Celestial Mechanics and rings

Bruno Sicardy, Paris Observatory, LTE

Conference in honour of Sylvio Ferraz-Mello
Paris Observatory, April 8-10, 2026

1 min

creeping tide in Normandy…

EXTRAIT DES SERVICES

Nom et prénoms : SICARDY, Bruno — N° d'immatriculation au recrutement : 78.060.10689
né le 29.03.1958 à Pont de Beauvoisin (Isère)

DÉTAIL DES SERVICES ET MUTATIONS DIVERSES

SERVICES ET MUTATIONS	DATE	RÉFÉRENCE
Report d'incorporation accordé jusqu'à 22 ans		
NOMMÉ élève de l'École Polytechnique par arrêté du 02/08/78. J.O. du ... En tant qu'élève officier de Réserve Sursis en Situation d'activité à compter du	16/08/78	élève officier 16 août 1978
...	01.09.78	
SERVICE COMPTANT du	01.9.78	
Rayé des contrôles le	19.09.78	... Polytechnique ... lettre ... de 16/09/78
Remis Report		rayé des contrôles 16 septembre 1978
A participé aux opérations de sélections du 18 au 20.12.79		
CLASSE Apte par la commission locale d'Aptitude de	21.1.80	
REPORT EXPIRÉ LE	29.08.80	
Maintenu en report d'incorporation jusqu'au	30.09.80	
Admis au bénéfice des dispositions de l'Art. L 9 du Code Service National Transfére pour Administration au Bureau du Service National de LYON, le	1 AOUT 1980	
Astreint à effectuer 16 mois de service actif		
Incorporé au titre de la coopération le	05.12.83	
pour être mis à la disposition Bureau Commun Service National Coopération		
Affecté au Bureau Commun SN Coop	05.12.83	
A rejoint le :		
Services comptant du	01.12.83	
Affecté en qualité de chercheur en Astrophysique à Rio (Brésil)		
Mis en route par VA le	27.12.83	débarqué à Rio
Débarqué à RIO le	28.12.83	le 28 décembre 1983
Rejoint son Poste le d.t jour		
Rapatrié pour fin de séjour par VA		
A subi la visite Médicale de libération le	04.12.84	
Rayé des contrôles du Service de la Coopération et Libéré du Service National actif par le Ministère des Relations Extérieures le	01.04.85	rapatrié le 4 avril 1985

DATE DE PASSAGE DANS LA		DATE DE LIBÉRATION DÉFINITIVE DU	
DISPONIBILITÉ	RÉSERVE	SERVICE MILITAIRE	SERVICE NATIONAL

DOMICILES OU RÉSIDENCES SUCCESSIFS

DATE	COMMUNE et département	GENDARMERIE	1 Domicile 2 Résidence

CAMPAGNES

DU	AU	TAUX	DÉSIGNATION
NÉANT			

citations et récompenses: néant

CITATIONS ET RÉCOMPENSES

Blessures: néant

BLESSURES (1)

Certifié exact le 3 avril 2014
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par ordre
l'adjoint administratif principal de 1ère classe
Denise NIGEOU – rédacteur

DÉCORATIONS

décorations: néant

(1) Indiquer la date, le lieu, la nature de la blessure et l'agent vulnérant. Mentionner "blessure de guerre" ou "service commandé"

Code 600.40.21

VEDERE Serge (IDENT) 06/10/2008

- Officer cadet, Polytechnique school, 16 August 1978
- Resigned from Polytechnique, barred from the controls 19 September 1978
- Affected as a researcher in Astrophysics at Rio (Brazil)
- Carried by Air Transport : 27 December 1983
- Debarked at Rio: 28 December 1983
- Reached his post the same day
- Rapatriated by Air Transport: 4 April 1985
- Has undergone the Liberation Medical visit

citations and awards: none

wounds: none

medals: none

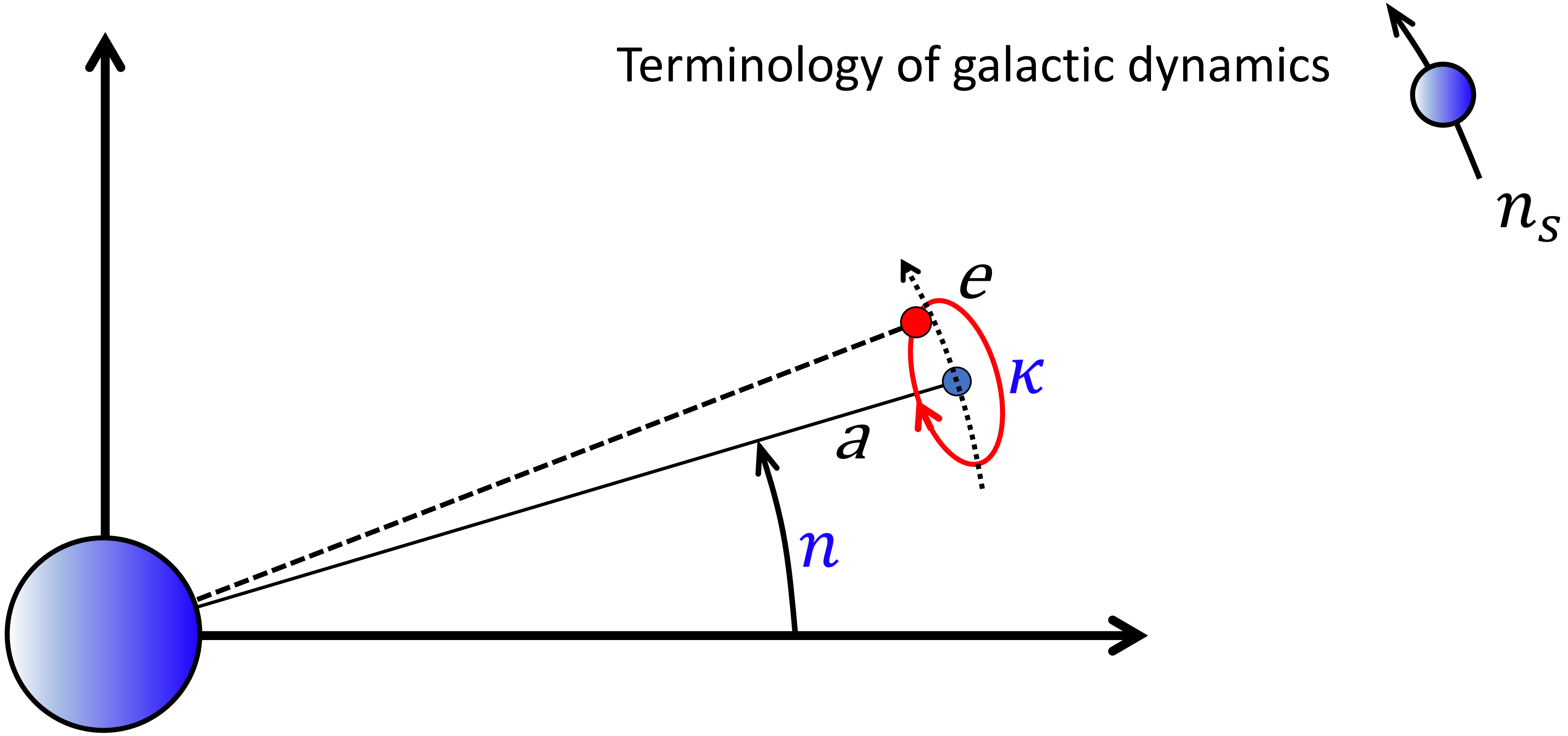
desembarque Brasil
Divinópolis, MG
maio 1982

Sylvio Ferraz Mello
IAG São Paulo

(everything you want
to know about resonances)

Question: what is a resonance?

Terminology of galactic dynamics



Planar restricted 3-body problem:

2-degree of freedom system →
2 fundamental frequencies, n (mean motion) and κ (epicyclic)

Lindblad resonances

Simple if problem circular, restricting ourselves to the radial degree of freedom (associated with κ)

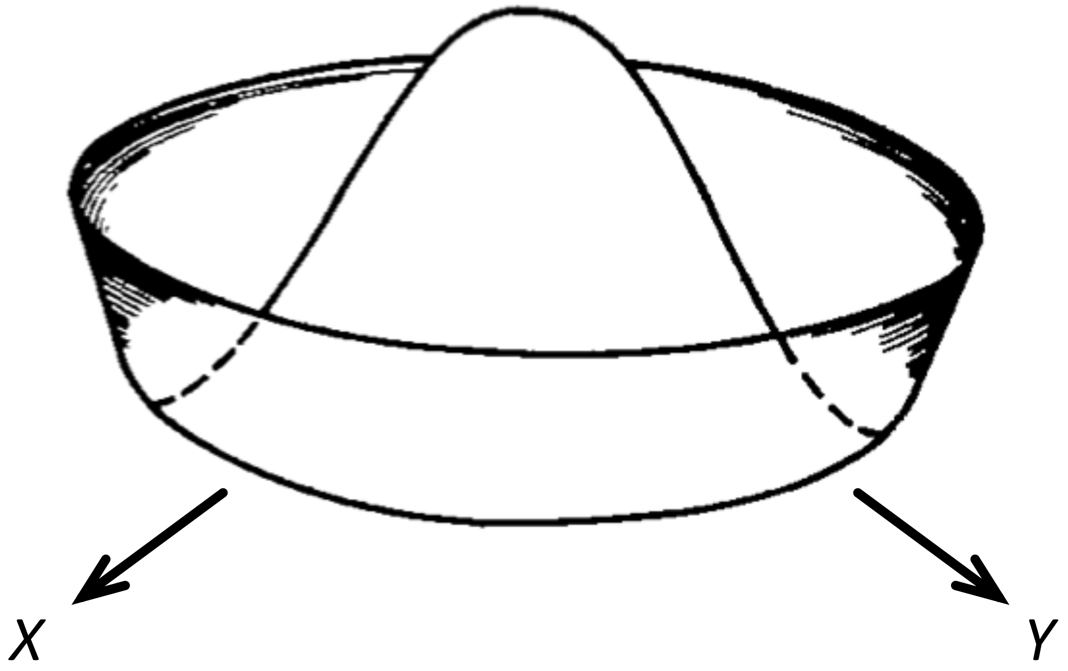
→ integrable

$$\phi = m\lambda_s - (m-1)\lambda - \varpi$$

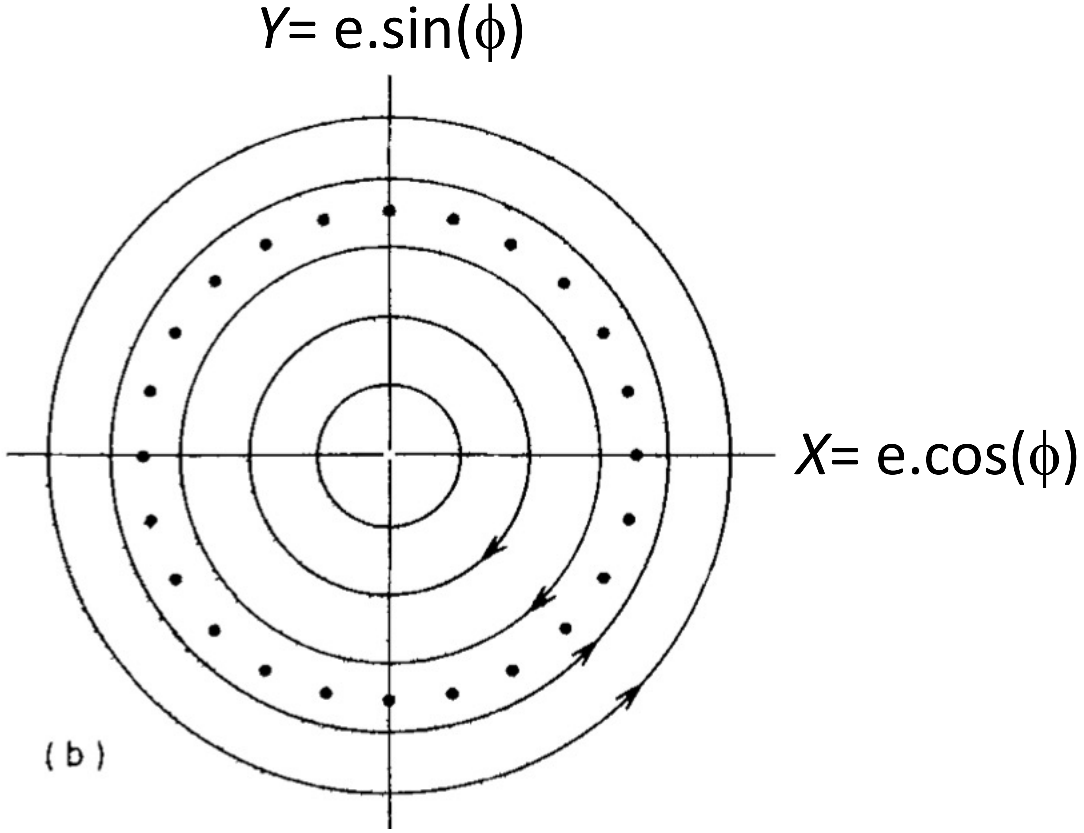
(called Lindblad resonances in ring dynamics)

A nice geometrical view of a 1st-order resonance has been provided by...

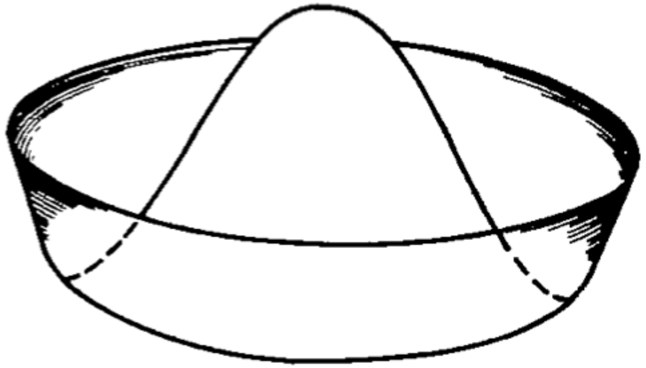
…"A morphogenic analysis of the orbits in the case of a first-order resonances"
(Ferraz-Mello, *CelMec* 1985)



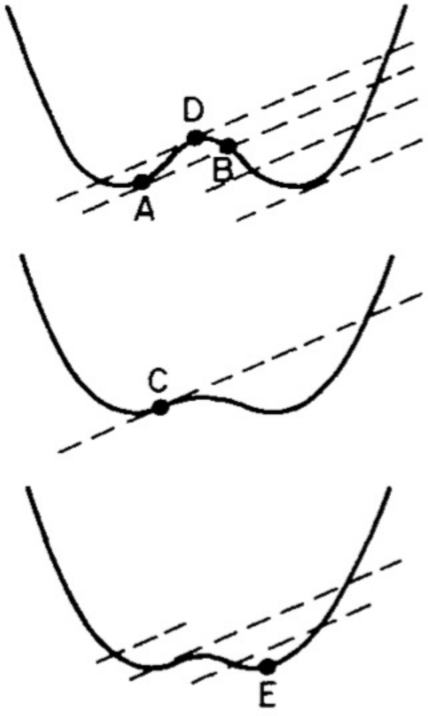
"Mexican hat" part of the Hamiltonian $\mathcal{H}(X,Y)$



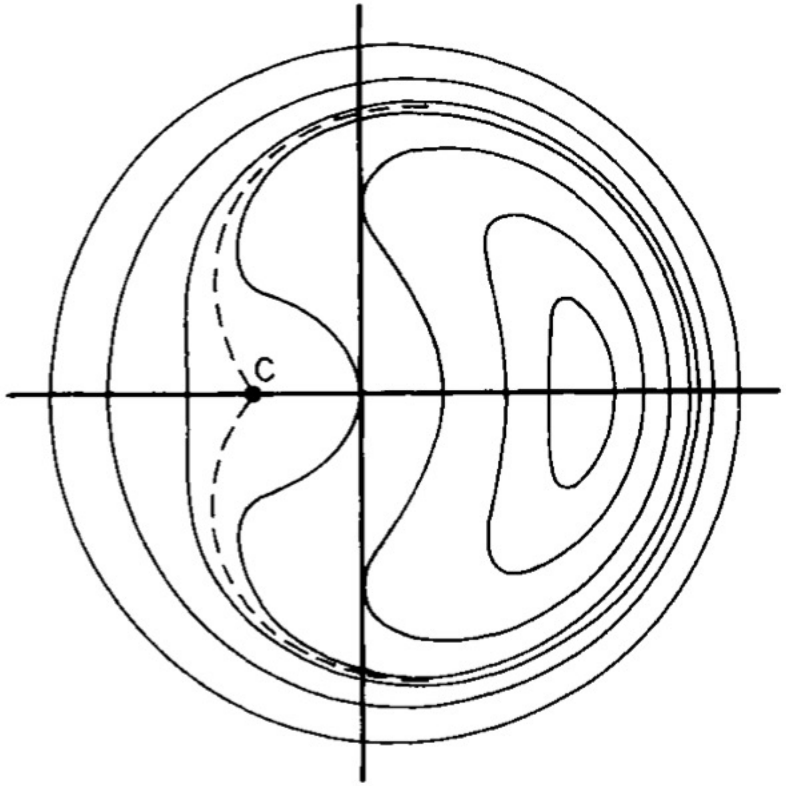
Ferraz-Mello *CelMec* (1985)



Mexican hat



1^{st}-order resonance
→ Mexican hat
cut by a plane



See also Henrard & Lemaitre
"A second fundamental model for resonance", CelMec (1984)

Let’s be crazy…

what about a “more extended fundamental model for resonance“?

i.e. at $m/(m-j)$ j^{th}-order resonance?

Let's be crazy: what happens at a *m/(m-j) j*th order resonance?

$X = e\cos(\phi)$ and $Y = e\sin(\phi)$, where resonant angle is $\phi = m\lambda_s - (m-j)\lambda - \varpi$

Jacobi constant (parameter)

$$\mathcal{H}(X,Y) = -\frac{3}{2}\left[\Delta J - \left(\frac{m-j}{2j}\right)\left(X^2+Y^2\right)\right]^2 + \epsilon P(X,Y)$$

Mexican hat (unperturbed) resonance perturbation

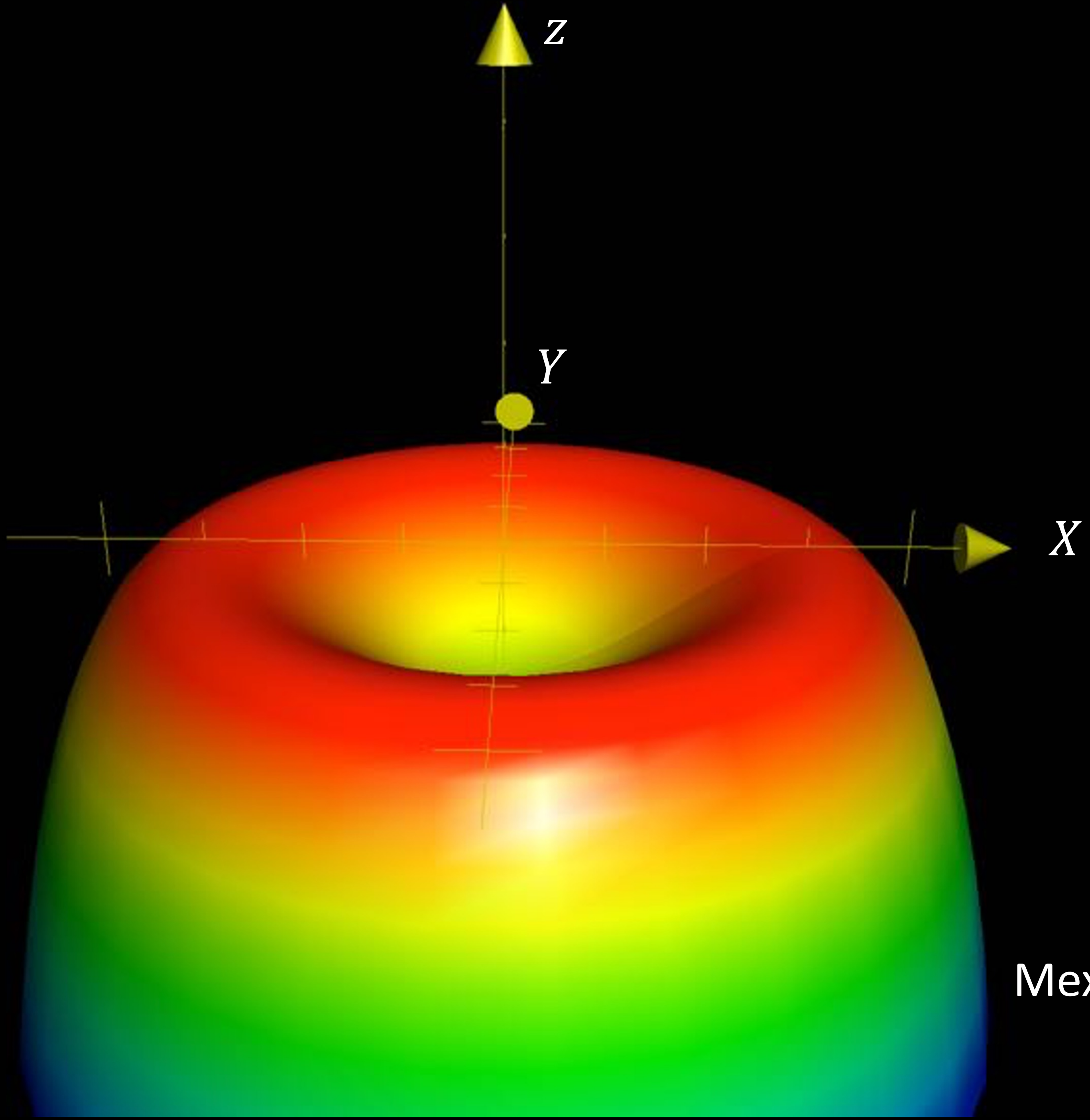
P(X,Y): homogeneous polynomial of degree *j*

$$P(X,Y) = \sum_{k=0}^{\mathrm{int}(j/2)} (-1)^k C_j^k X^{j-2k} Y^{2k} \rightarrow$$

Resonance order j	$P(X,Y)$
1	X ⬅
2	$X^2 - Y^2$
3	$X^3 - 3XY^2$
4	$X^4 + Y^4 - 6X^2Y^2$
5	$X^5 - 5X^3Y^2 + 10XY^4$

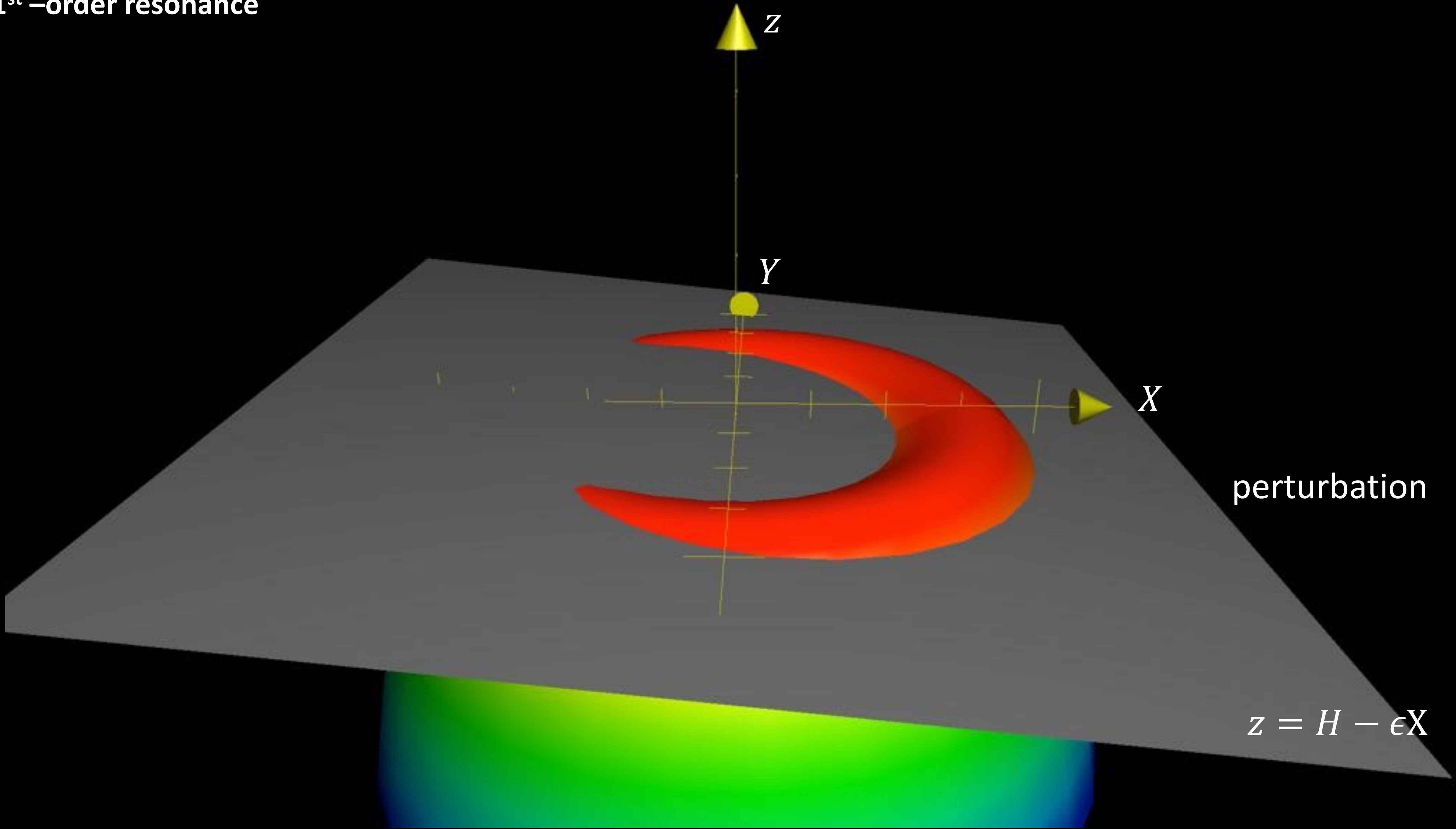
Sicardy et al (2025)

1st –order resonance

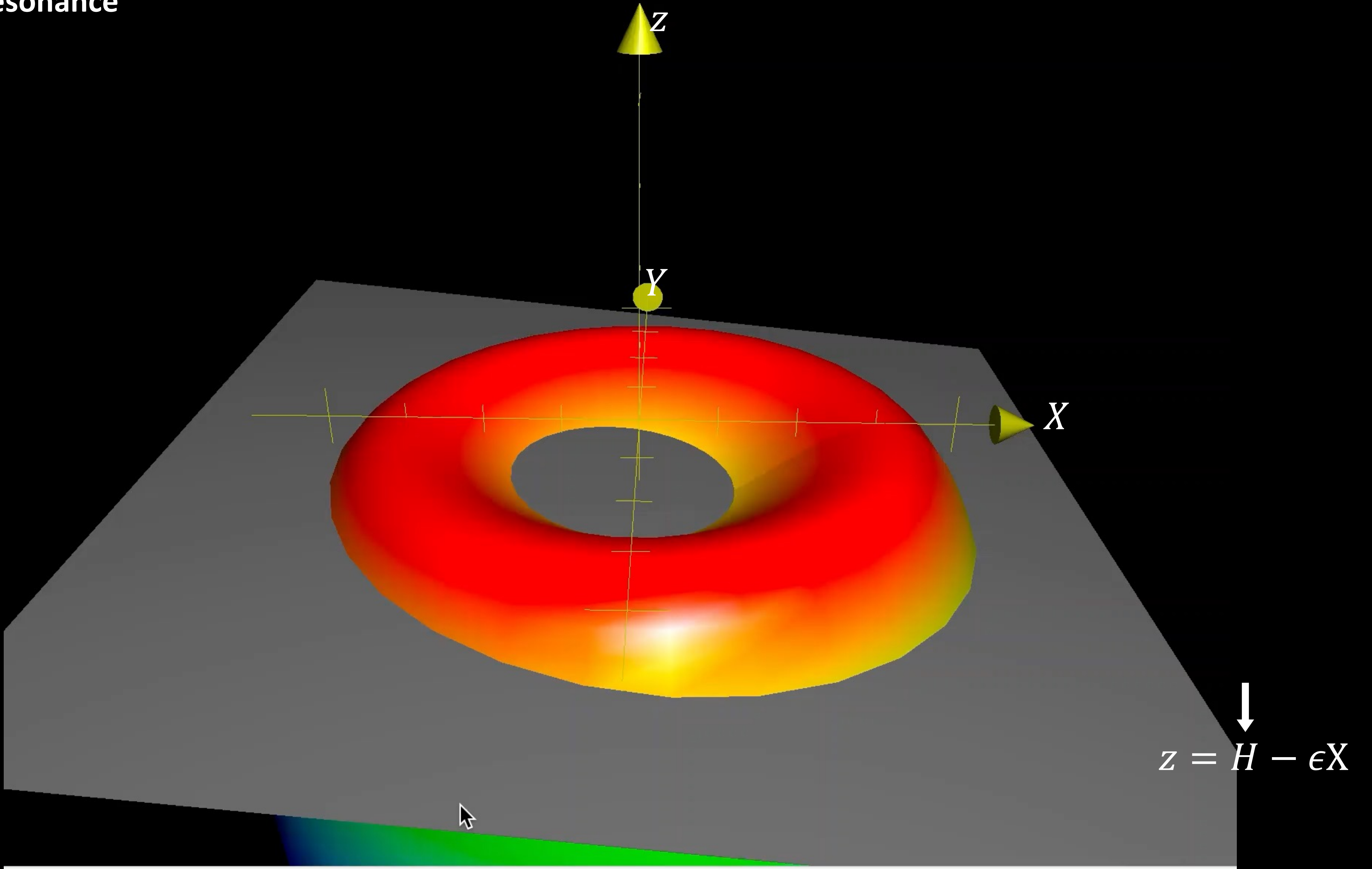


Mexican hat

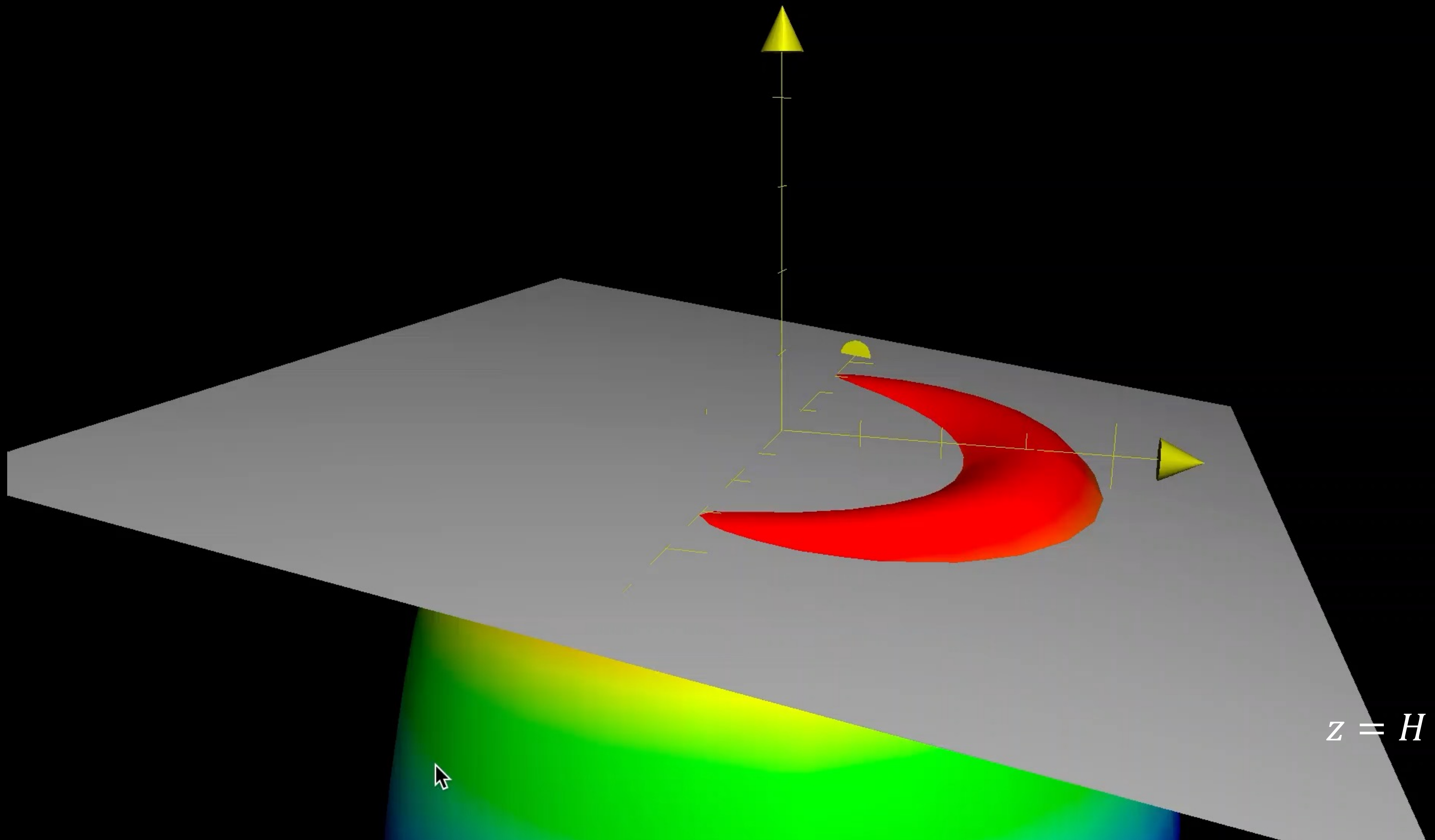
1st –order resonance



1st –order resonance

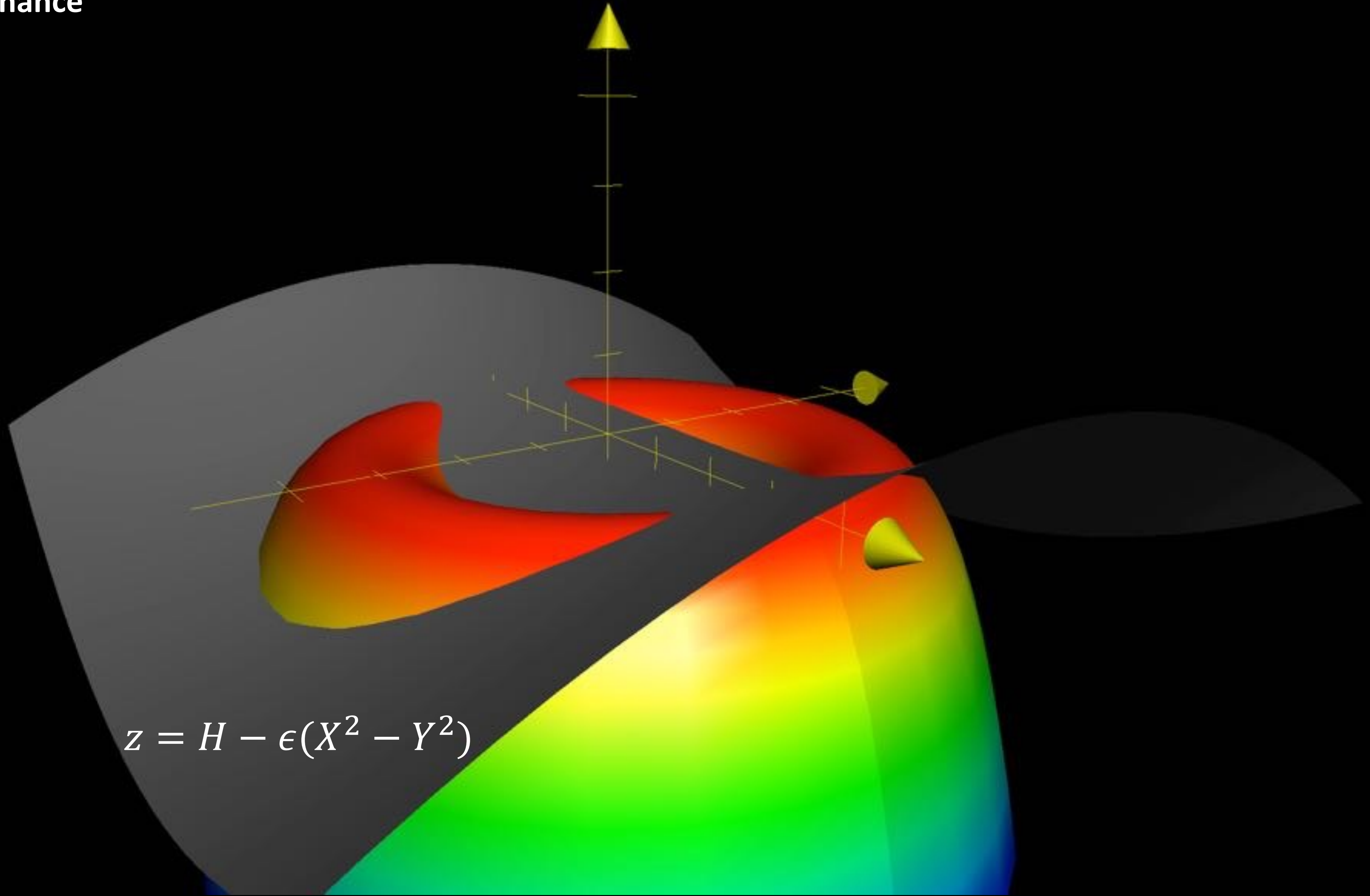


1^{st} –order resonance



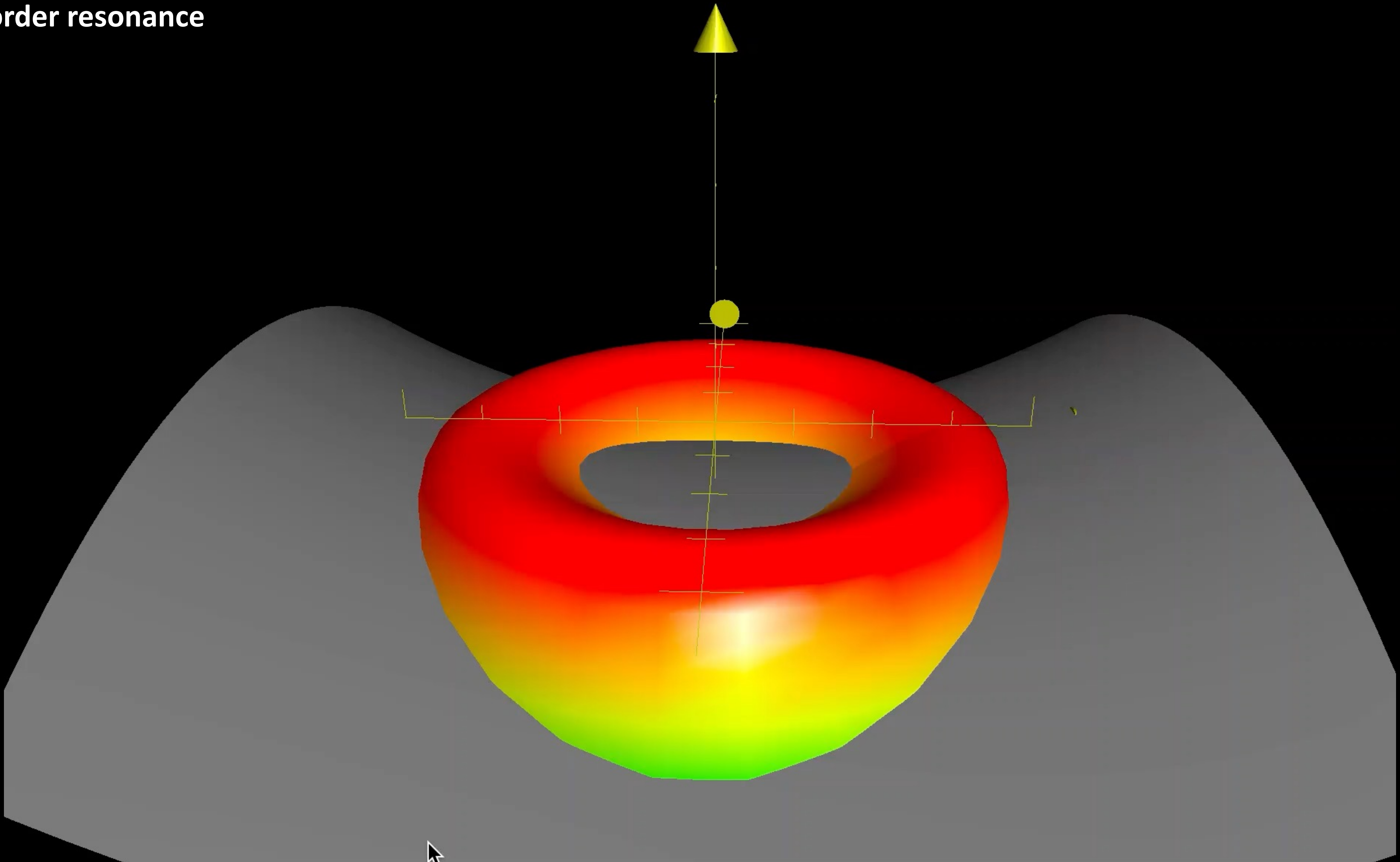
$z = H - \epsilon X$

2^{nd}-order resonance

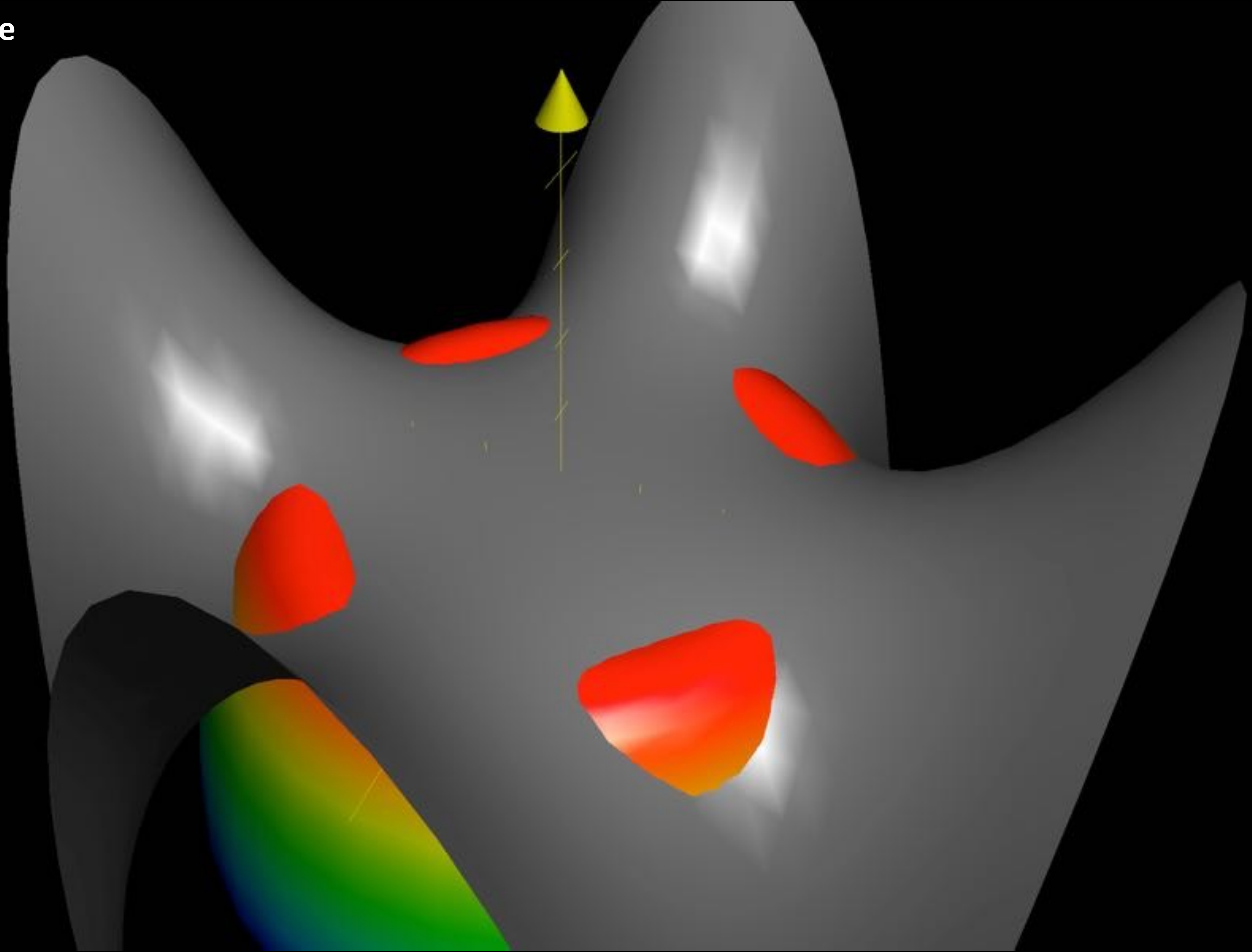


$$z = H - \epsilon(X^2 - Y^2)$$

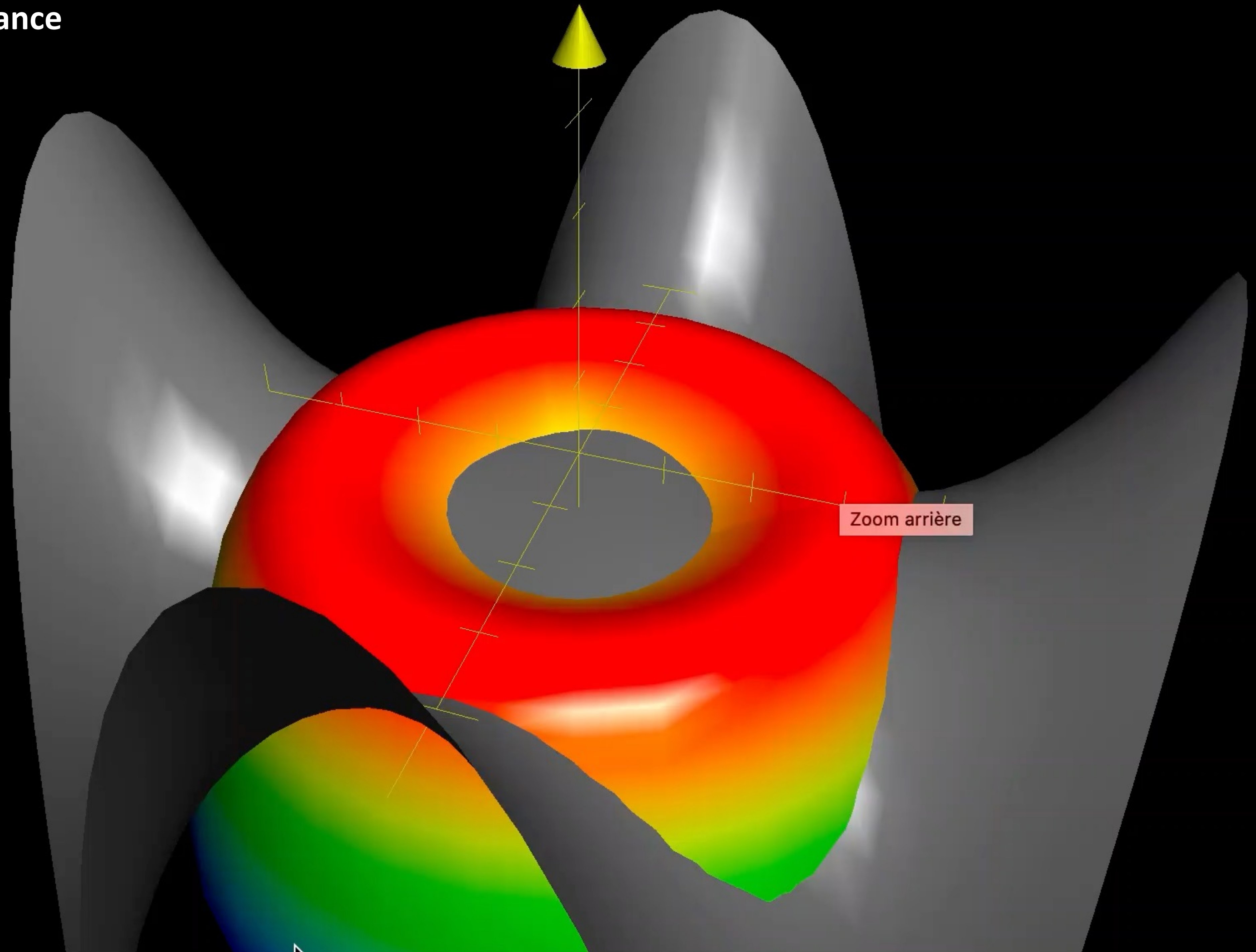
2^{nd}-order resonance

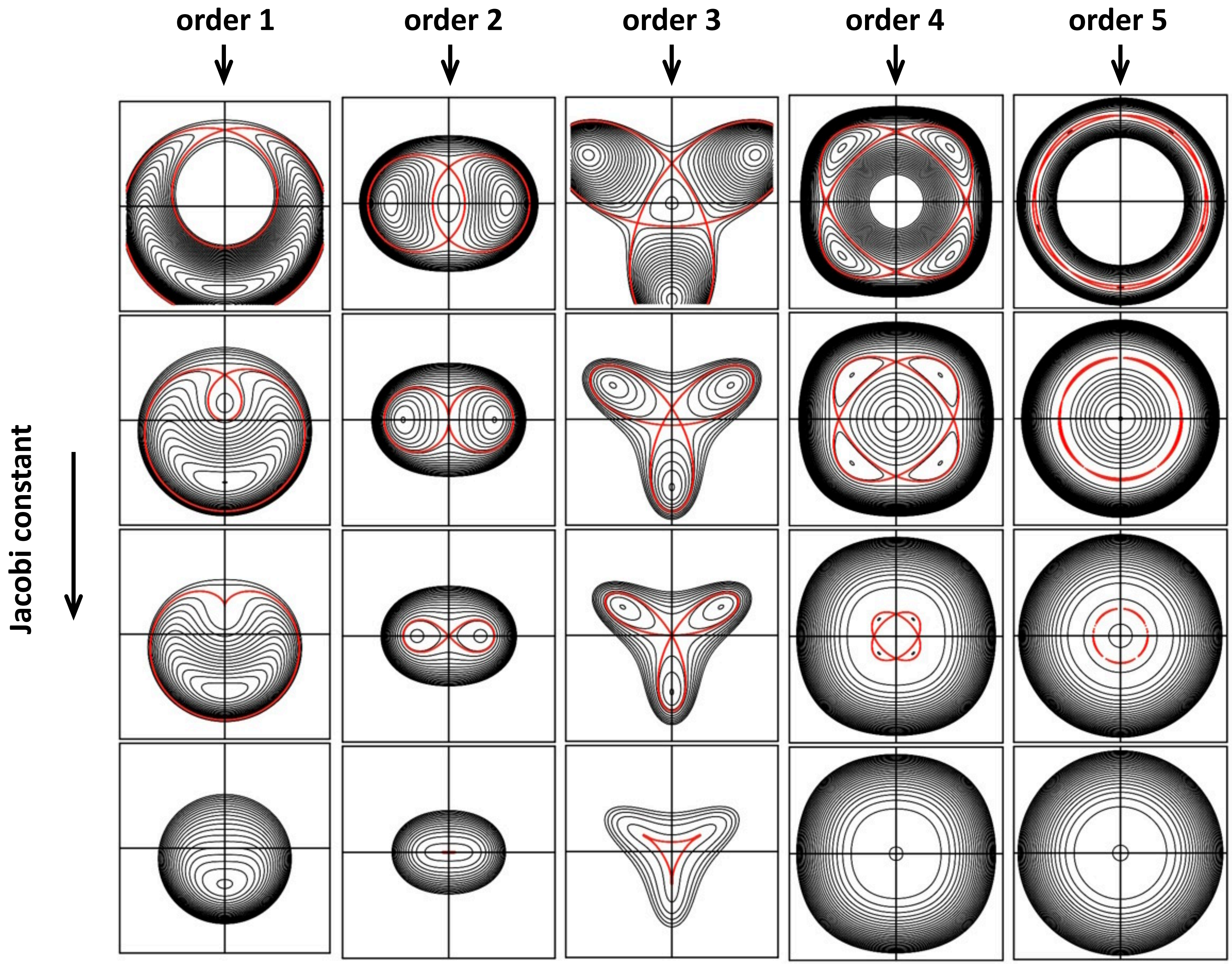


4^{th}-order resonance

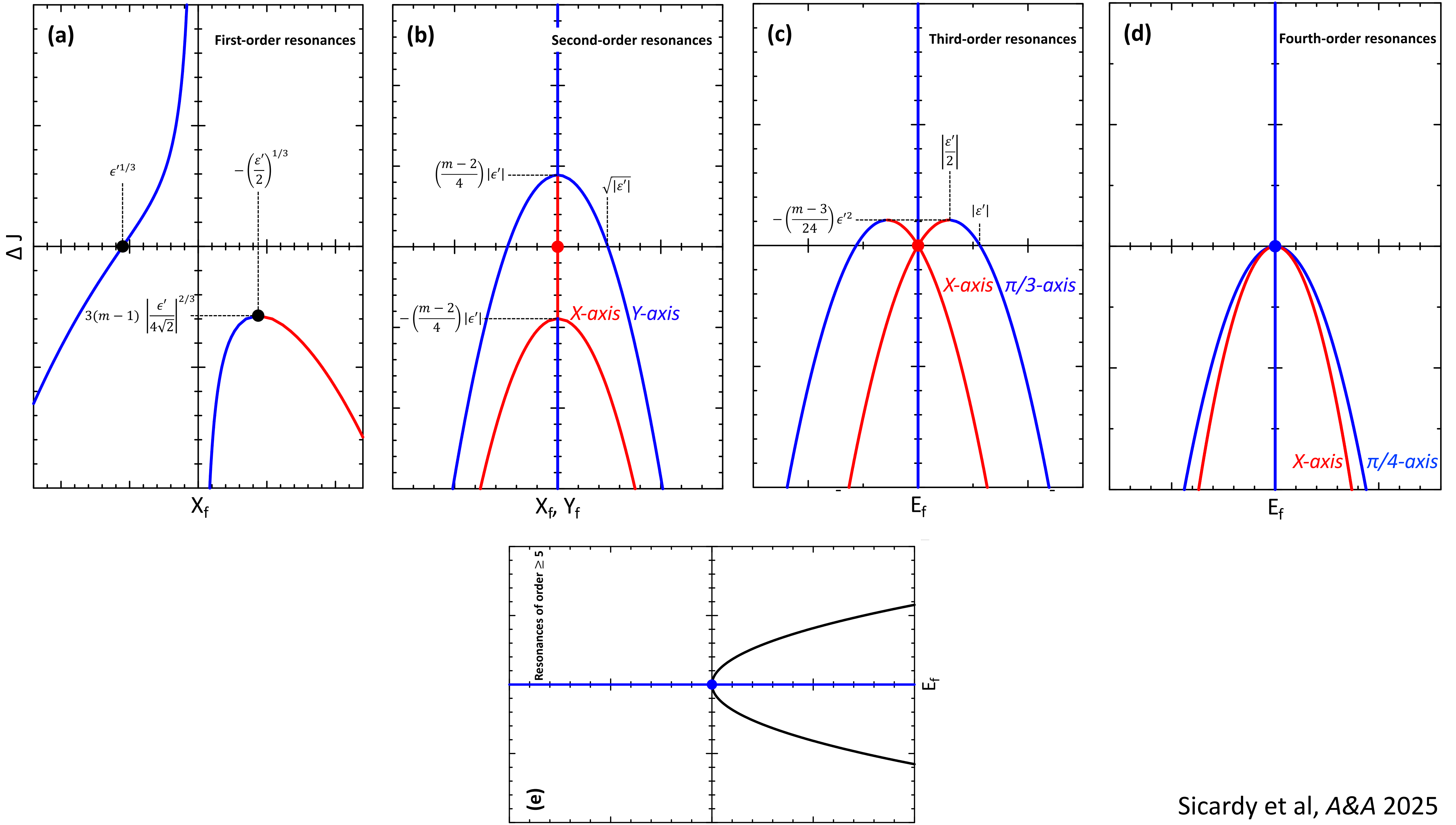


4th-order resonance





Sicardy et al, *A&A* 2025



(a)
First-order resonances
ΔJ
$\epsilon'^{1/3}$
$-\left(\frac{\varepsilon'}{2}\right)^{1/3}$
$3(m-1)\left|\frac{\epsilon'}{4\sqrt{2}}\right|^{2/3}$
X_f
(b)
Second-order resonances
$\left(\frac{m-2}{4}\right)|\epsilon'|$
$\sqrt{|\varepsilon'|}$
$-\left(\frac{m-2}{4}\right)|\epsilon'|$
X-axis
Y-axis
X_f, Y_f
(c)
Third-order resonances
$\left|\frac{\varepsilon'}{2}\right|$
$-\left(\frac{m-3}{24}\right)\epsilon'^2$
$|\varepsilon'|$
X-axis
π/3-axis
E_f
(d)
Fourth-order resonances
X-axis
π/4-axis
E_f
(e)
Resonances of order ≥ 5
E_f
Sicardy et al, *A&A* 2025

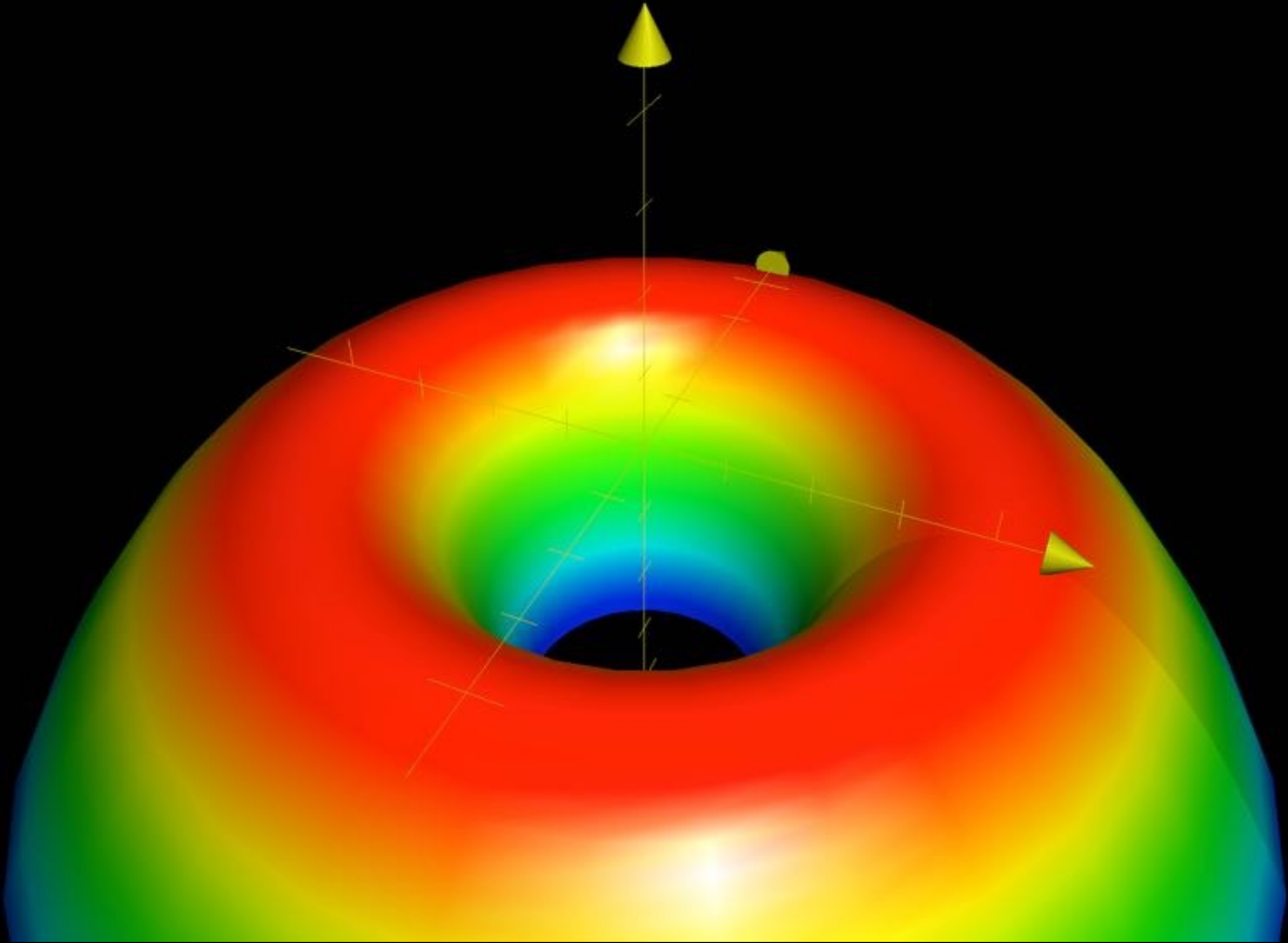
corotation resonances

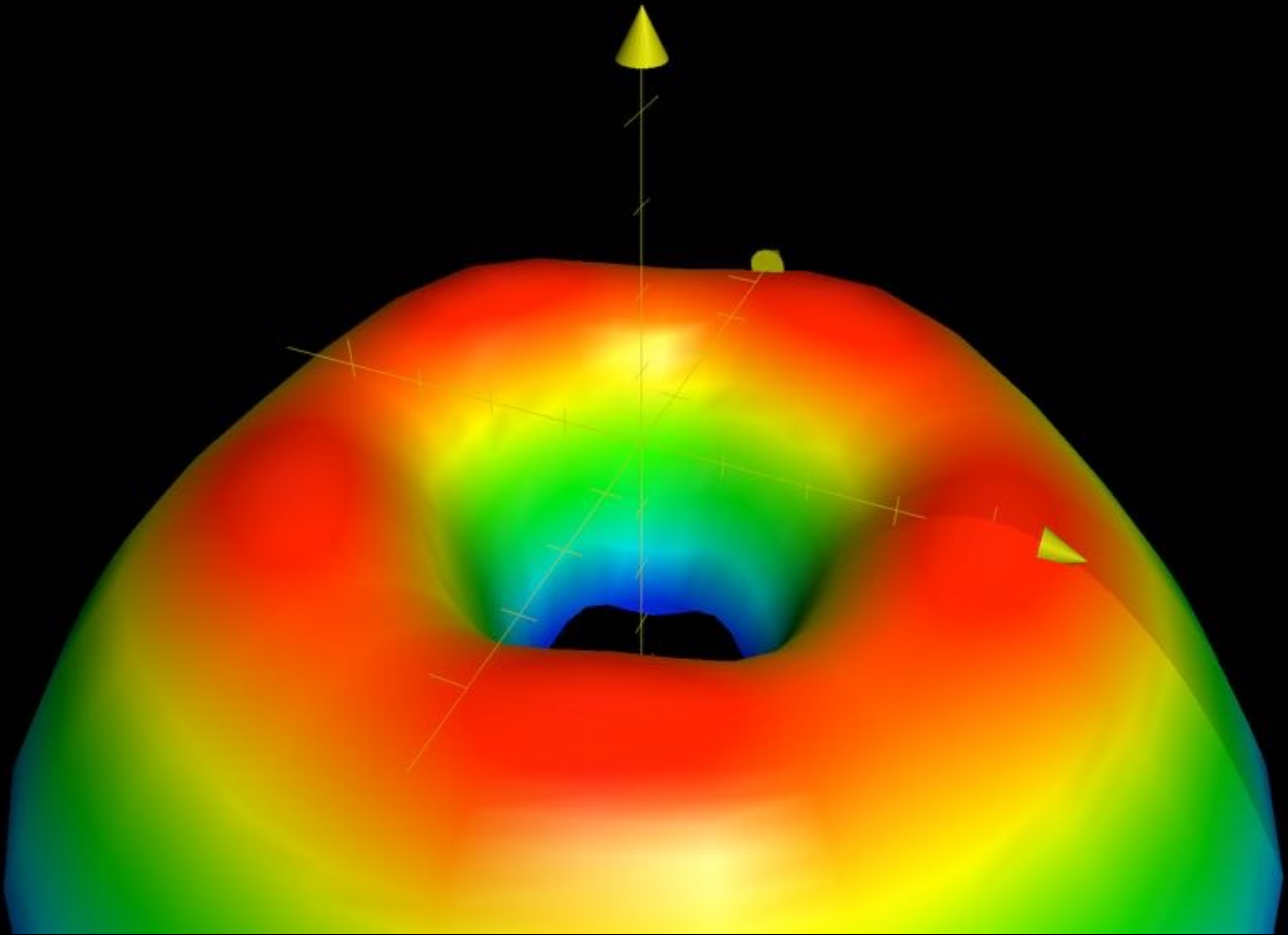
What if we focus on the longitudinal degree of freedom (associated with n , ignoring the d.o.f. associated with κ)?

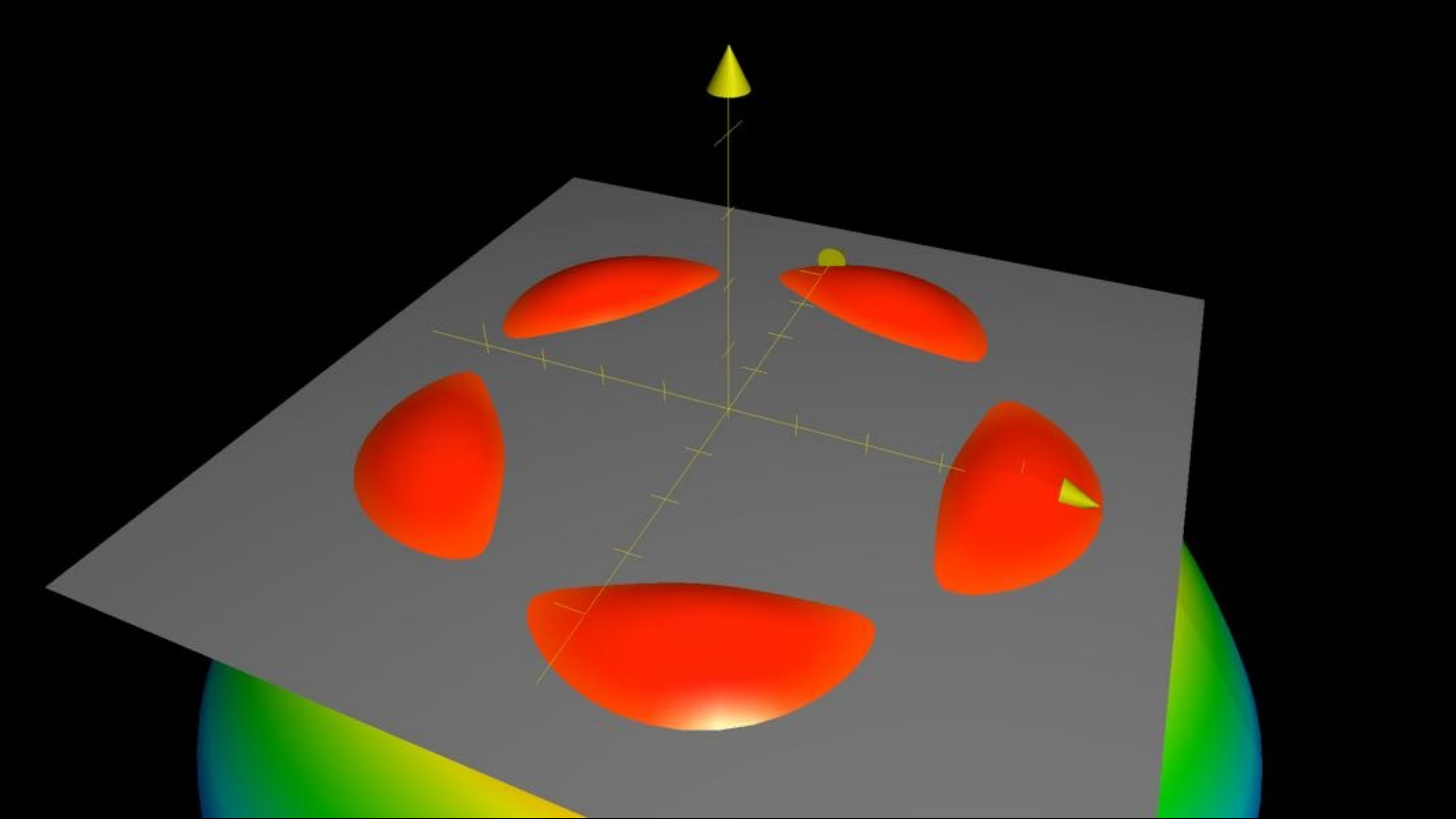
→ integrable

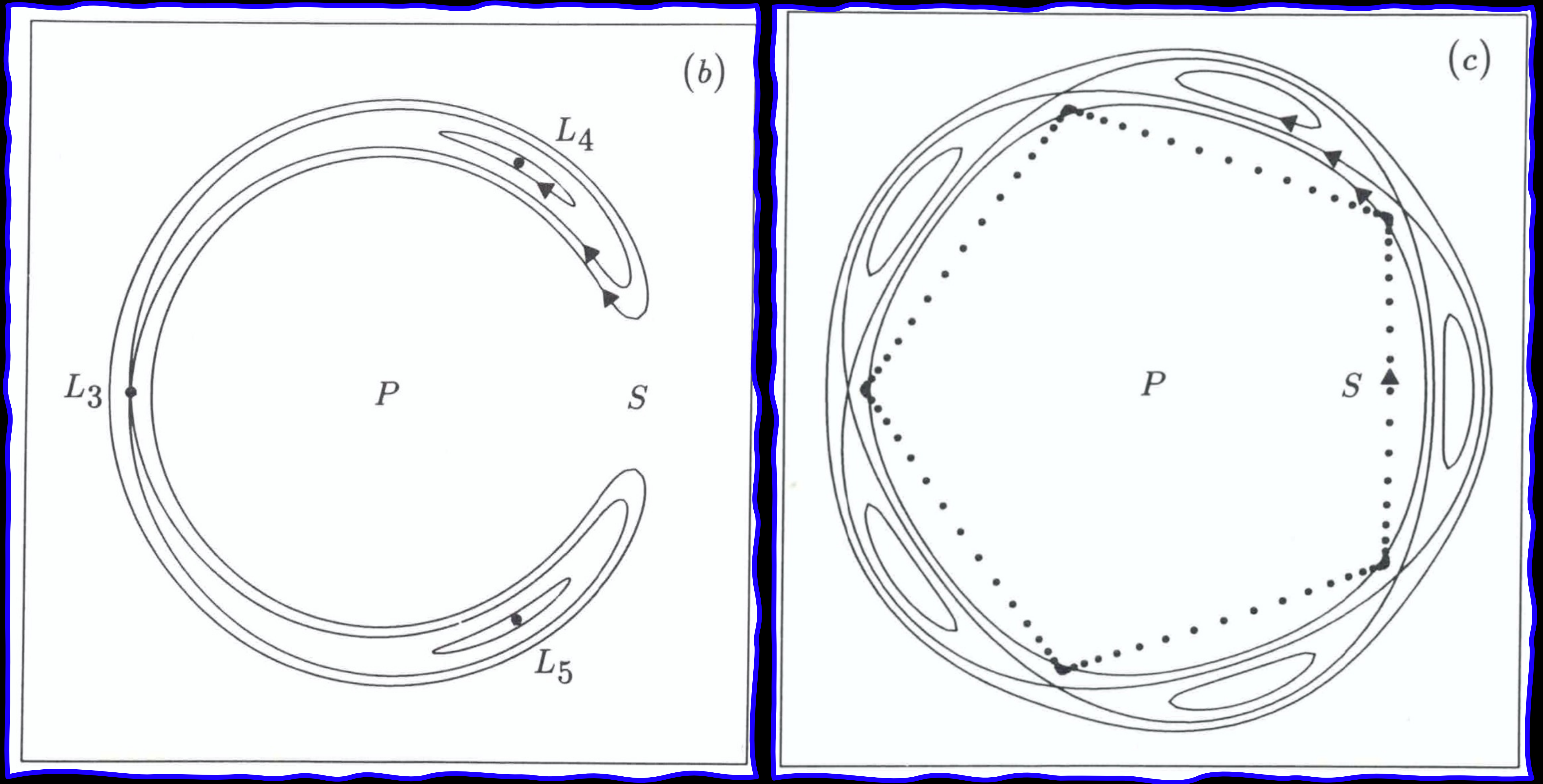
$$\phi' = m\lambda_s - (m-j)\lambda - \varpi_s$$

(called corotation resonances in ring and galactic dynamics)

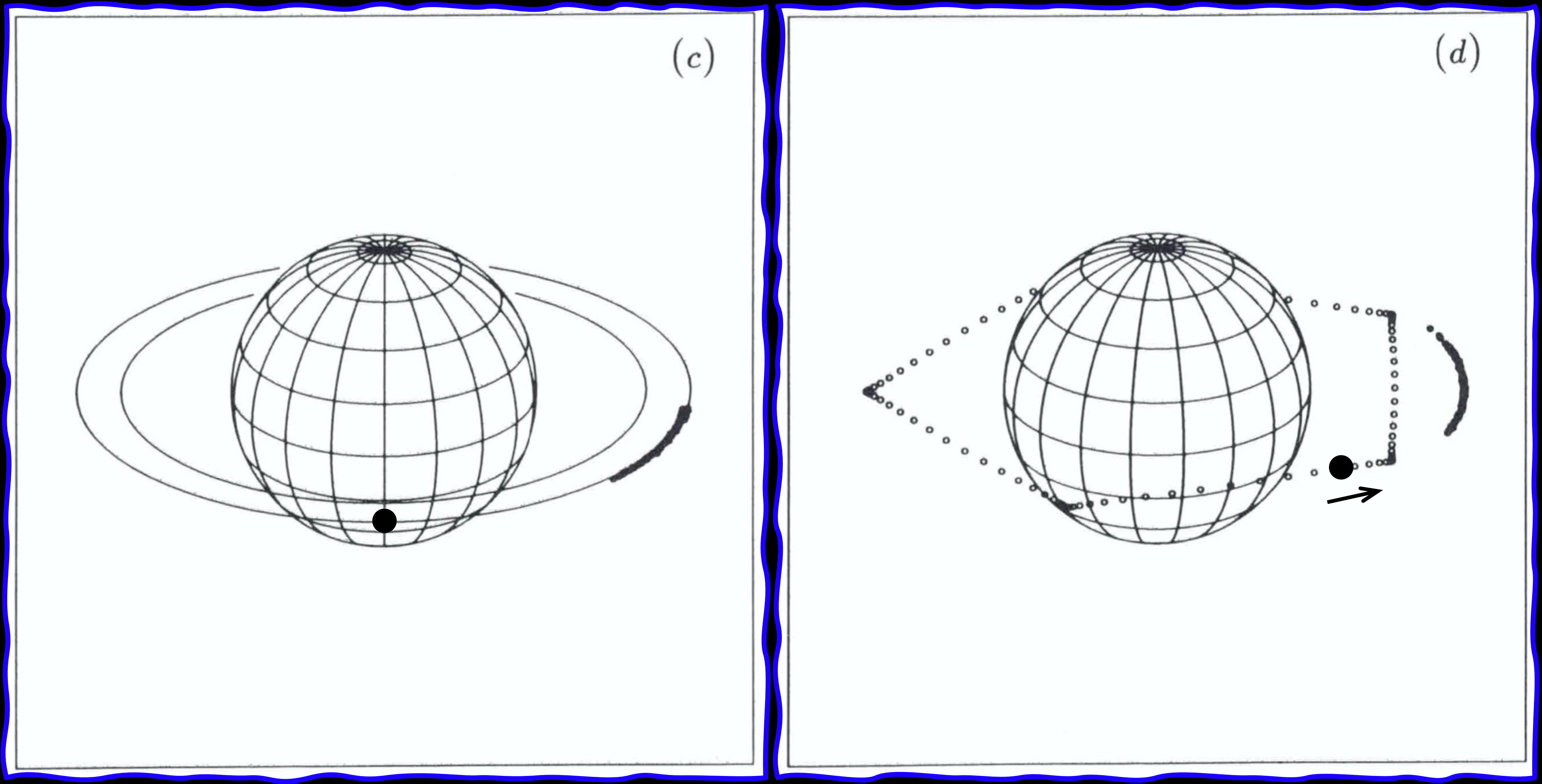






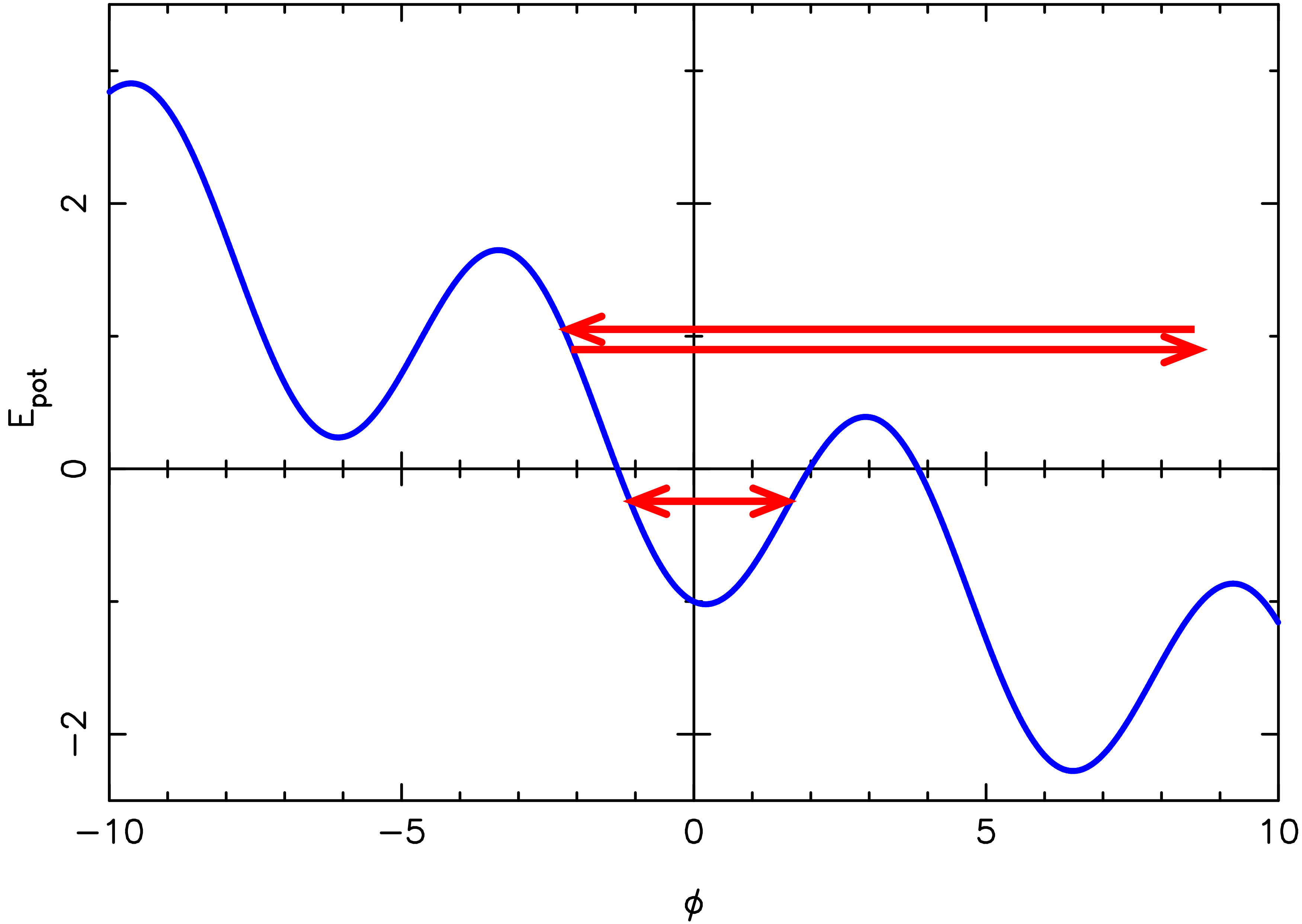


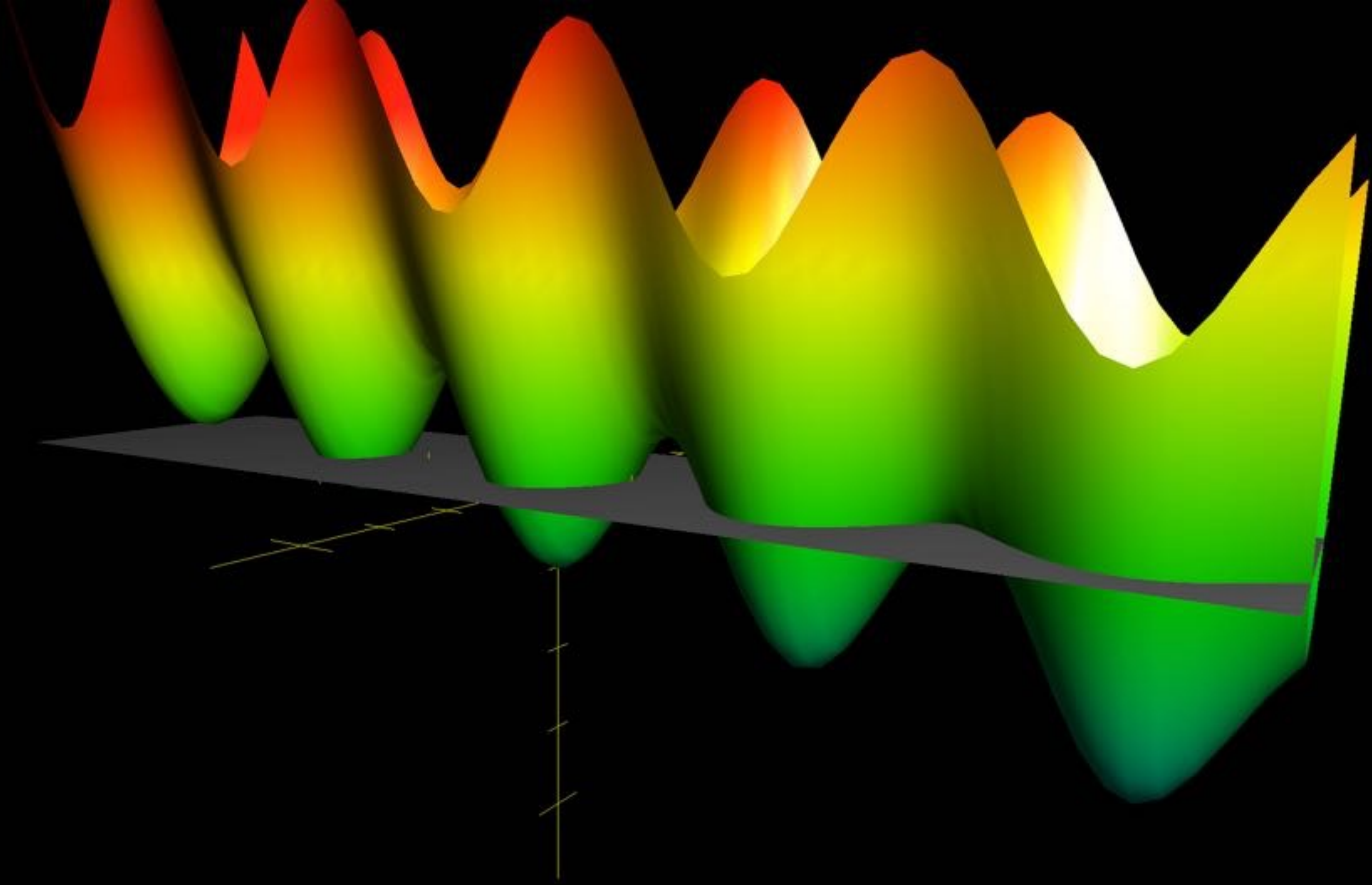
Sicardy *Icarus* (1991)

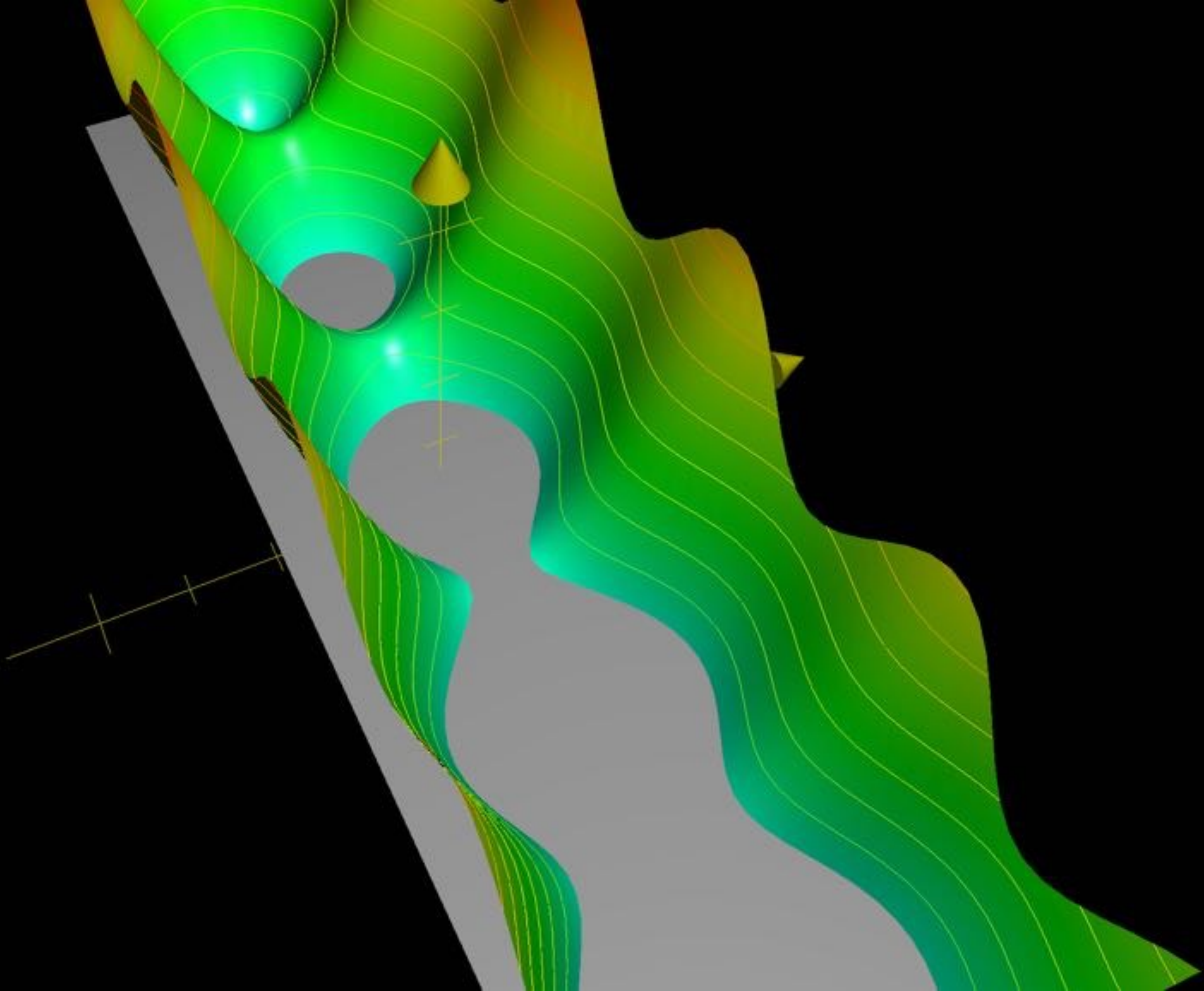


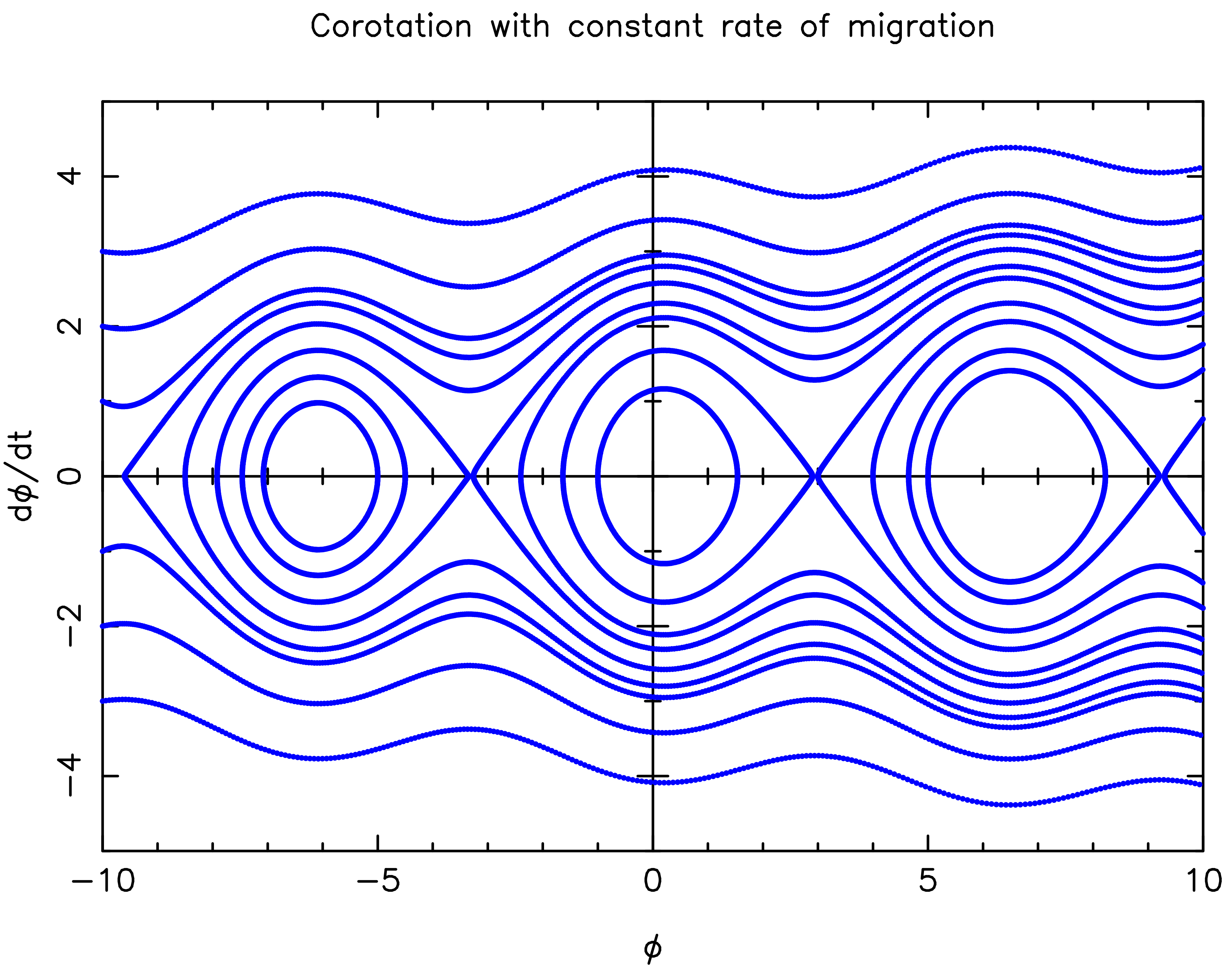
Sicardy *Icarus* (1991)

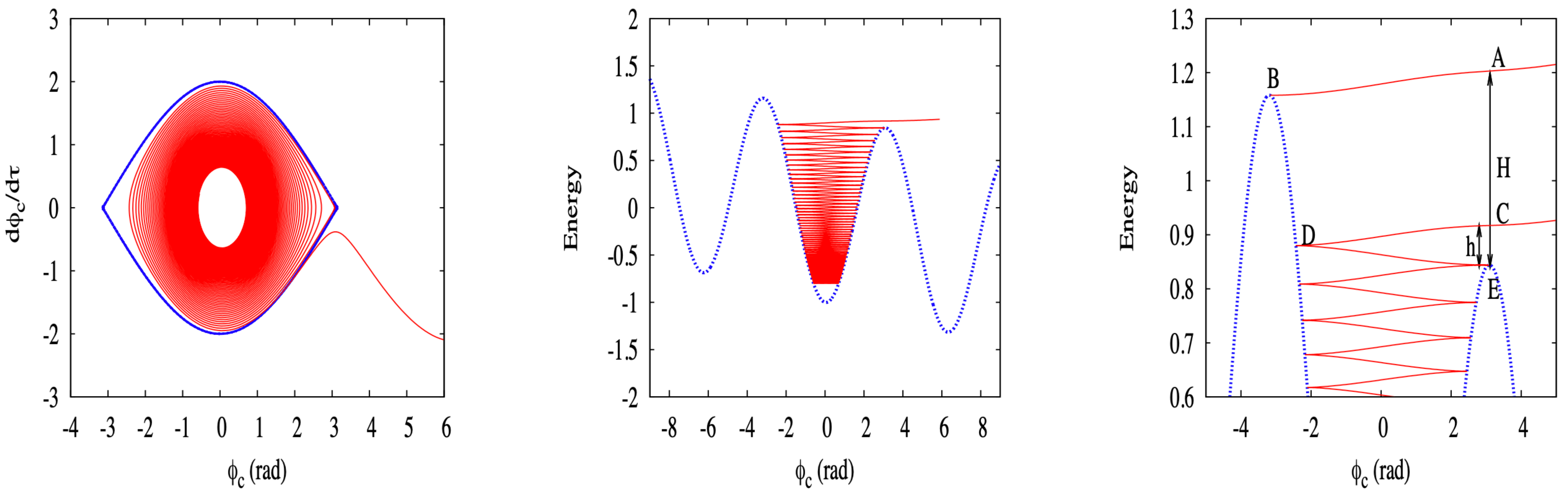
Corotation with constant rate of migration





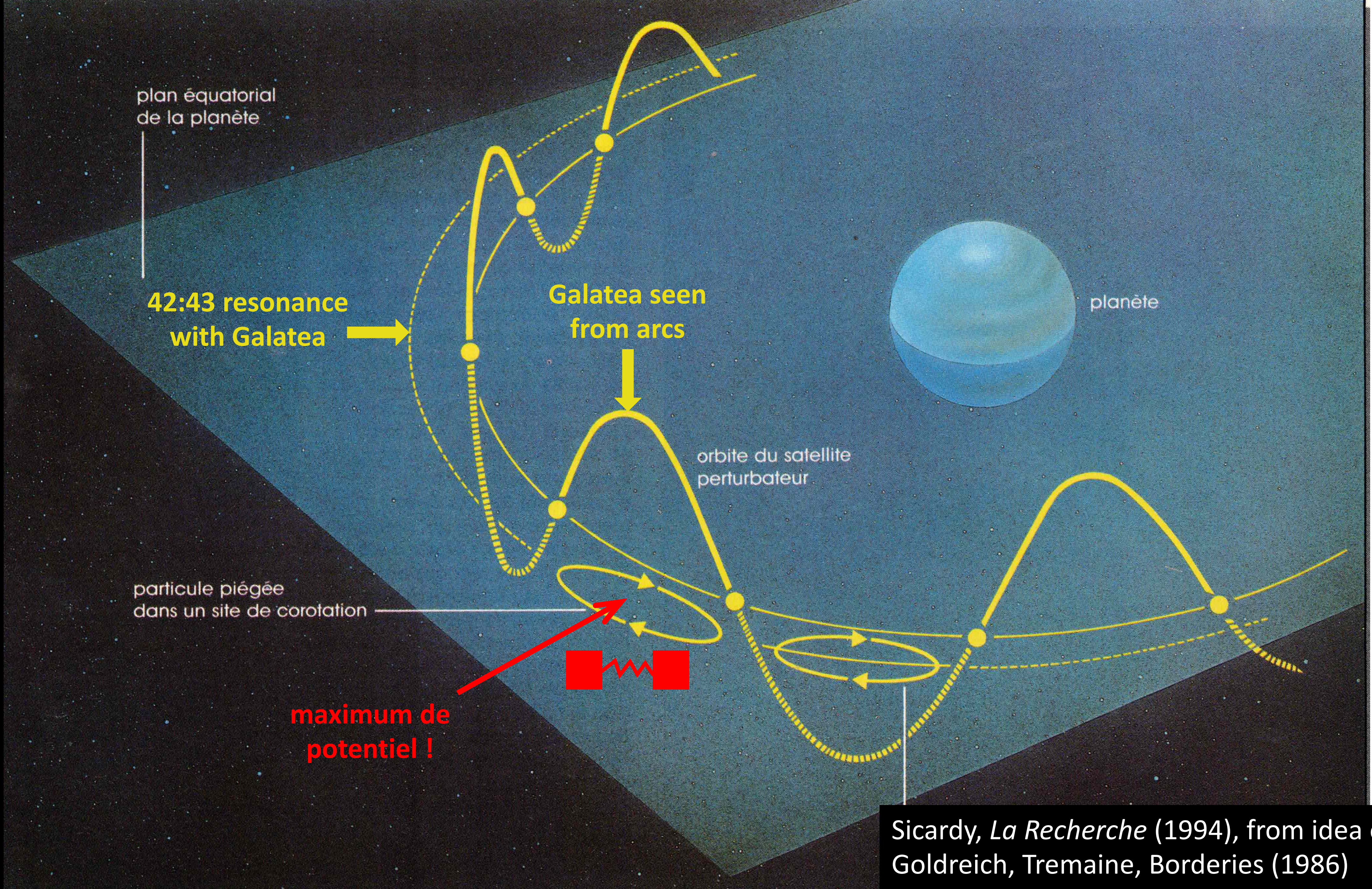



Corotation with constant rate of migration
dφ/dt
φ
4
2
0
−2
−4
−10
−5
0
5
10



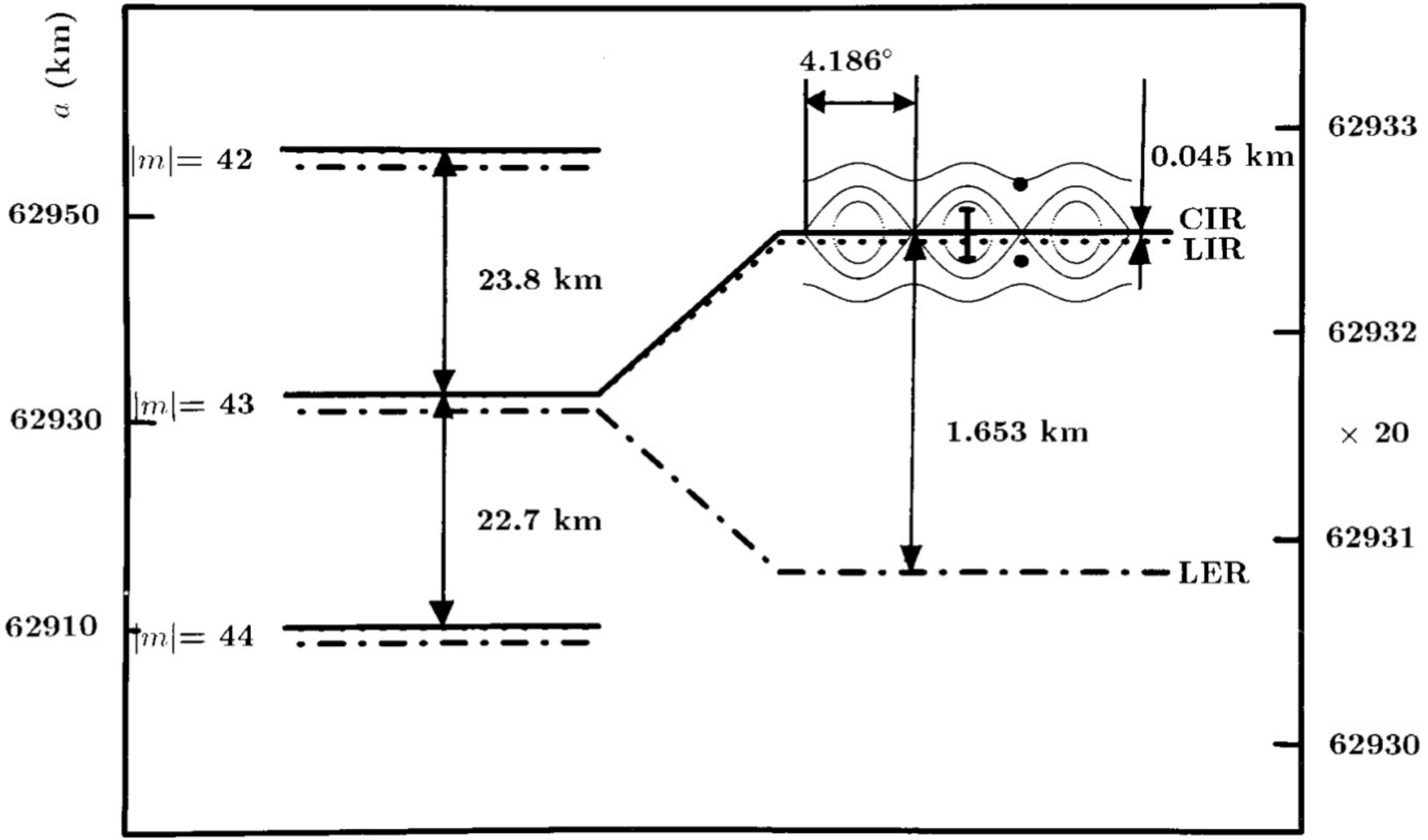
El Moutamid, Sicardy, Renner CelMec (2017)

Lindblad and corotation resonances (the elliptic restricted 3-BP)



Sicardy, *La Recherche* (1994), from idea of Goldreich, Tremaine, Borderies (1986)

NEPTUNE'S ARCS DYNAMICS

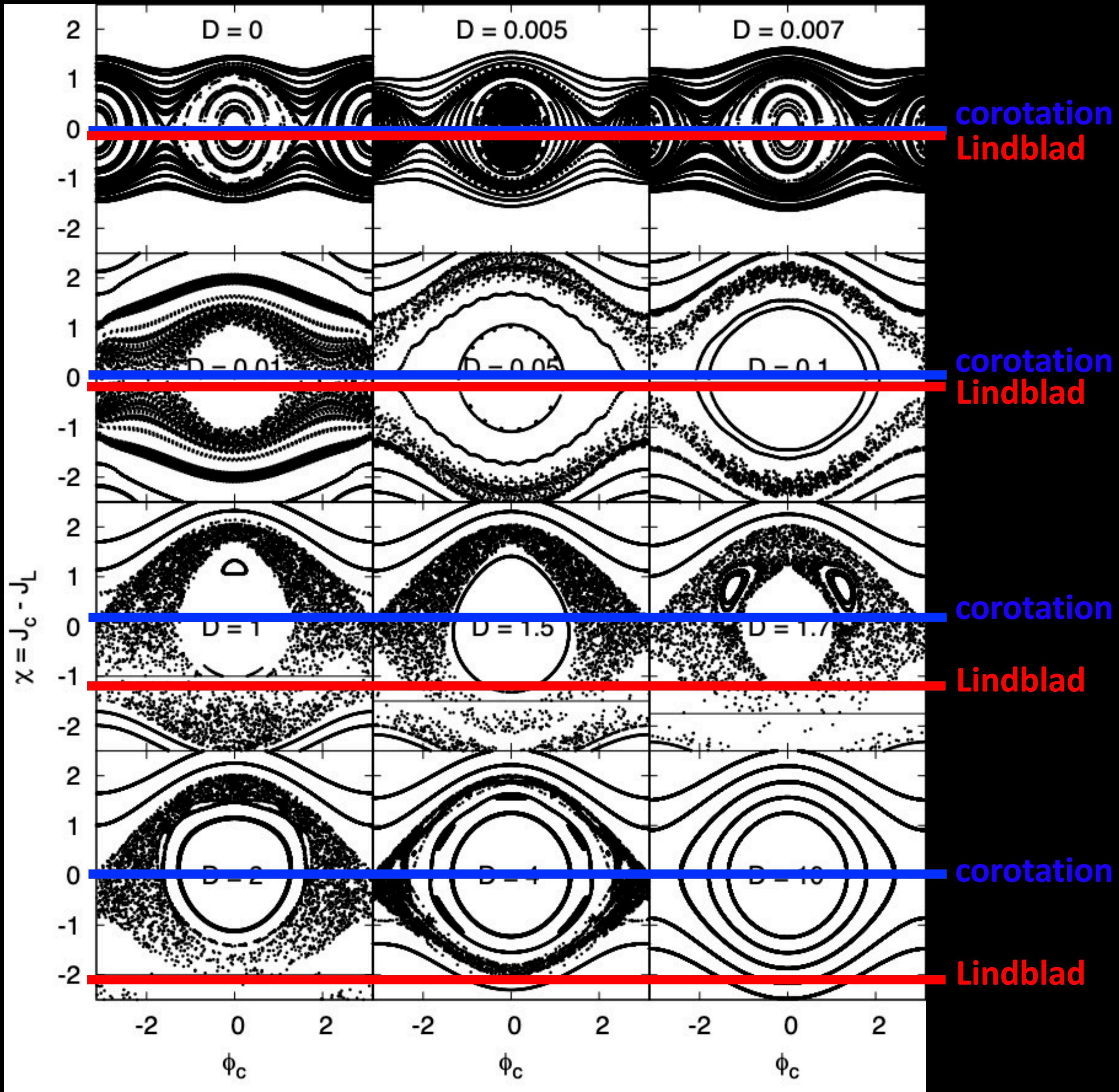


Foryta & Sicardy (1996)

A supplementary constant of motion exists
→ the Keplerian elliptical restricted problem is integrable! (Sessin & Ferraz-Mello 1985, Wisdom 1986)

for non-Keplerian elliptical restricted problems, this constant is destroyed, They are non longer integrable!

The "Coralin model"
El Moutamid, Sicardy & Renner Cel'Mec (2014)



MOTION OF TWO PLANETS WITH PERIODS COMMENSURABLE IN THE RATIO 2 : 1
SOLUTIONS OF THE HORI AUXILIARY SYSTEM

W. SESSIN
Instituto Tecnológico de Aeronáutica, Departamento de Astronomia,
12200-São José dos Campos, SP, Brazil

and

S. FERRAZ-MELLO
Universidade de São Paulo, Departamento de Astronomia, 04301-São
Paulo, SP, Brazil

CelMec 1984

The Elliptic Planar restricted problem has two degrees of freedom, i.e. two critical arguments, yet it is integrable due to Sessin's constant of motion

Why?!

CANONICAL SOLUTION OF THE TWO CRITICAL ARGUMENT PROBLEM

JACK WISDOM
Department of Earth, Atmospheric and Planetary Sciences Massachusetts Institute of Technology

CelMec 1986

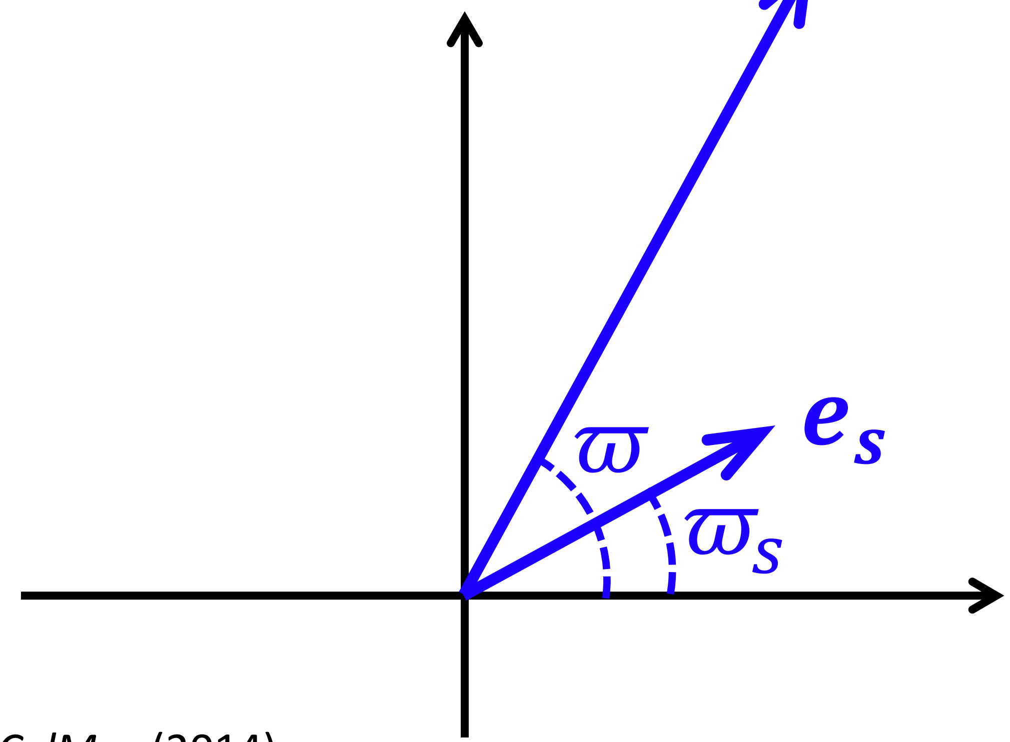
Circular RP3BP, Jacobi constant:

$$J_c = \frac{\Delta a}{a_0} + me^2 = \text{constant}$$

Elliptic RP3BP, *relative* Jacobi quantity:

$$J_{c,rel} = \frac{\Delta a}{a_0} + m(\boldsymbol{e} - \alpha \boldsymbol{e_s})^2$$

$$\boldsymbol{e} = [\, e\cos\varpi, e\sin\varpi]$$
$$\boldsymbol{e_s} = [\, e_s\cos\varpi_s, e_s\sin\varpi_s]$$



Keplerian case: no secular change of ϖ and ϖ_s, then it can be shown that $J_{c,rel}$ is a constant of motion →

The 2-degree of freedom system is integrable

However, in general, secular change of ϖ and ϖ_s → $J_{c,rel}$ varies → system not integrable

El Moutamid et al. *CelMec* (2014)

Relaxing the restricted hypothesis:→

$$J_{c,rel} = A^2 \frac{\Delta a}{a_0} - A'^2 \frac{\Delta a_s}{a_{0s}} + m(|A|\boldsymbol{e} - |A'|\boldsymbol{e}_s)^2$$

Hill's problem: $A \approx A'$ →

$$J_{c,rel} = \frac{\Delta a}{a_0} - \frac{\Delta a_s}{a_{0s}} + \beta(\boldsymbol{e} - \boldsymbol{e}_s)^2$$

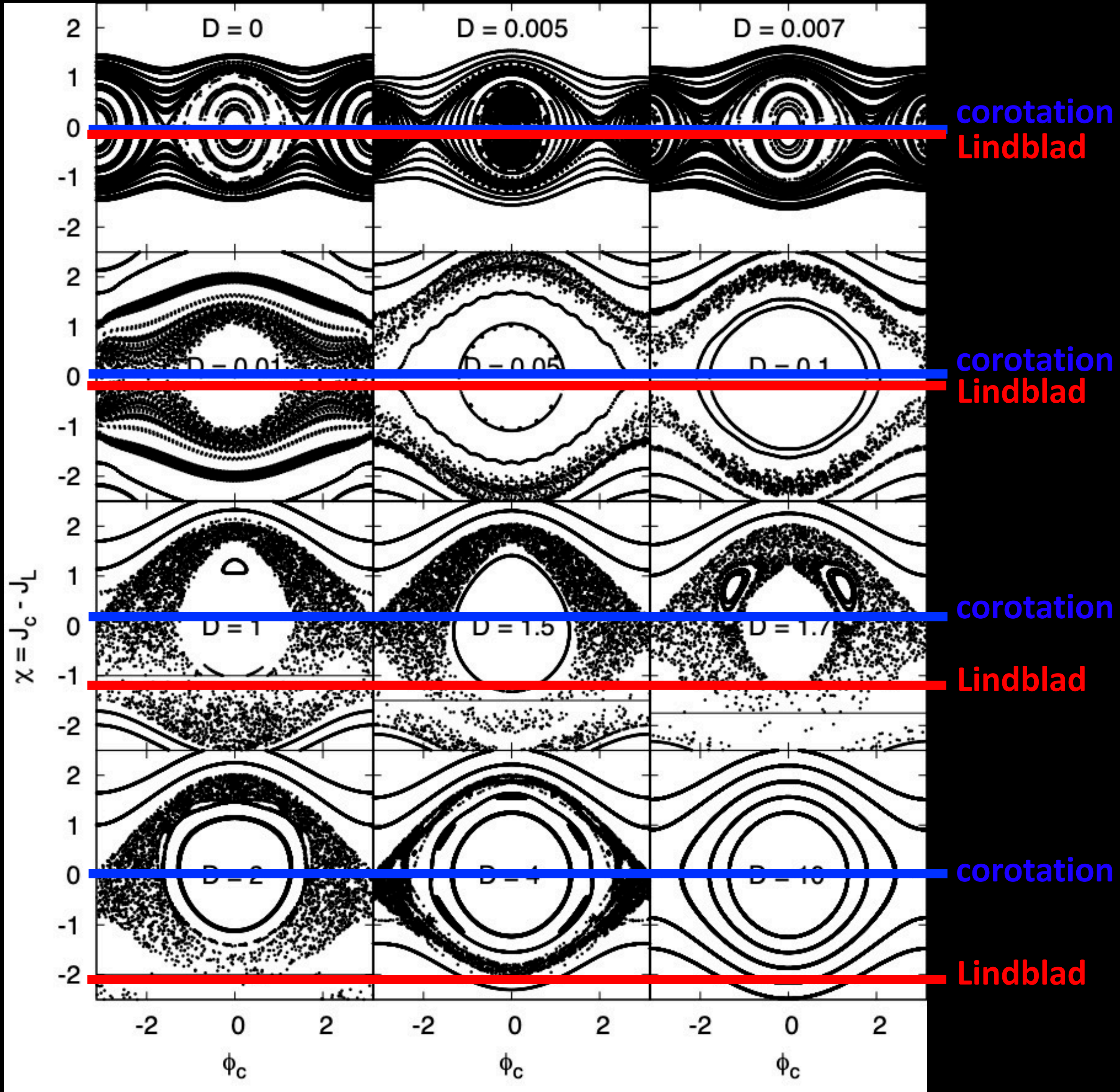
is conserved (Hénon & Petit *CelMec* 1986)

Questions: $J_{c,rel}$ is conserved near a *m/m-1* resonance and for the asymptotic Hill's problem. So, is it true *everywhere*?

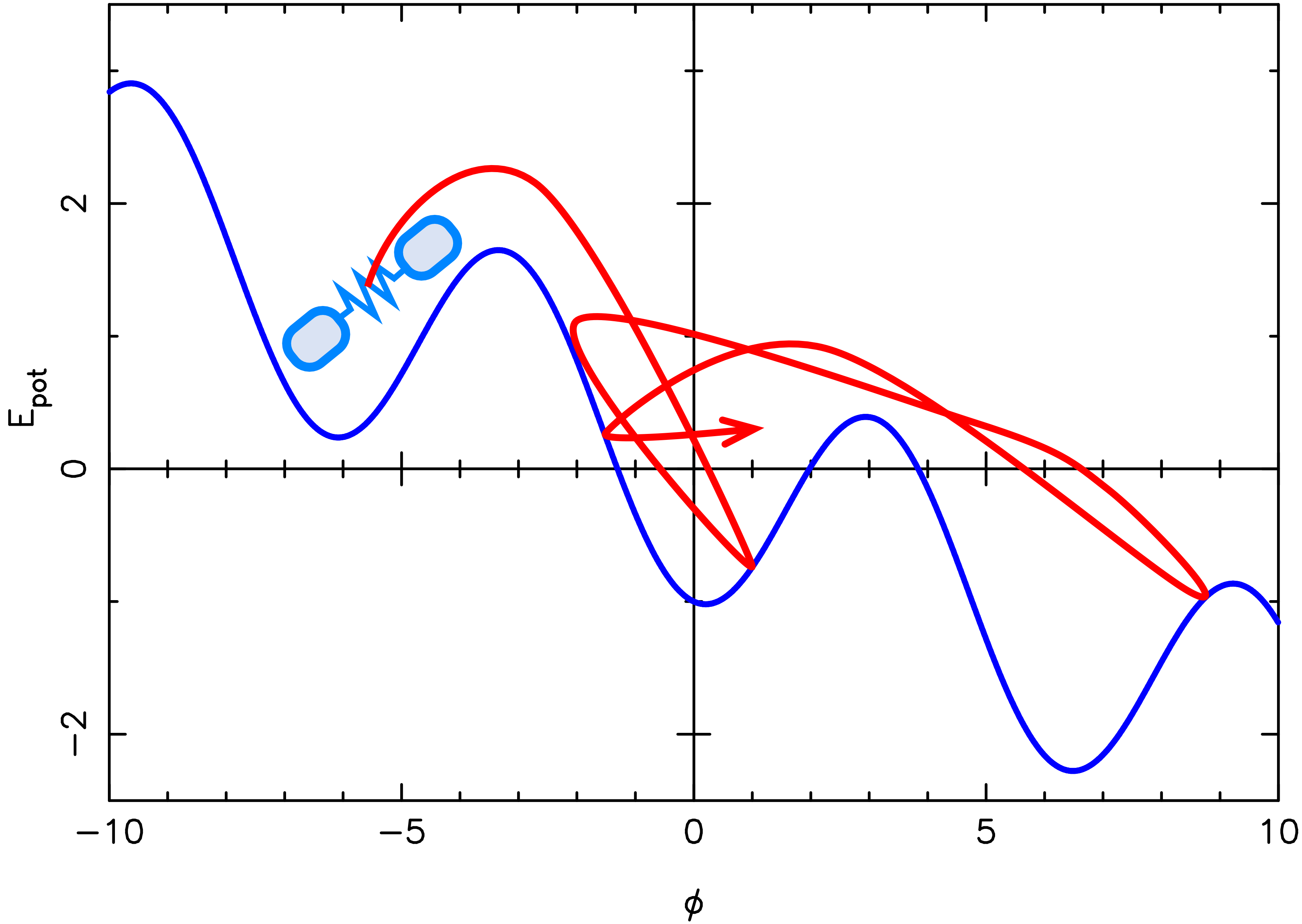
A supplementary constant of motion exists → the Keplerian elliptical restricted problem is integrable! (Sessin & Ferraz-Mello 1985, Wisdom 1986)

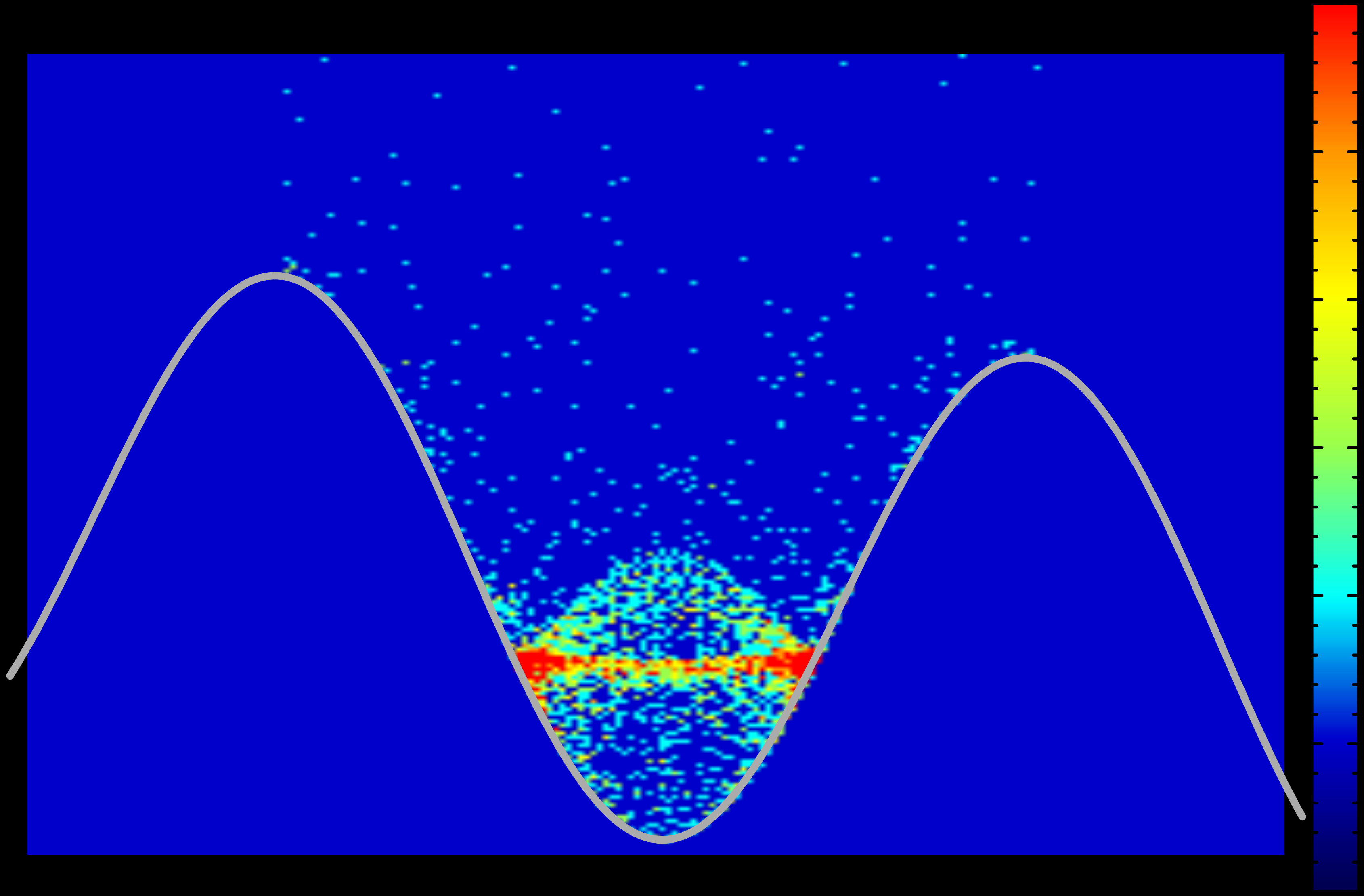
for non-Keplerian elliptical restricted problems, this constant is destroyed, They are non longer integrable!

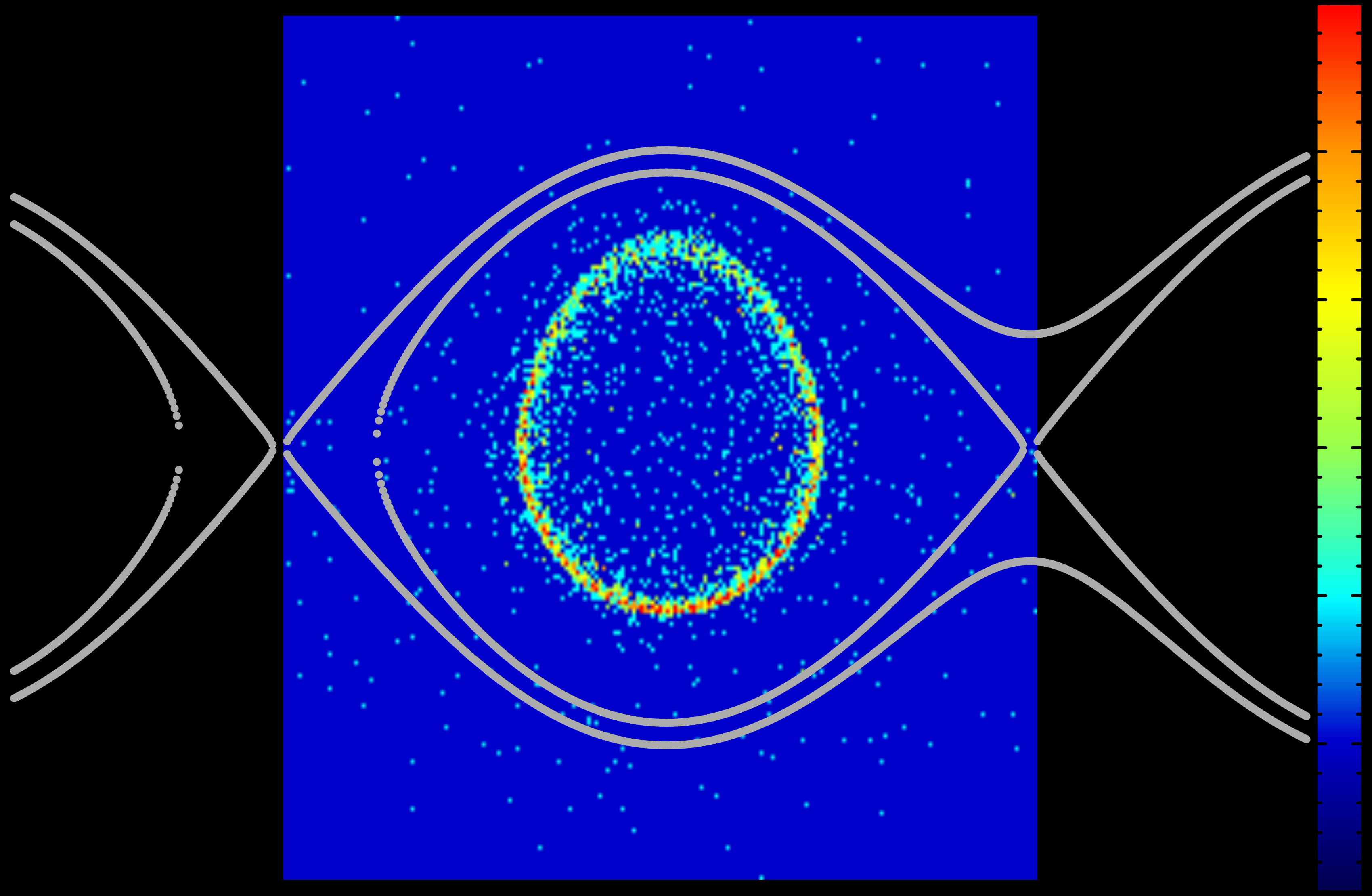
The “Coralin model“
El Moutamid, Sicardy & Renner Cel’Mec (2014)

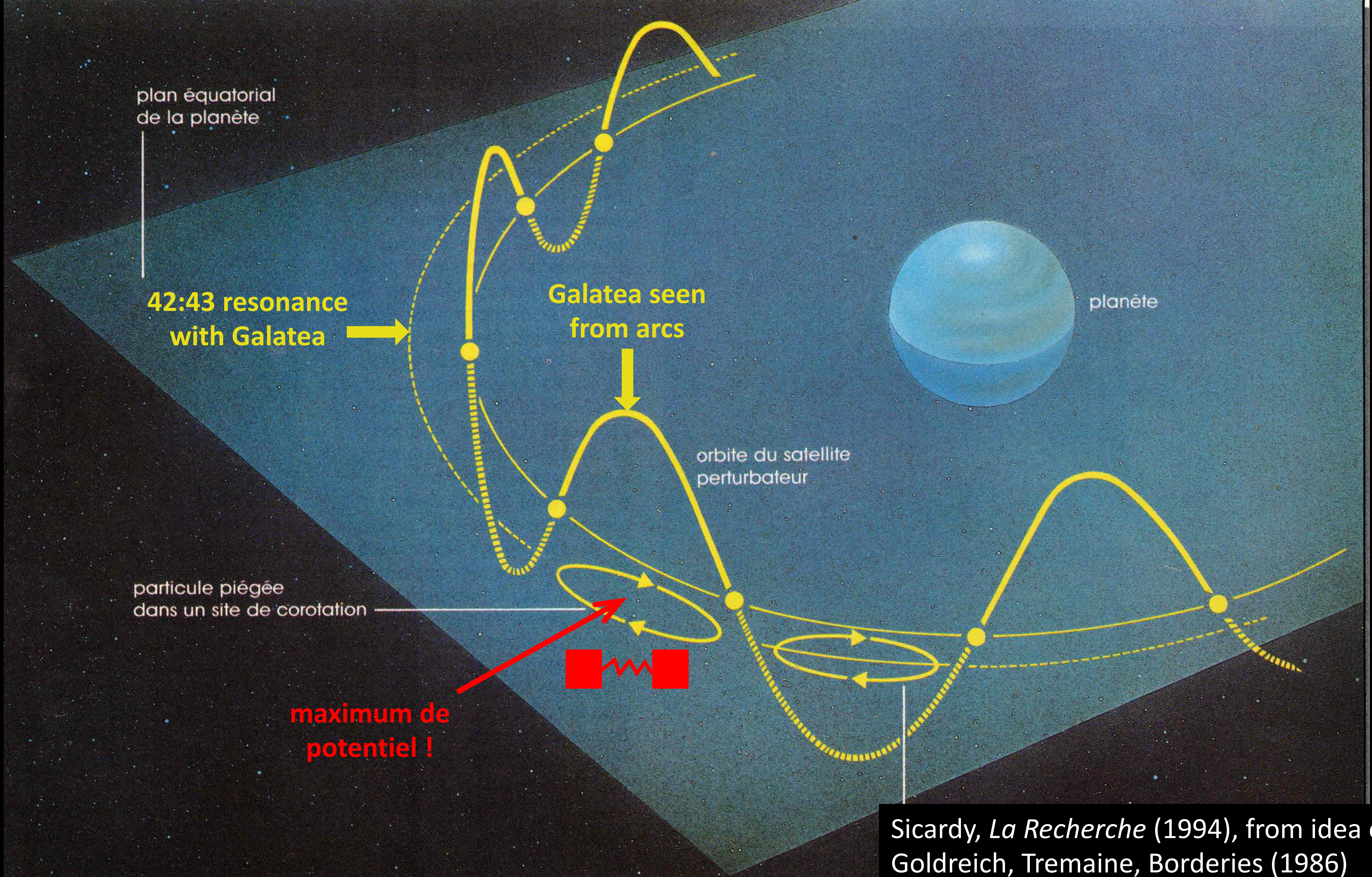


Corotation with constant rate of migration

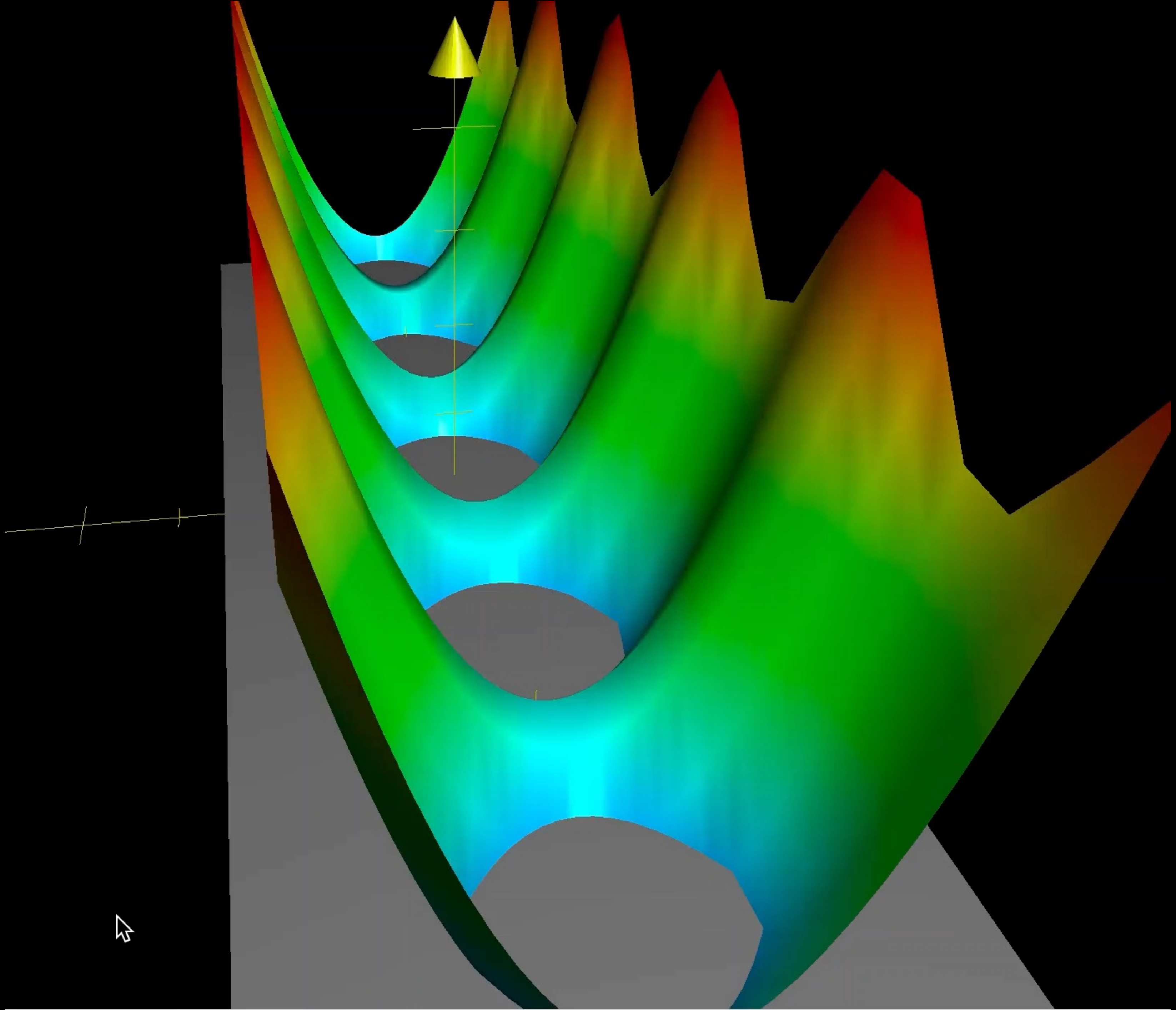


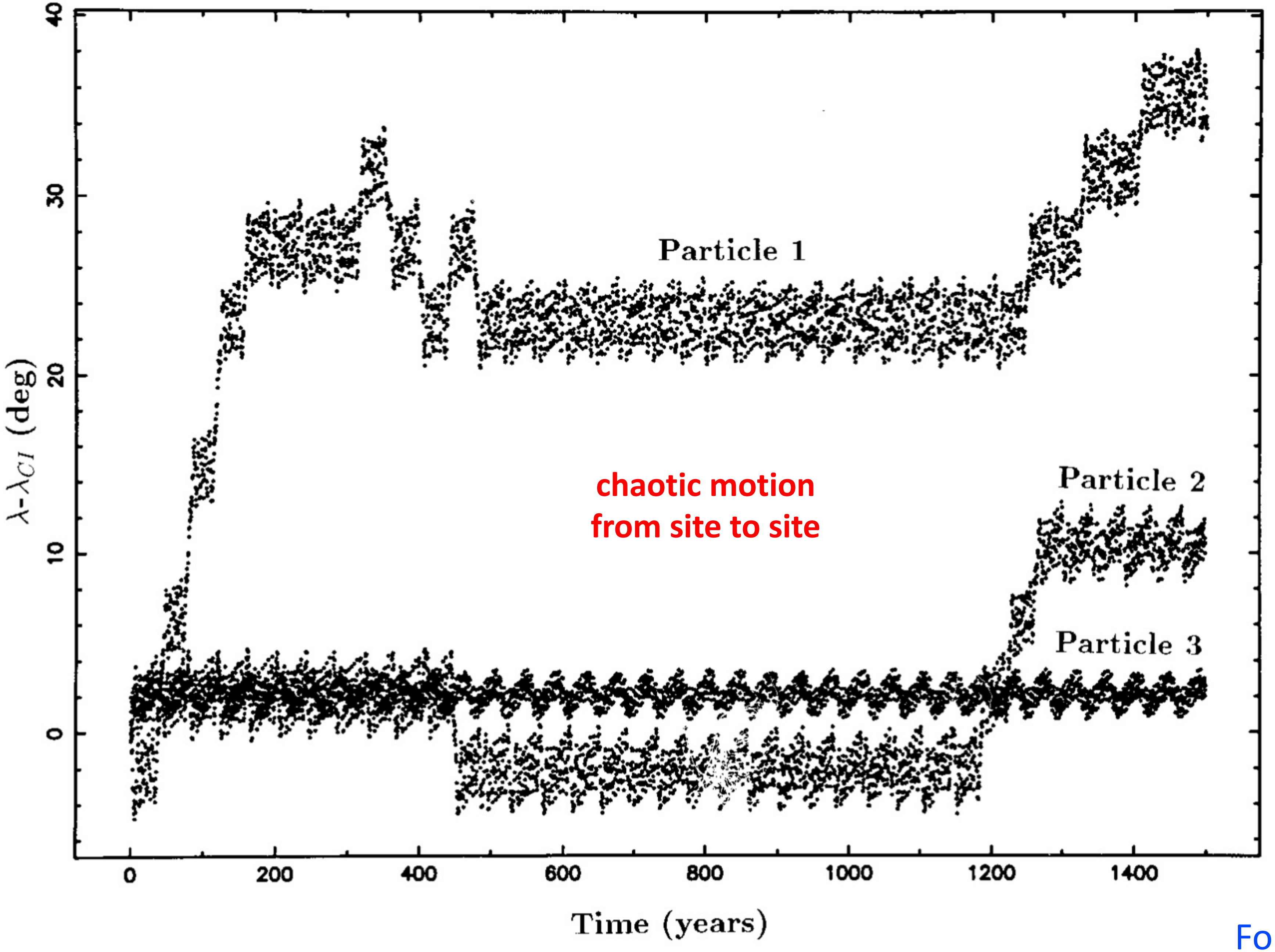




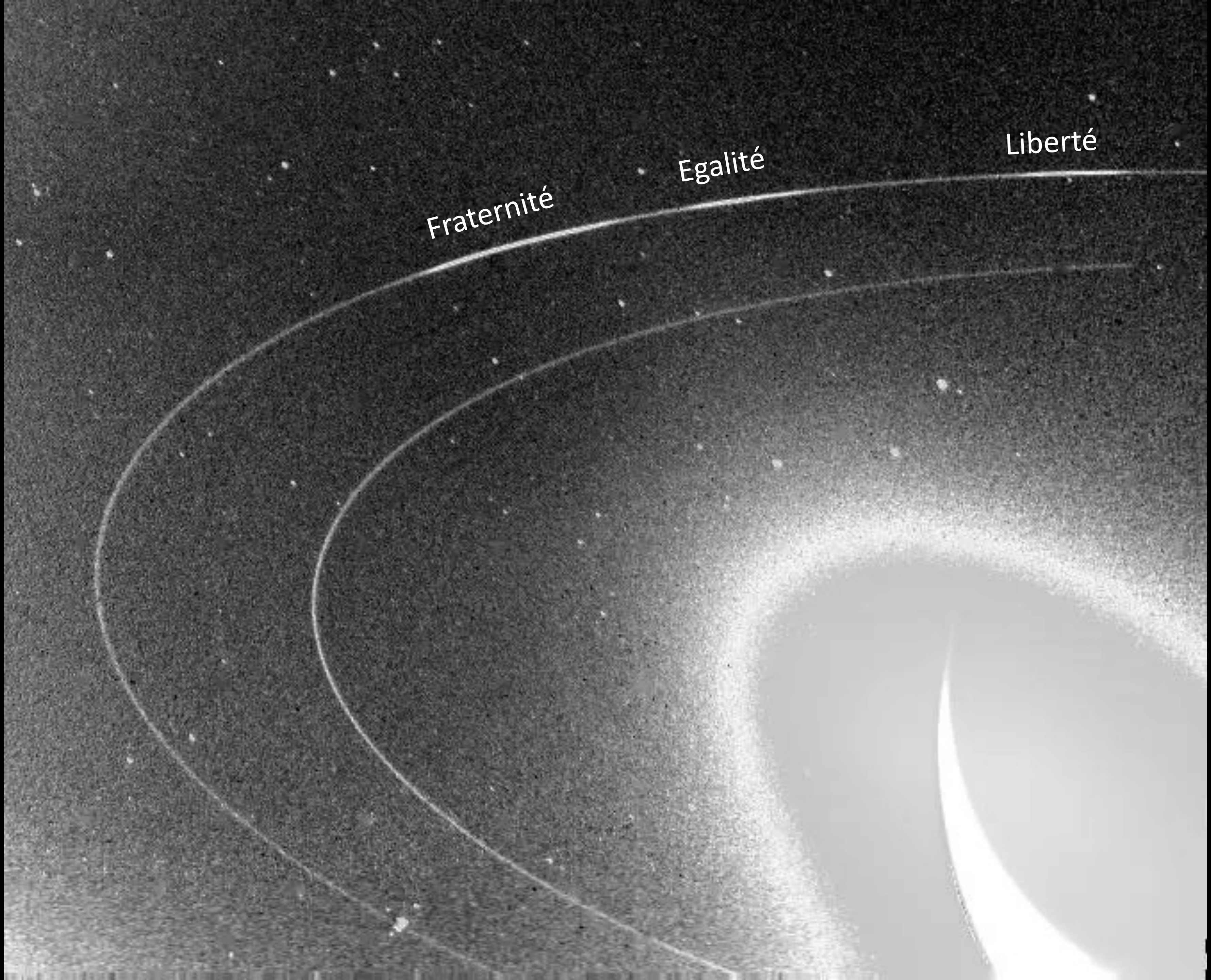


Sicardy, *La Recherche* (1994), from idea of Goldreich, Tremaine, Borderies (1986)

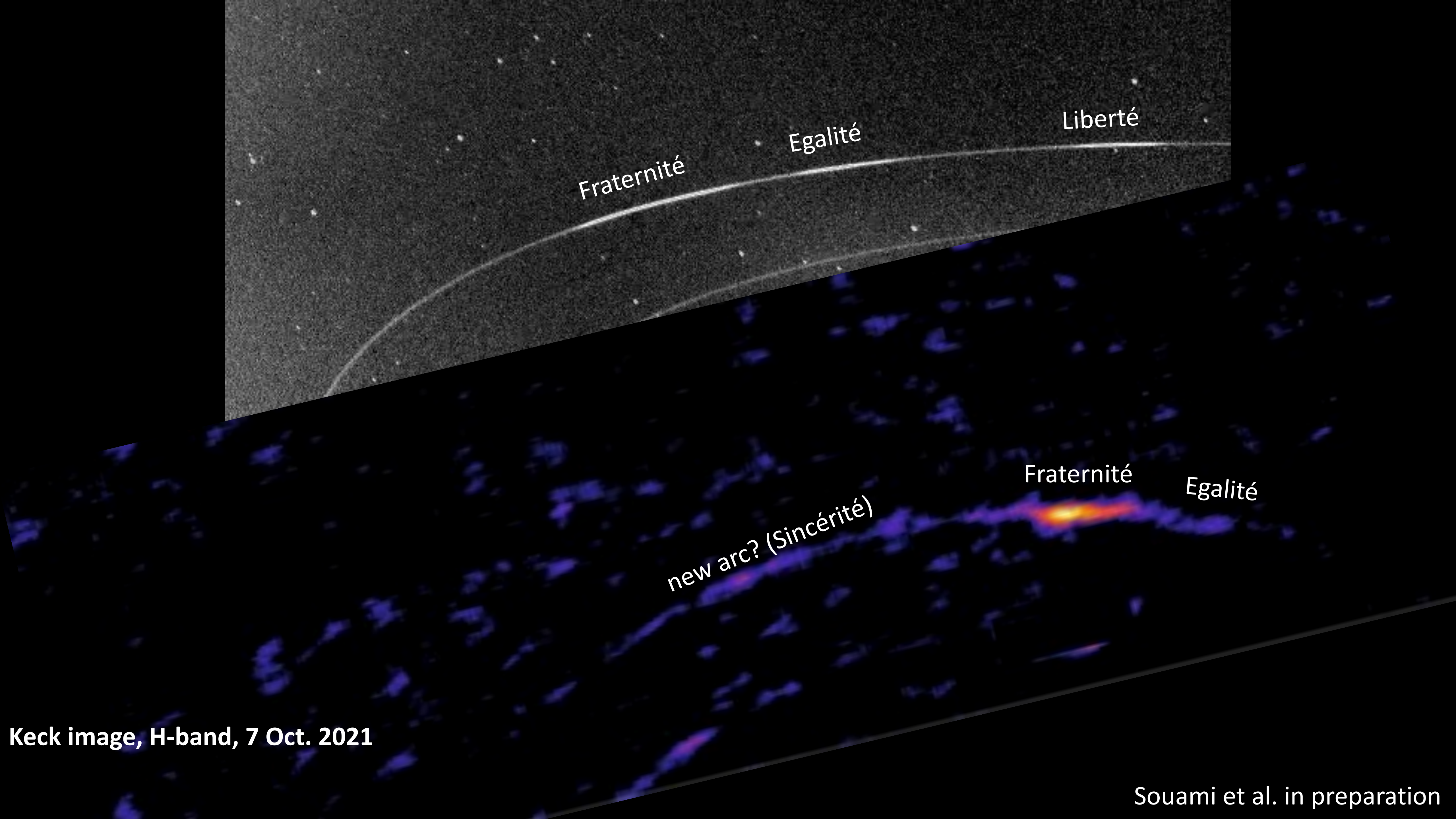




Foryta & Sicardy 1996



NASA/Voyager
August 1989


Liberté
Egalité
Fraternité
Fraternité
Egalité
new arc? (Sincérité)
Keck image, H-band, 7 Oct. 2021
Souami et al. in preparation

The connection between disks, resonances and tides…

MATHEMATICAL AND DYNAMICAL ASTRONOMY SERIES

Volume 3

RESONANCES IN THE MOTION OF PLANETS, SATELLITES AND ASTEROIDS

Edited by
S. FERRAZ-MELLO and W. SESSIN

CONTENTS

PLANETS AND GENERAL THREE-BODY PROBLEM

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DISK-SATELLITE INTERACTION: SOME REMARKS

BRUNO SICARDY

CNPq – Observatório Nacional, Rio de Janeiro, Brasil

ABSTRACT. The response of a disk orbiting a massive central body and perturbed by a satellite is described. It is shown heuristically why viscosity and autogravitation are expected to give the same exchange of angular momentum (torque) between the disk and the satellite. Another example of dissipation process given in this work provides again a similar expression for the torque ("standard formula") and some possible applications to the formation of the Kirkwood gaps are given. Preliminary results of a numerical simulation indicate that the standard formula applies as well to numerically simulated systems.

1. INTRODUCTION

In the recent past, disk dynamics received an increasing interest from the astrophysical community, due to the widespread presence of such objects in the universe. To quote only a few examples, we can think of galactic disks, accretion disks in close binary systems, the asteroidal belt in the Solar System, planetary rings, etc. In the last few months, a new and exciting class of such objects has been observed: circumstellar dust disks which are probably precursors of extra-solar planetary systems (Auman *et al.*, 1984; Smith and Terrile, 1984). More than a mere resemblance of morphology, these systems share common dynamics. A good illustration of this is given by recent Voyager flybys of Saturn's rings. The high resolution images show spiral structures whose formation and evolution had already been described more than a decade ago in the frame of galactic dynamics (Shu, 1984).

The aim of this paper is to present as simply as possible one aspect of disk dynamics, namely the disk-satellite interaction. The question to be answered is

S. Ferraz-Mello and W. Sessin (eds.), Resonances in the Motion of Planets, Satellites and Asteroids, 167-186. Universidade de São Paulo, São Paulo, 1985.

ICARUS 69, 157–175 (1987)

On the Physics of Resonant Disk–Satellite Interaction

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Within the framework of a single derivation, we study the transfer of angular momentum in a disk subjected to a linear perturbation at Lindblad resonance, whenever the physics include friction, nonstationarity, or self-gravitation, pressure, and viscosity. Each of the above physical processes can be described by one parameter which indicates the main physics at work and the resonance width. We show that dissipation or waves are not formally necessary for a torque to appear, but only for the problem to remain stationary and/or linear. In this framework, the torque exerted at an isolated resonance is independent of the particular physics at work. We consider applications to numerical simulations, the impulse approximation, planetesimal accretion, and edges and gaps in planetary rings. © 1987 Academic Press, Inc.

I. INTRODUCTION

Resonance effects between a disk and a satellite orbiting a massive body can dramatically shape the structure and evolution of the disk. Such effects are found in different objects such as galaxies perturbed by a central bar, accretion disks in close binary systems, planetary rings perturbed by satellites, or the asteroid belt subjected to Jupiter's influence. These systems share common dynamics, even though their physics and spatial scales are very different.

The aim of this paper is to present in a general way the exchange of angular momentum (torque) between the "satellite" and the disk in a typical case of resonances, namely the Lindblad resonances which occur when the mean motion of the disk particles and that of the satellite are in the ratio $m/(m \mp 1)$, where m is a positive integer.

Our starting point lies in some previous studies of Lindblad resonances for various astrophysical objects and in the connection of such problems to plasma physics. An intriguing point is that the expression of the torque always bears the same form in spite of the very different physics at work (Goldreich and Tremaine, 1982). To quote a few examples, we can think of galactic dynamics (Lynden-Bell and Kalnajs, 1972), accretion disks (Lin and Papaloizou, 1979), planetary rings (Goldreich and Tremaine, 1979), and numerical simulation (Sicardy, 1985). Also, the sign of this torque is always the same, so that the satellite always repels the disk. The torque per unit of surface density does *not* depend on the physical parameters of the disk, but only on the mass of the satellite, the mass of the central body, and the order m of the resonance (cf. Eq. (16) of this paper).

Greenberg (1983) proposed to answer that puzzle by assuming that the particles are subjected to friction effects which suppress the singularity at exact resonance. As the damping decreases, the perturbed motion of the particle increases but the width of the resonance shrinks, so that the two effects compensate and the torque is independent of the damping coefficient.

However, other mechanisms such as density waves (Goldreich and Tremaine, 1979) can create a torque without explicit or implicit dissipation. We show in this paper that a wide class of physical processes can actually displace the resonance frequency in the complex plane, so that the torque density function (torque per unit radius) has

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Two-stage formation of the Moon from accreting fragmentation and resonance captures

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Meyer-Vernet, N. & Sicardy, B. 1987, Icarus, 69, 157
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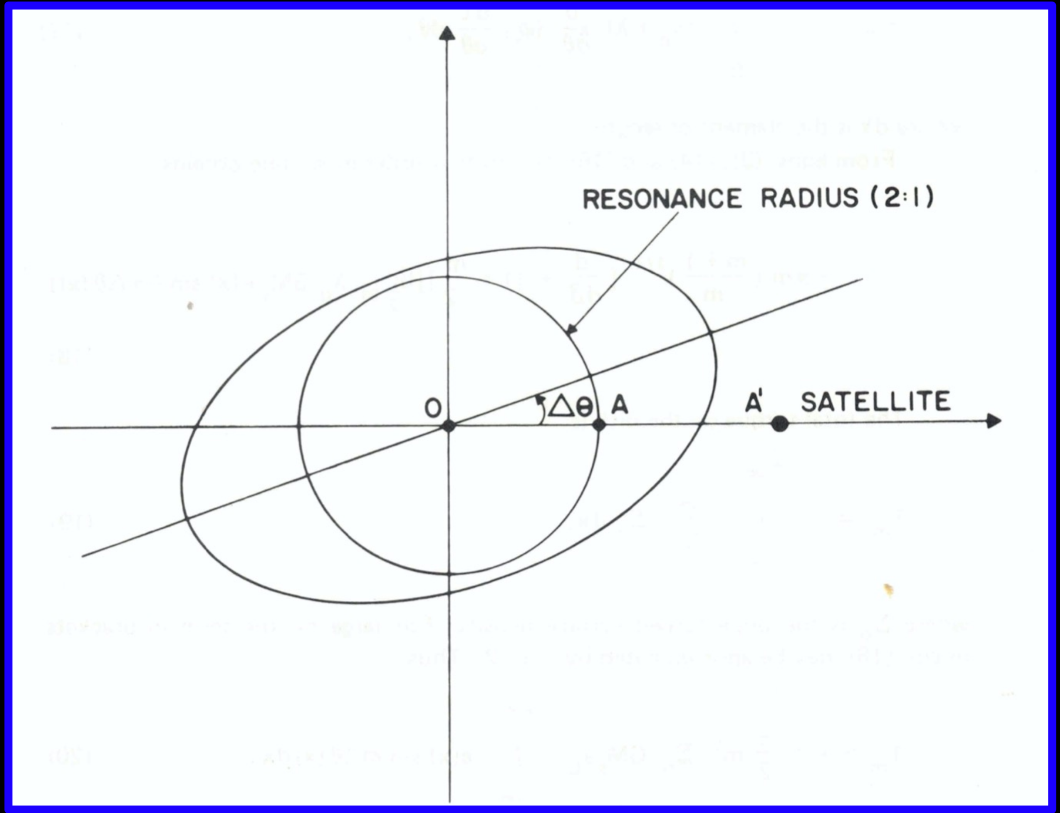
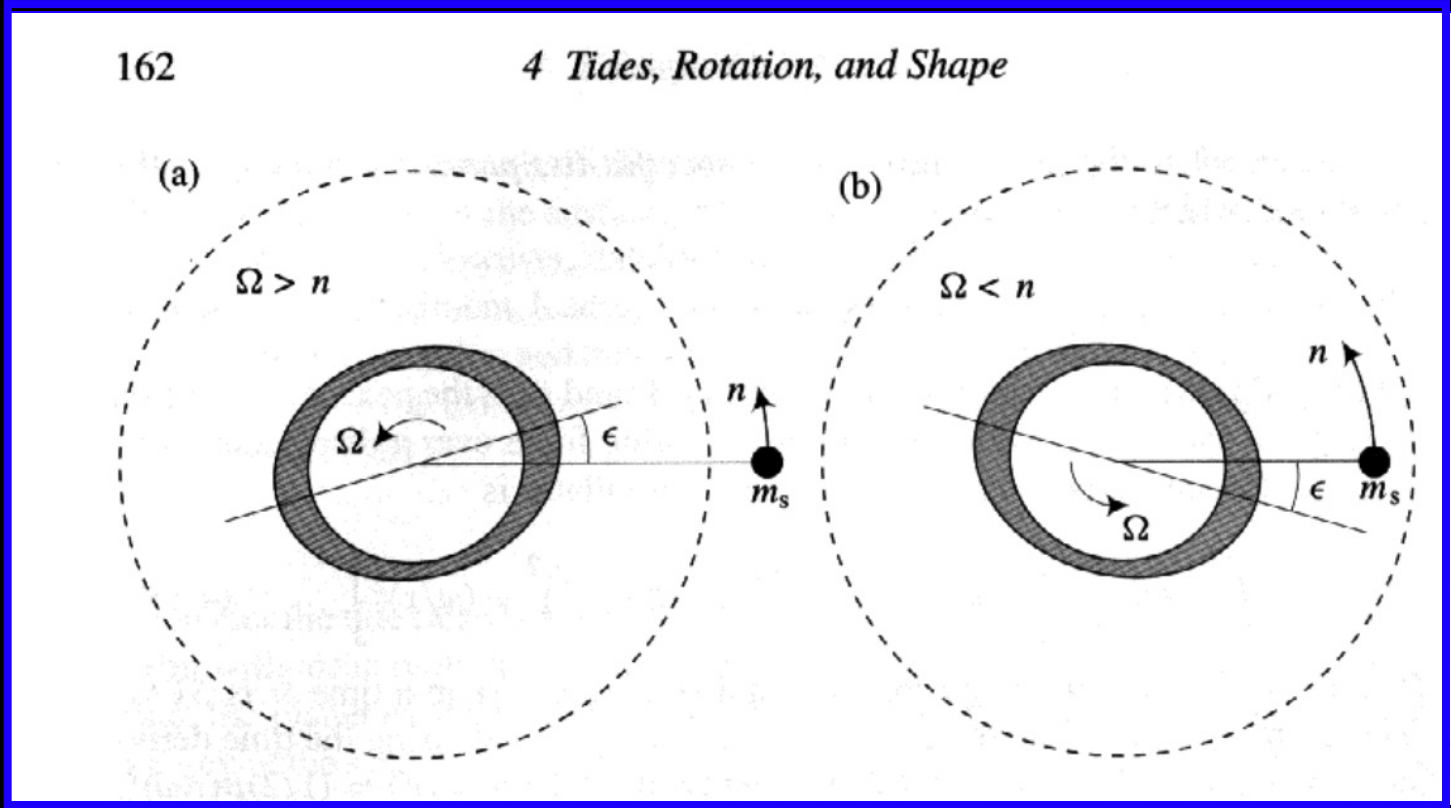
The response of a disk to a perturber is a kind tidal effect!

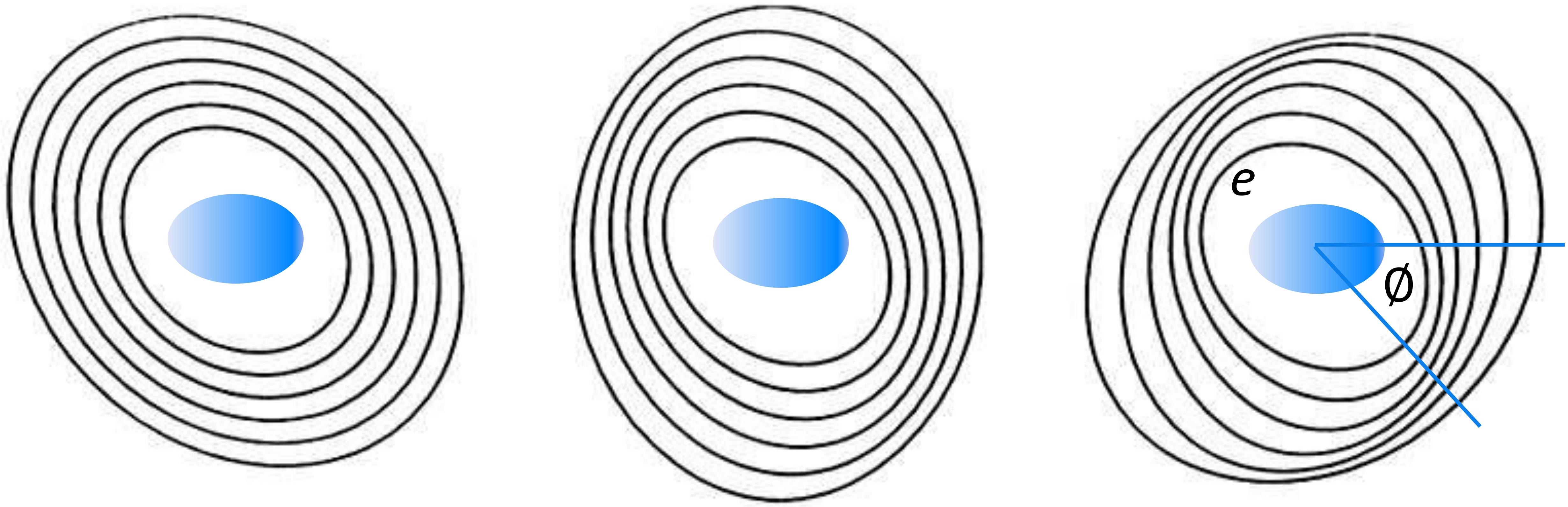


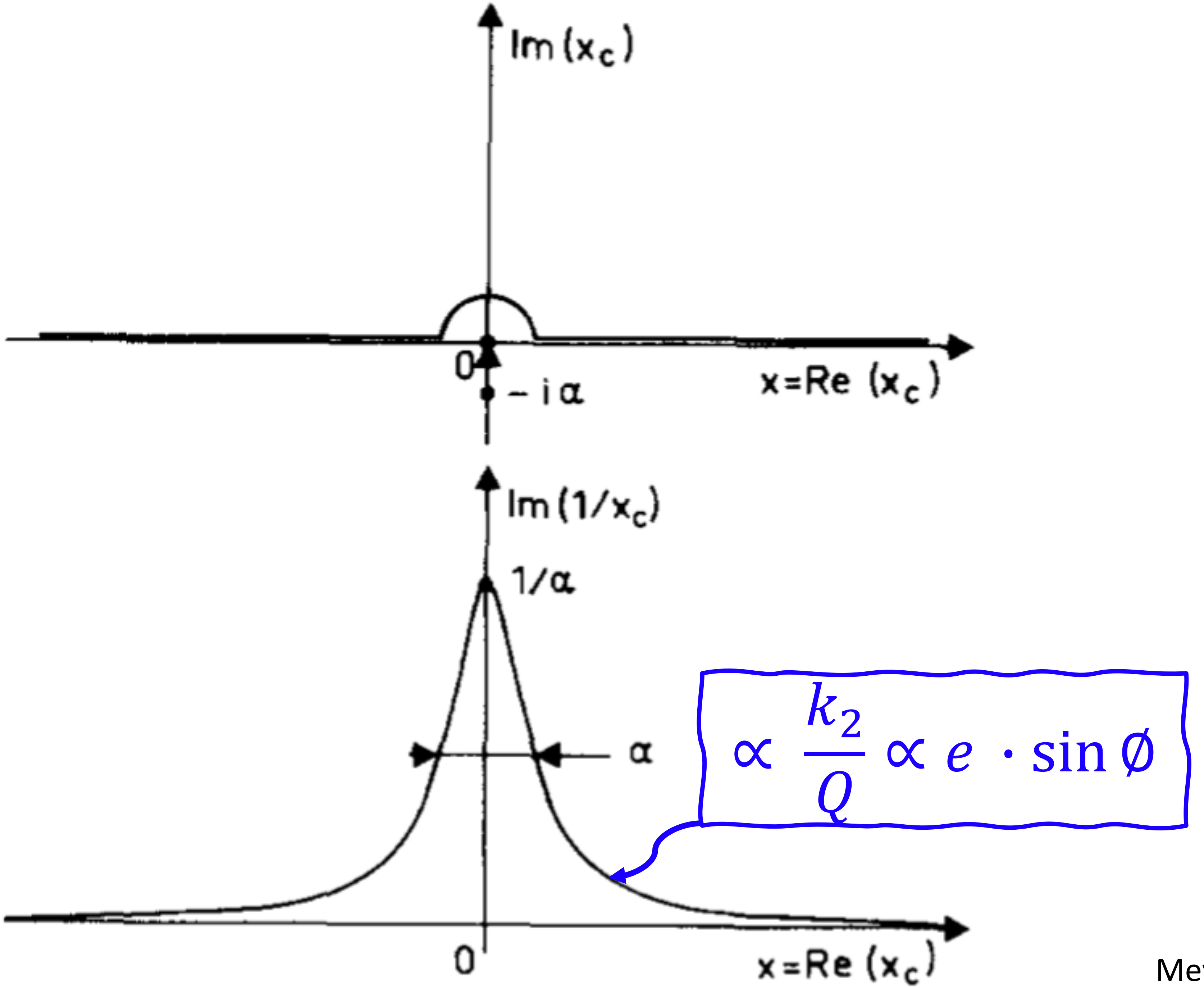


Sicardy 1985

Murray & Dermott 1999

In particular, a tidal torque: $\Gamma \propto e \cdot \sin\phi \propto \frac{k_2}{Q}$





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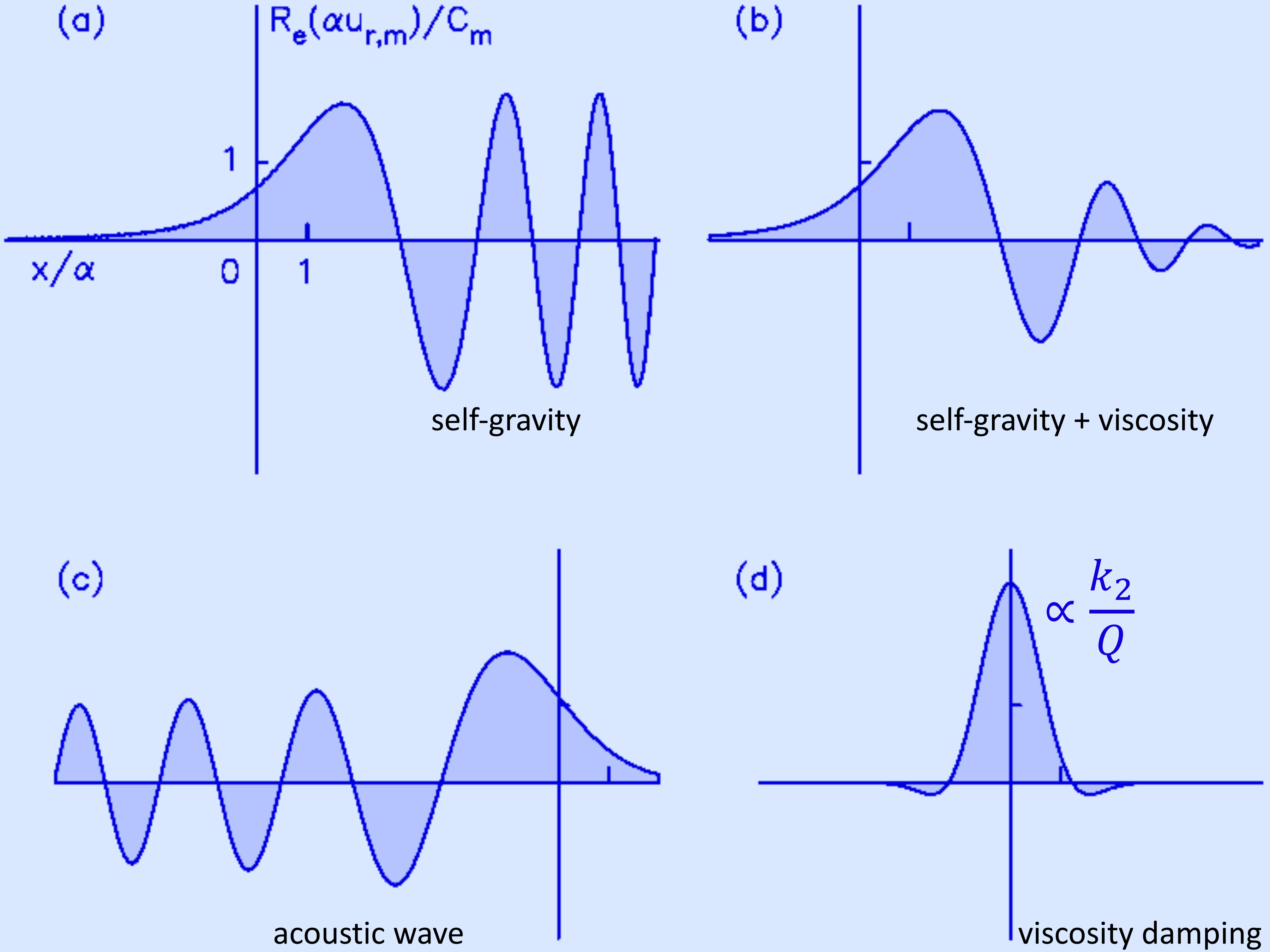

Meyer-Vernet &
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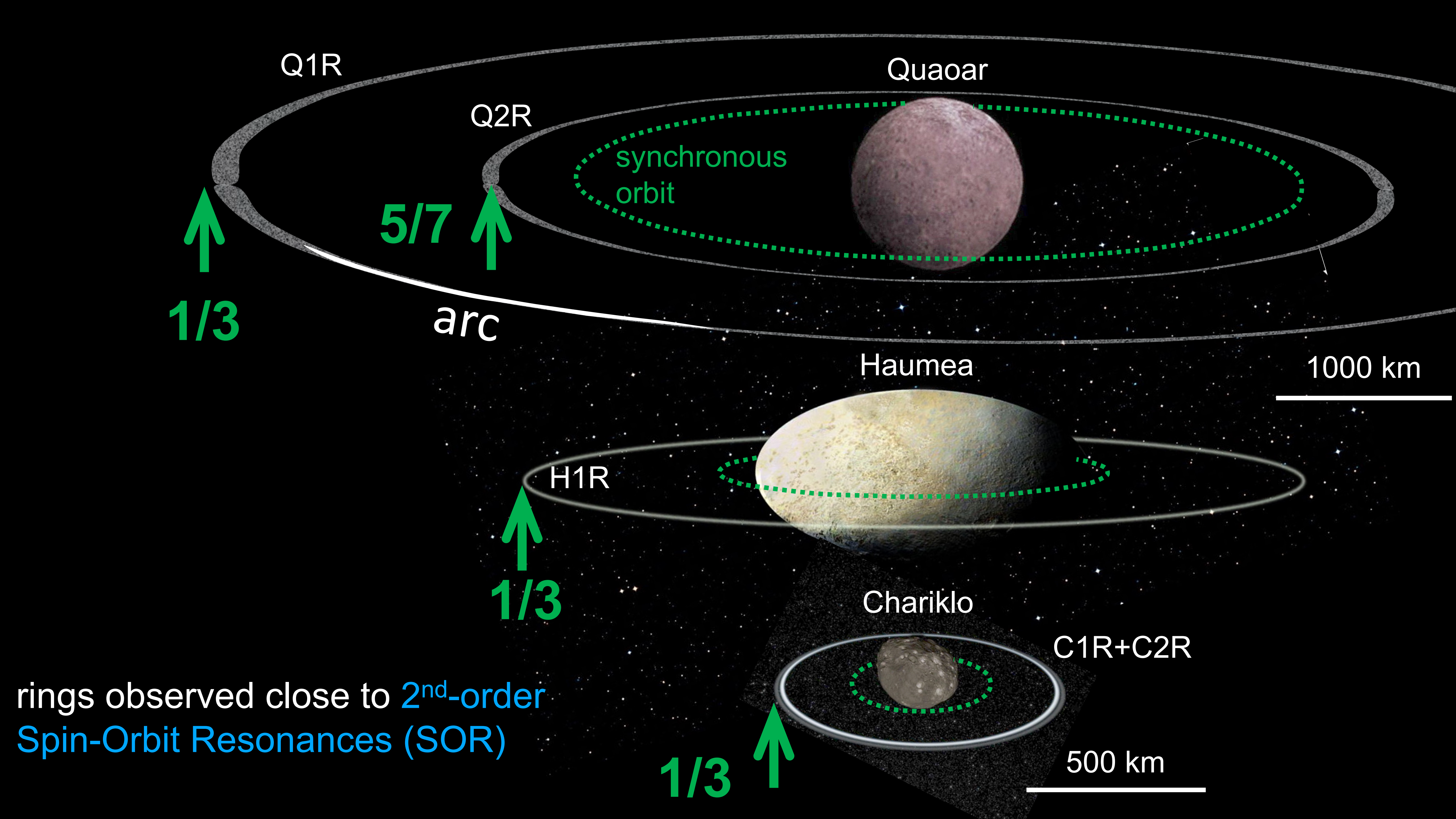
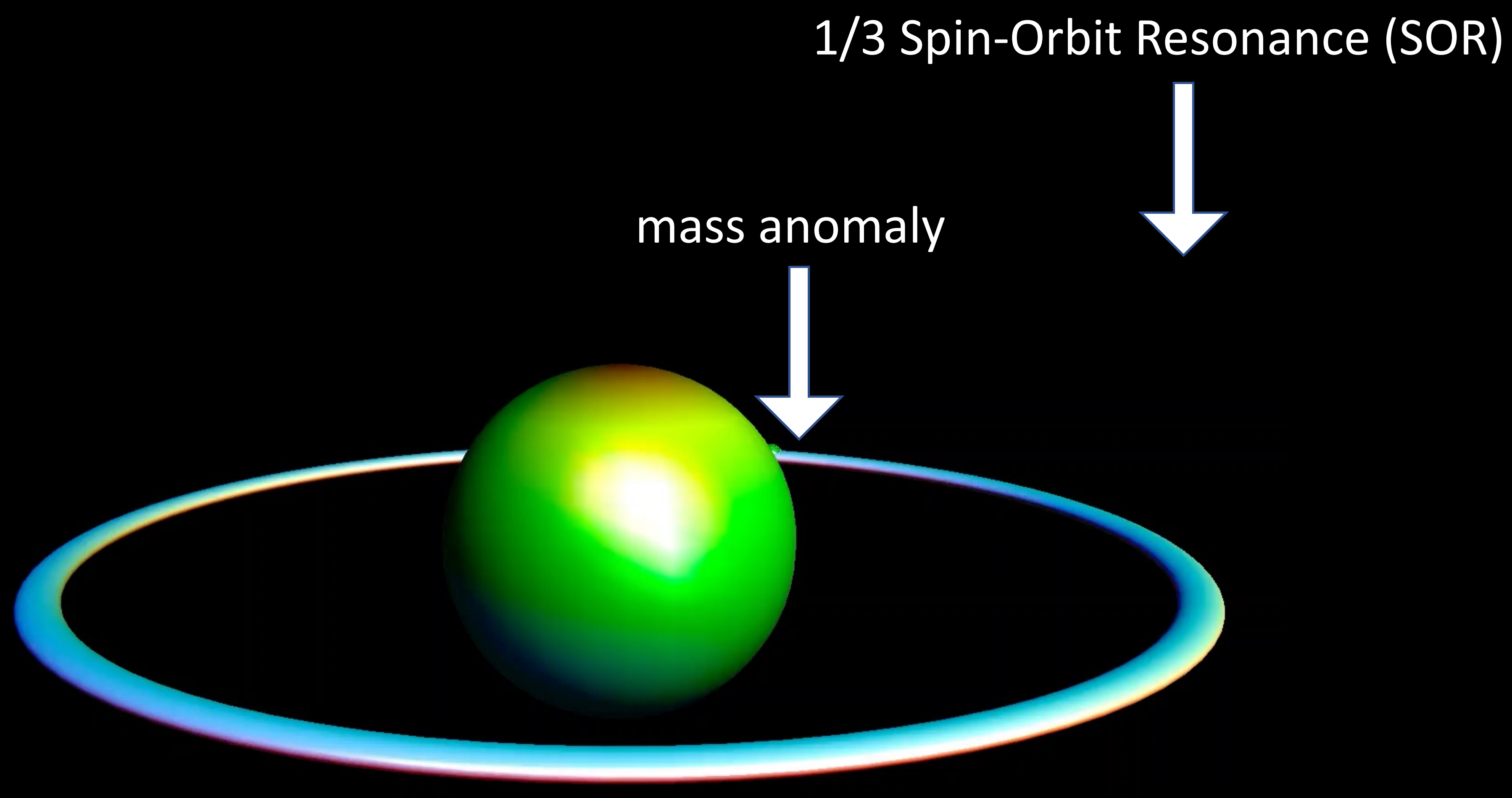
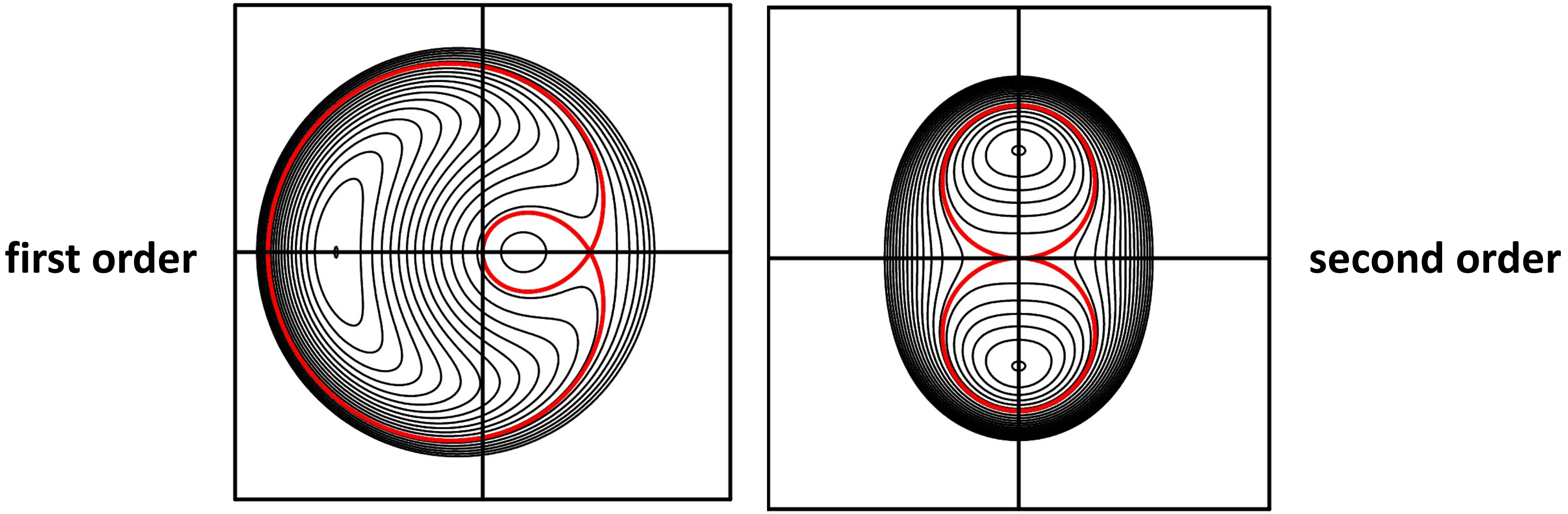
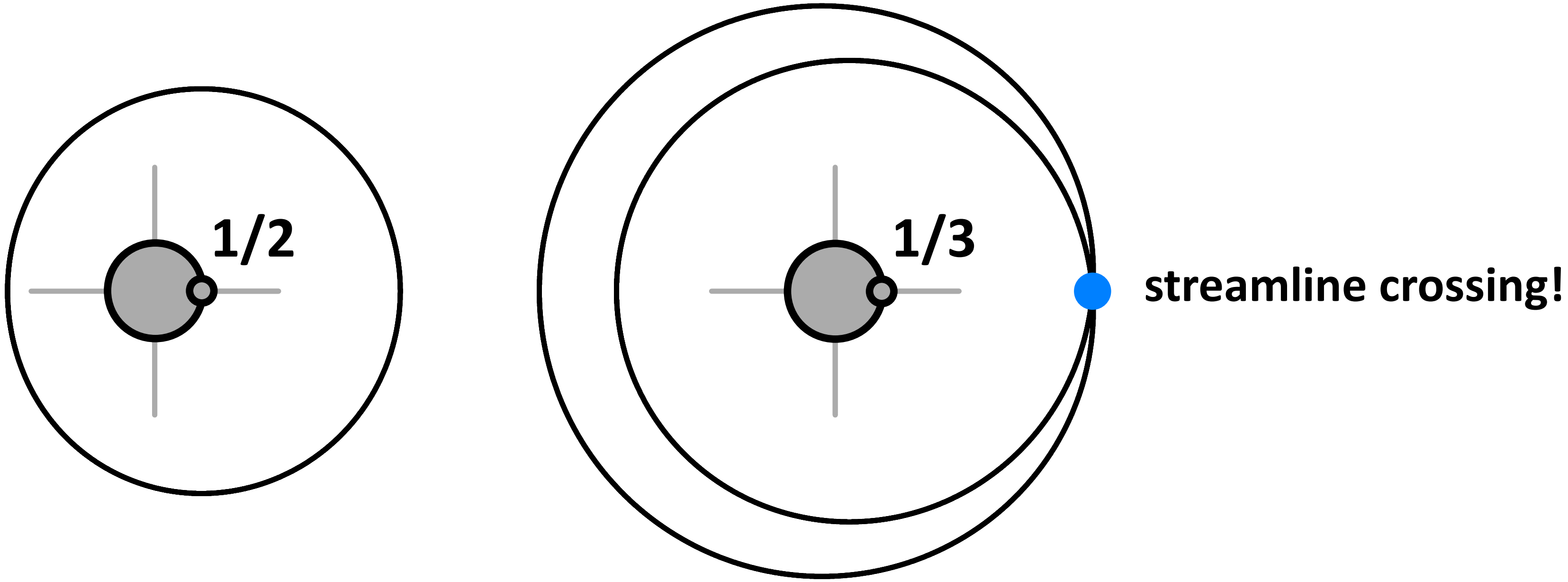
capture of rings in resonances
around small objects:
a surprising result



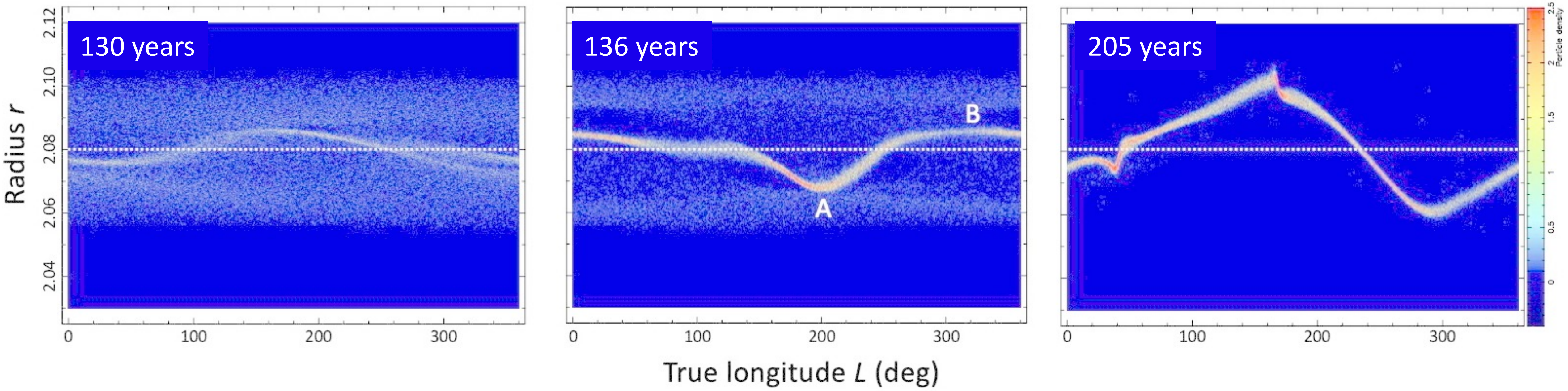
Q1R
Quaoar
Q2R
synchronous
orbit
5/7
1/3
arc
Haumea
1000 km
H1R
1/3
Chariklo
C1R+C2R
rings observed close to 2nd-order
Spin-Orbit Resonances (SOR)
1/3
500 km


1/3 Spin-Orbit Resonance (SOR)
mass anomaly


first order
second order

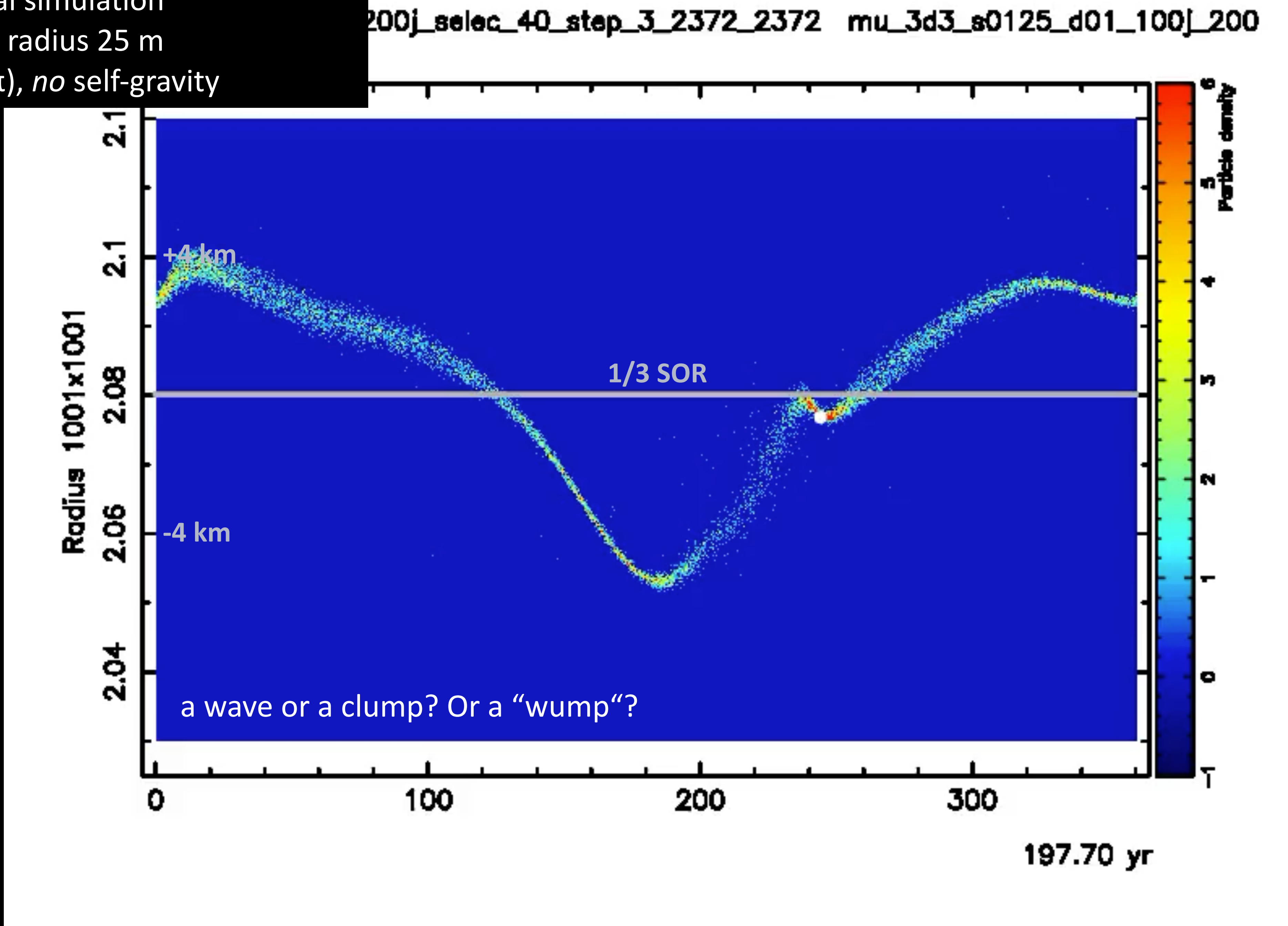

1/2
1/3
streamline crossing!


the 1/3 resonance:
the recipee for a disaster…



130 years
136 years
205 years
Radius r
2.12
2.10
2.08
2.06
2.04
0
100
200
300
A
B
True longitude L (deg)
Particle density

Mass anomaly μ= 0.003
N-body collisional simulation
40,000 particles, radius 25 m
complete (3D, 2π), *no* self-gravity



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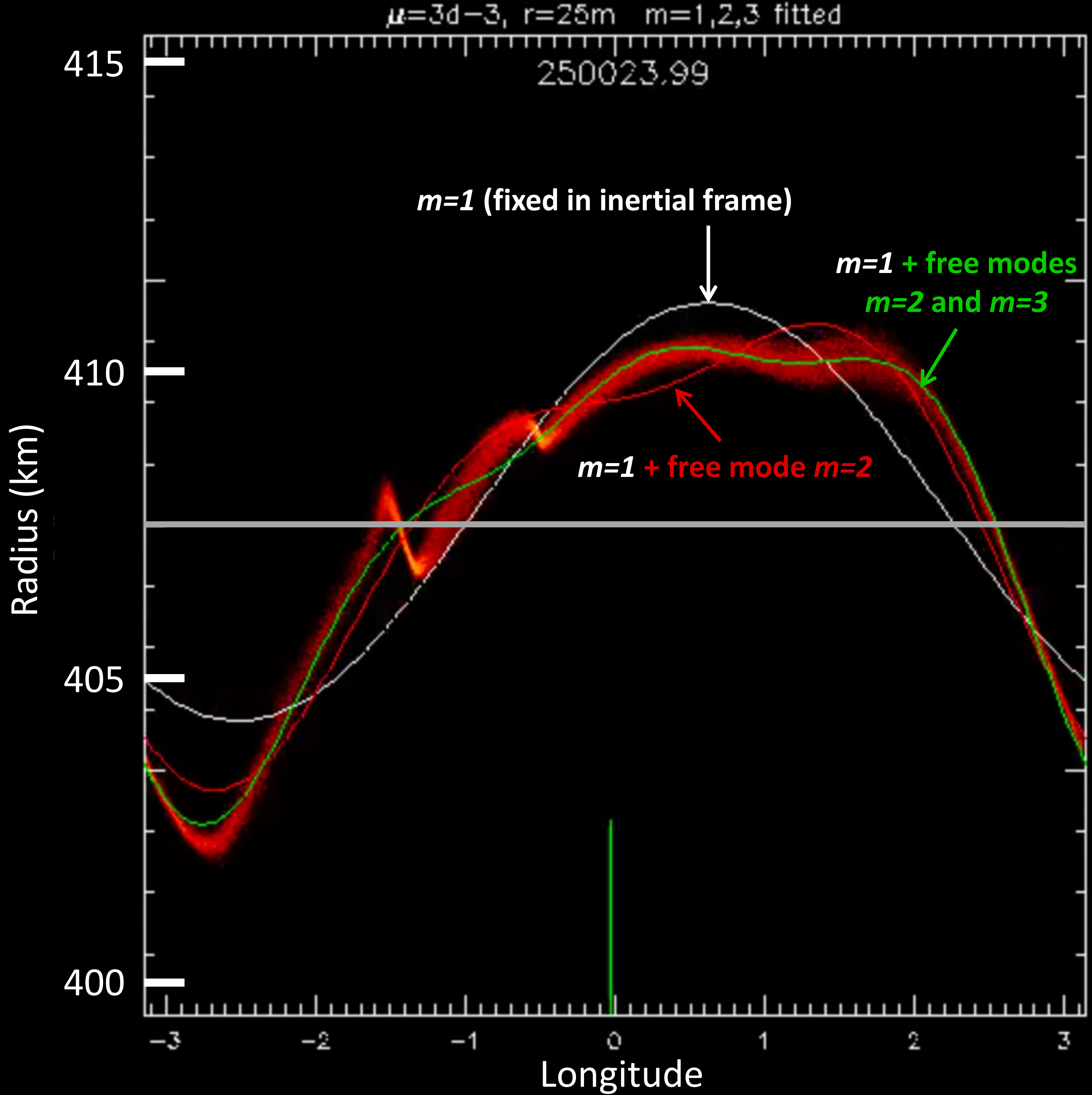

Free modes:
Lindblad-type
m-lobed oscillations

Salo &
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μ=0.003 1/3 resonance

final ringlet stage

rot=1/3 (co–rotating with the ringlet)

After subtracting the *m*= 1, 2 , 3 modes →

1 km

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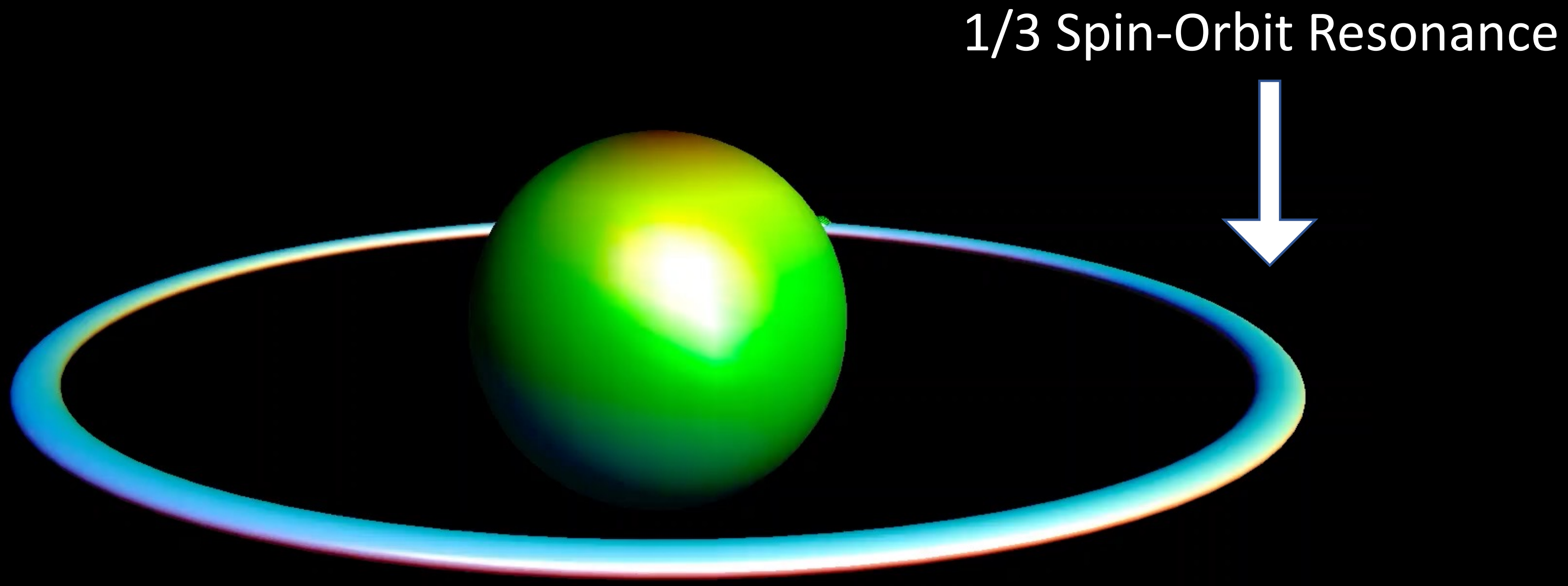
The ring *is* confined at the
1/3 Spin-Orbit Resonance,
but does *not* have a 1/3 resonant behavior!



obrigado gente!