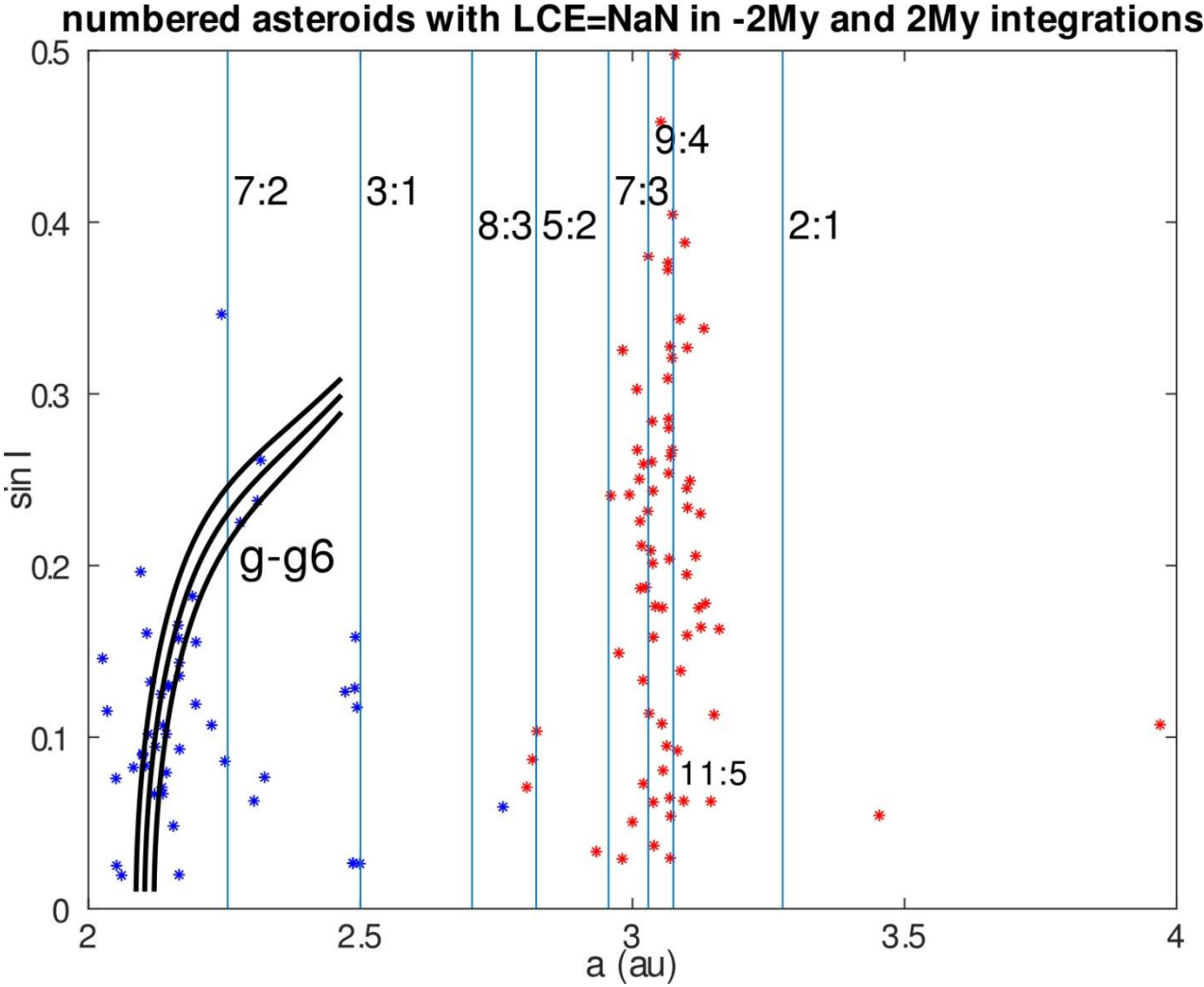


Asteroid
(590879)
locked in
5:2 MMR

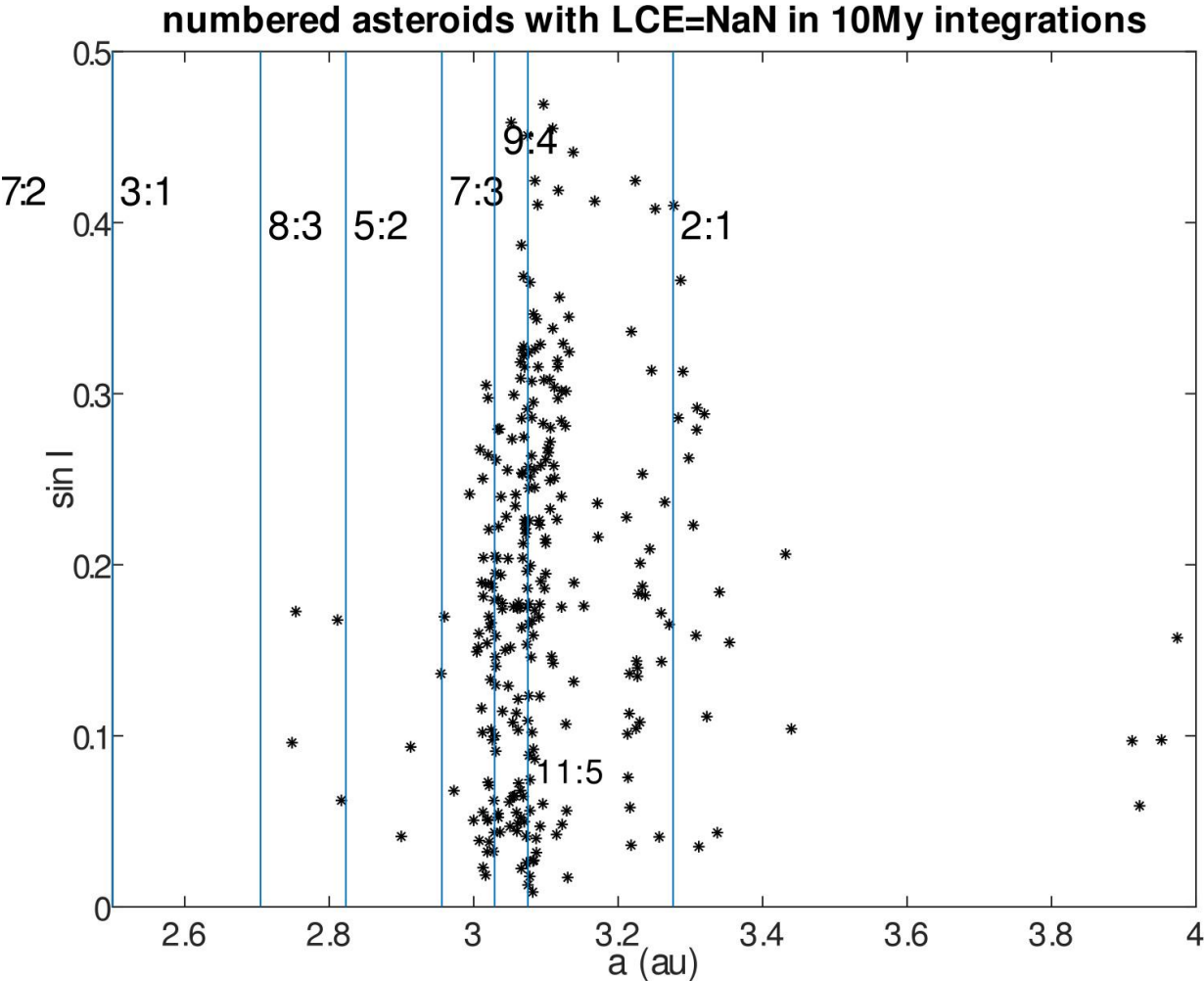
Overview of the Main Belt (numbered asteroids)



Overview of the Main Belt (numbered asteroids)

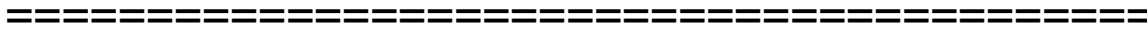


Overview of the Main Belt (numbered asteroids)



Statistics: multiopposition asteroids

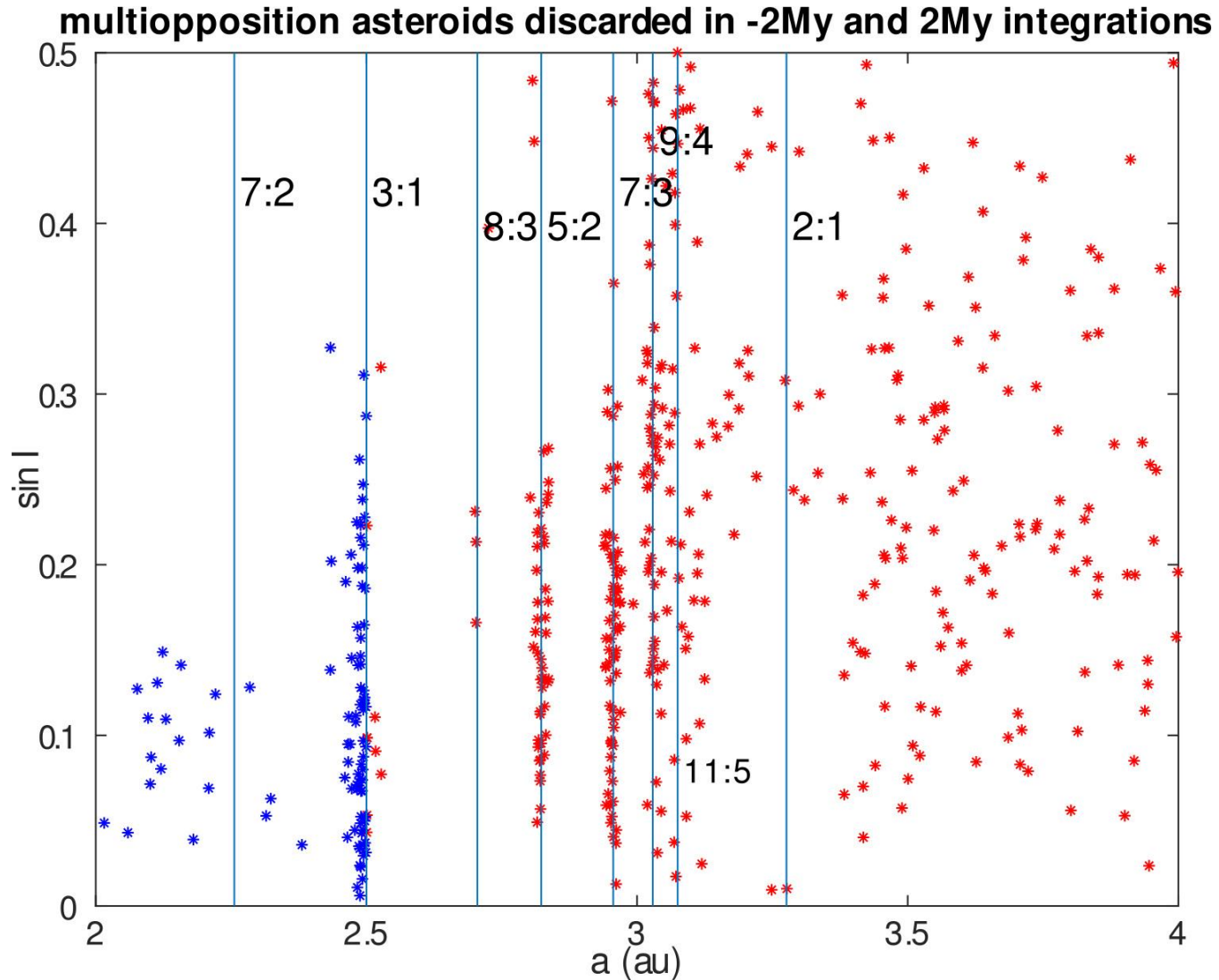
Total of 528913 asteroids (as of June2024)



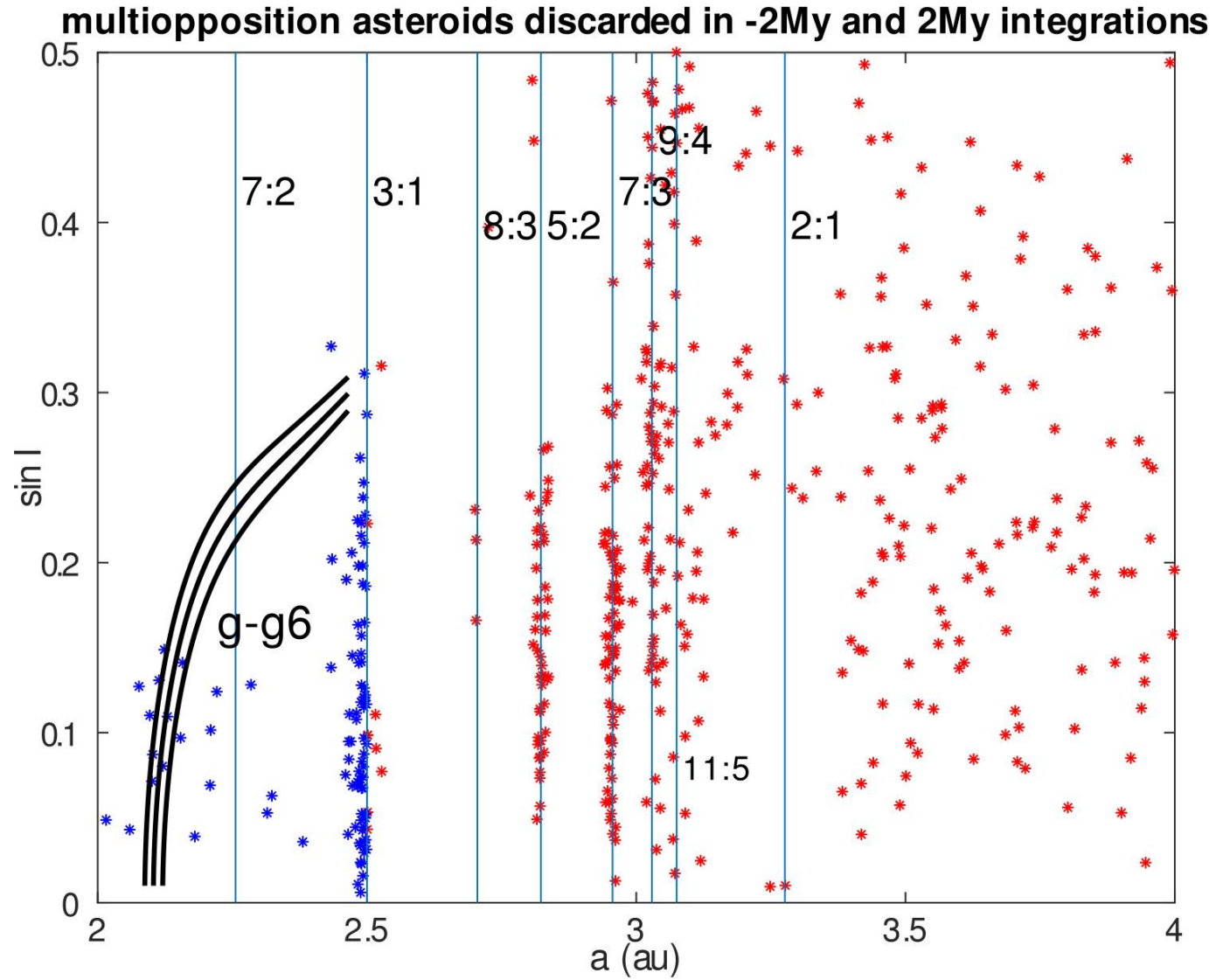
404 outer belt discarded in -2 My integration (16 for format problems)
1441 outer belt discarded in 10 My integration (20 for format problems)
99 inner belt discarded in +-1My (2) integration (1 for format problems)

330 outer belt with LCE set to 999.99 in -2 My integration
975 outer belt with LCE set to 999.99 in 10 My integration
115 inner belt with LCE set to 999.99 in +-1 My integration

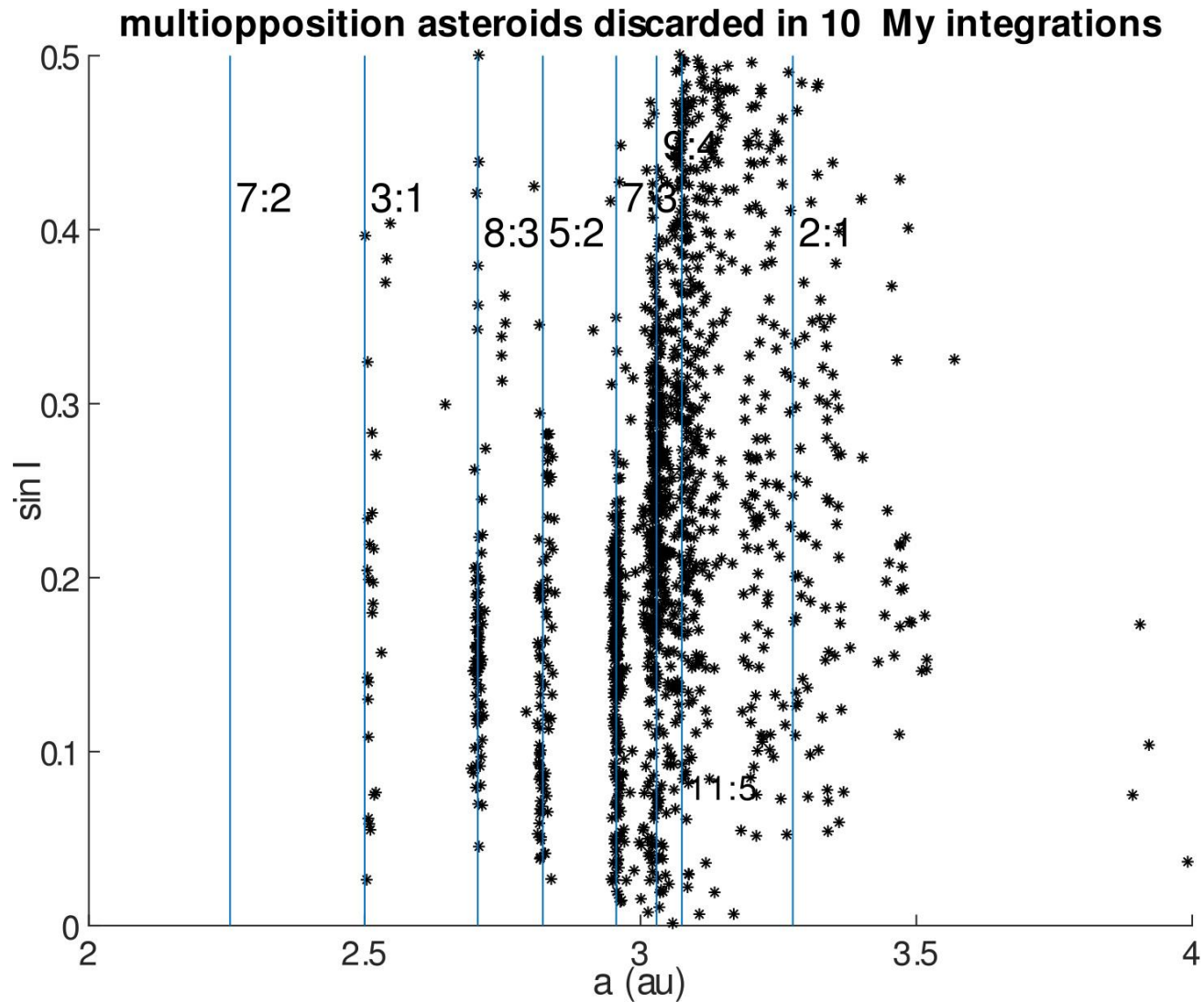
Overview of the Main Belt (multiopposition asteroids)



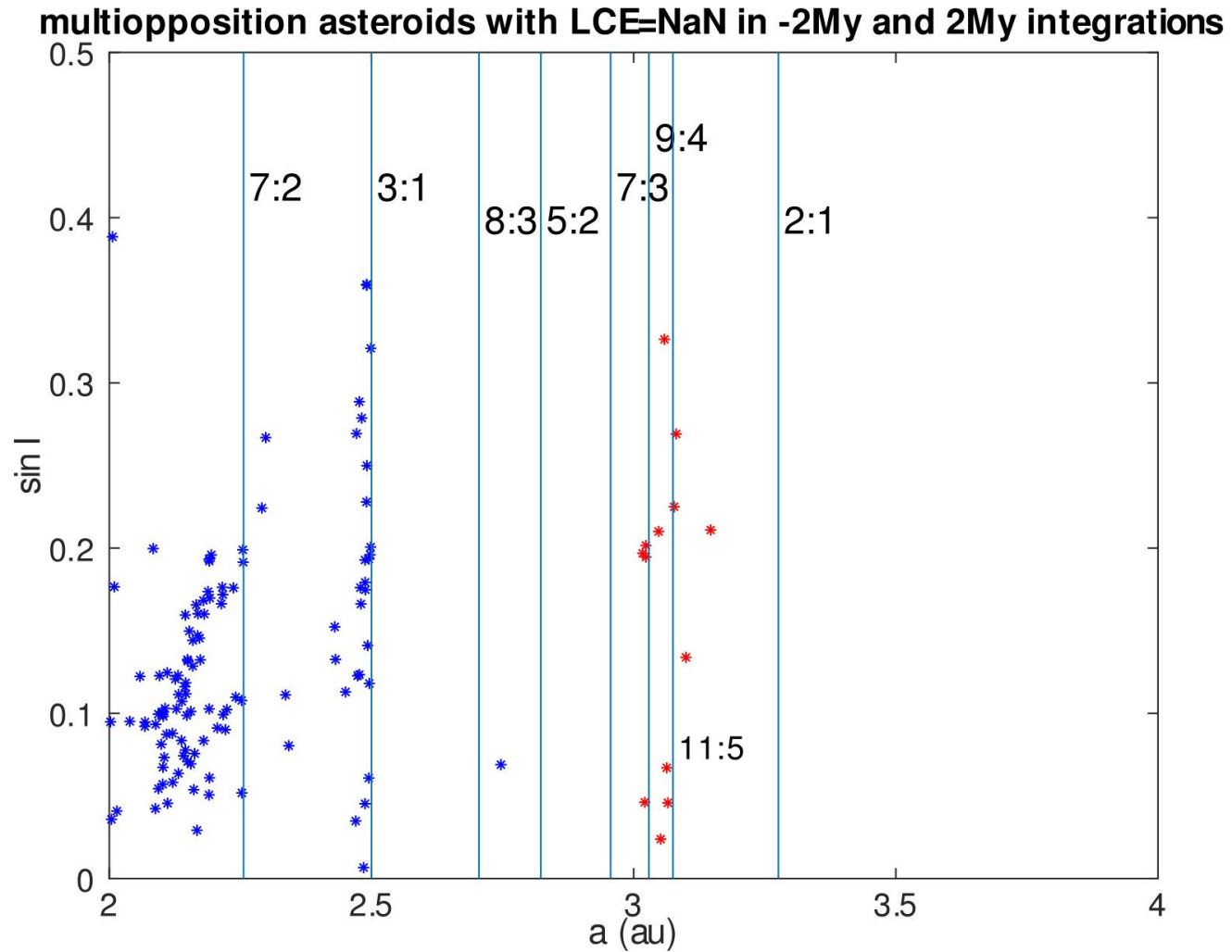
Overview of the Main Belt (multiopposition asteroids)



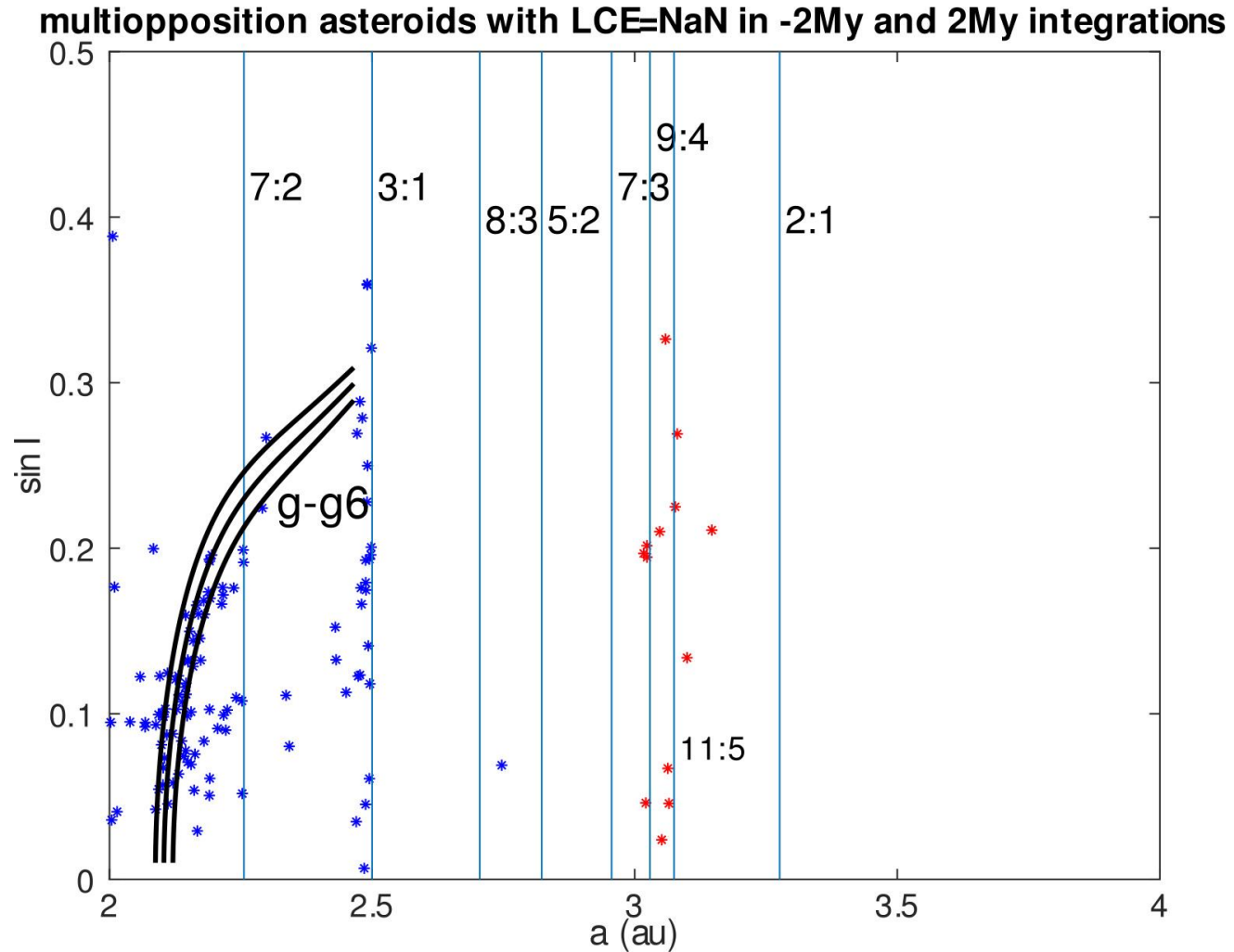
Overview of the Main Belt (multiopposition asteroids)



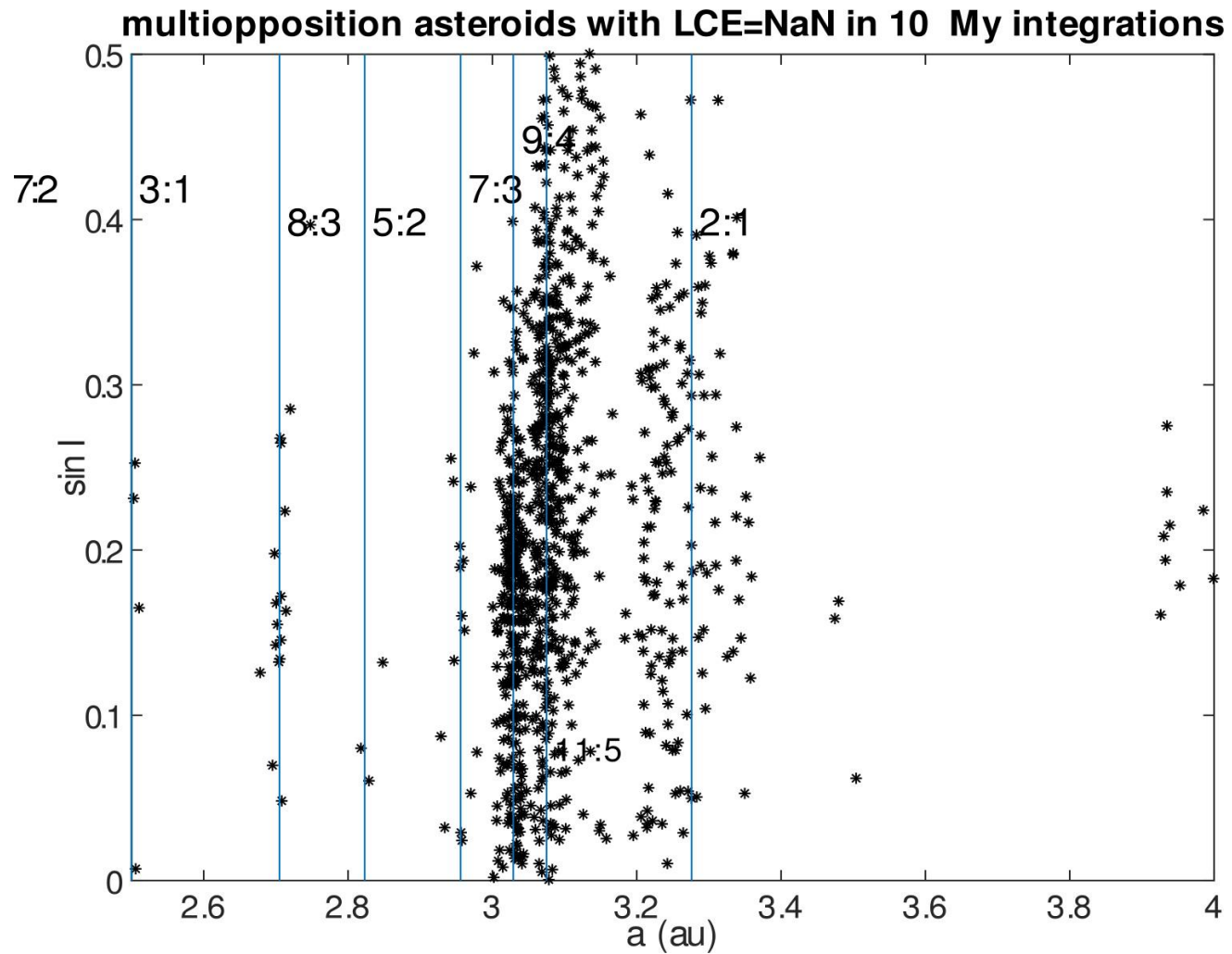
Overview of the Main Belt (multiopposition asteroids)



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Overview of the Main Belt (multiopposition asteroids)



Optimization of computation of proper elements: the time span of orbit propagation and complexness of the dynamical model

AstDyS (orbfit):

outer belt, Trojans 5/50My, TNOs 100My;

dynamical model with 4 outer planets (barycentric correction)

Hungaria -2/10My, inner belt ± 1 My,

with 7 planets (except Mercury, barycentric correction)

AFP (orbfit):

all for 10 My

all with 7 planets (as in AstDyS)

PDS (orbfit like):

all for 10 My

all with 8 planets, largest asteroids, relativistic correction

Accuracy of proper elements: the average standard deviations of proper elements and frequencies in the AstDyS and AFP data bases

Data	σa (au)	σe	$\sigma \sin I$
AstDys all	0.000174	0.001429	0.000454
AFP all	0.000320	0.001620	0.000600
AstDyS ± 1 My	0.000190	0.000693	0.000456
AstDys -2 My	0.000062	0.000391	0.000144
AstDys 10 My	0.000544	0.006399	0.001546

σg ("/y)	σs ("/y)
0.2415	0.1645
0.2963	0.1747
0.0387	0.0428
0.0782	0.0365
1.1779	0.8325

10000 asteroids Nos. 494657 - 509585

$2.5 < a < 3.3; T_L > 10^5 \text{ y} \rightarrow 7450/7448$

Integration for -2 My with 4 planets

Integrated for 10 My with 4 planets
and for -2 My with 7 planets

Proper elements computed using short (−2 My) and long (10 My) integration time span, and the dynamical models including either 4 or 7 planets

Asteroid	a (au)	e	$\sin I$	n (deg/y)	g ("/y)	s ("/y)	My	pl
494657	2.720173	0.247555	0.158558	80.22406	53.82266	−71.25520	−2	4
494657	2.720174	0.247899	0.158592	80.22399	53.83364	−71.15010	10	4
494657	2.720179	0.249468	0.158464	80.22447	54.08581	−71.62373	−2	7
494662	2.590734	0.221024	0.061548	86.31341	53.18891	−61.00469	−2	4
494662	2.590727	0.240721	0.061816	86.31335	53.45665	−63.47093	10	4
494662	2.590741	0.225743	0.061585	86.31384	53.50577	−61.55593	−2	7

2 My vs. 10 My

Mean	-2 My	10 My	$\sigma_{-2} > \sigma_{10}$	$\sigma_{-2} < \sigma_{10}$
$\bar{\sigma}_a$	0.0000020	0.0000081	2763(37.1%)	4685(62.9%)
$\bar{\sigma}_e$	0.0019252	0.0019915	2884(38.7%)	4564(61.3%)
$\bar{\sigma}_{sI}$	0.0001811	0.0003102	5367(72.1%)	2081(27.9%)
$\bar{\sigma}_g$	0.1432	0.1349	2001(26.9%)	5447(73.1%)
$\bar{\sigma}_s$	0.1415	0.1681	1914(25.7%)	5534(74.3%)

4 planets vs. 7 planets

Mean	4 planets	7 planets	$\sigma_4 > \sigma_7$	$\sigma_4 < \sigma_7$
$\bar{\sigma}_a$	0.0000020	0.0000125	3085(41.4%)	4365(58.6%)
$\bar{\sigma}_e$	0.0019252	0.0017843	3948(53.0%)	3502(47.0%)
$\bar{\sigma}_{sI}$	0.0001811	0.0001900	3376(45.3%)	4074(54.7%)
$\bar{\sigma}_g$	0.1432	0.1588	4295(57.6%)	3155(42.4%)
$\bar{\sigma}_s$	0.1415	0.1276	4541(61.0%)	2909(39.0%)

Conclusions

- different types of failures identified;
 - hyperbolic orbits occur for 0.3% asteroids over 10My;
 - failures distributed along strong MMRs in the asteroid belt;
 - typically, close approaches to Jupiter give rise to instabilities;
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- ▶ proper elements obtained in different runs are nearly the same;
 - ▶ no significant improvement of the overall accuracy of results when using the longer integration or more complex dynamical model;
 - ▶ from the point of view of the application of these elements in classification of asteroids into families, they can be considered as practically equal.

Thank you !