Arches of chaos, heteroclinic connections of first-order MMRs and the chaotic transport of small bodies in the Sun-Jupiter system

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1. Summary

We discuss how the transport of small bodies through the orbit of Jupiter in the Solar System is governed by the heteroclinic intersections between the stable and unstable manifolds of the unstable periodic orbits corresponding to each one of the main mean motion resonances between the body's and Jupiter's orbits.

These manifolds have been extensively discussed in the literature in the case of the co-orbital resonance, but to a lesser extent for other important mean motion resonances.

Results

We show how a global visualization of these manifolds can be achieved through the computation of short time Fast Lyapunov Indicator maps [2], allowing to depict their underlying intricate heteroclinic dynamics (Figure 1 and 2).

Besides the classical heteroclinic channels established by Koon et al.([3]), we give evidence of heteroclinic connections between the manifolds of the short-period orbits around L3 and of the periodic orbits associated with interior or exterior first order MMRs. Moreover, we observe direct heteroclinic connections between the manifolds of the interior with exterior MMRs, which do not involve the manifolds of any of the periodic orbits of the co-orbital resonance.

Most of our results are obtained in the framework of the planar circular RTBP. However, through FLI maps we show that the manifold connections observed in the circular problem persist in the elliptic problem as well.

2. Method

The *FLI* (Fast Lyapunov Indicator) maps are obtained by integrating orbits forward and backward in time, while varying the initial values of φ and p_φ, in the planar circular restricted three-body problem given by the Hamiltonian:

$$H = \frac{1}{2}\left(p_r^2 + \frac{p_\varphi^2}{r^2}\right) - \Omega_J p_\varphi - \frac{GM_S}{\sqrt{(r\cos\varphi - x_S)^2 + (r\sin\varphi - y_S)^2}} - \frac{GM_J}{\sqrt{(r\cos\varphi - x_J)^2 + (r\sin\varphi - y_J)^2}}$$

where (r, φ) are cylindrical coordinates in the barycentric synodic frame.
By computing the *FLI* values for short times, one can visually observe chaotic saddle structures created by the stable and unstable manifolds of different unstable periodic orbits characterizing the phase space.

The Lagrangian coherent structures formed by the union of the stable and unstable manifolds of several MMRs (e.g 1:1 (co-orbital), 3:2 (Hildas), the interior MMR 2:1, and the exterior MMR 2:3), both interior and exterior to the orbit of Jupiter, are shown in Figures 5 and 6. Furthermore, overlapping the backward and forward FLI maps reveals a rich network of heteroclinic connections created by the intersections of these manifolds (Figure 7).

Through an explicit computation by common numerical techniques of manifold computation in dynamical systems, we aim to specify the exact correspondence between the structures observed in the numerical stability maps and the manifolds associated with unstable periodic orbits. Figures 1 and 2 show the intricate set of heteroclinic connections between the manifolds of some unstable periodic orbits covering the entire outer essentially asteroidal belt beyond the 2:1 MMR, and reaching all exterior MMRs beyond the co-orbital resonance and up to 2:3.

Through the manifolds of the MMRs and the corresponding 'ridges' in the numerical FLI maps, we explain the 'arches-of-chaos' structures ([4]) found in FLI maps in the asteroid plane of orbital elements (a, e) (Figures 3 and 4).

3. Heteroclinic connections and arches of chaos

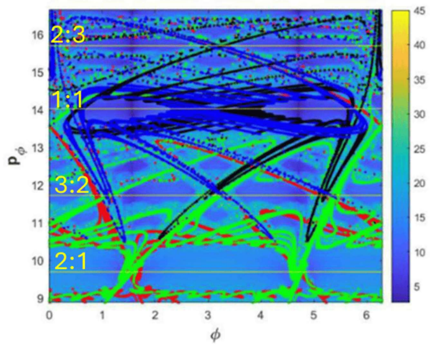

Figure 1. Overlap of FLI map and stable-unstable manifolds associated with the 1:1 and 2:1 resonances for $T = 2.96$ (the Tisserand parameter).

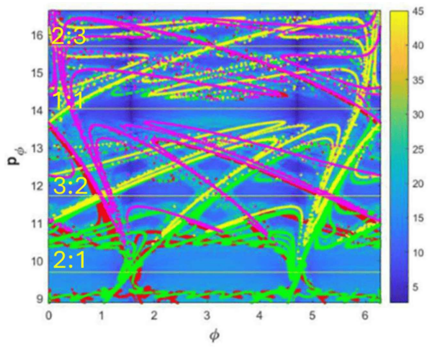

Figure 2. Overlap of FLI map and stable-unstable manifolds associated with the 2:1 and 2:3 resonances for $T = 2.96$.

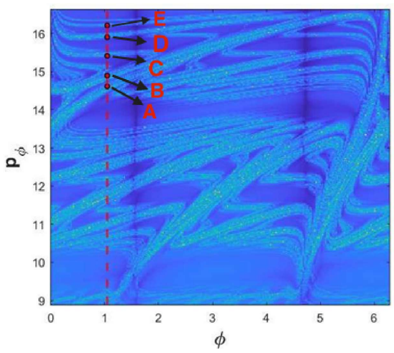

Figure 3. The FLI map for $E_J = 1.48$. The points A,B,C,D,and E are chosen at local ridges of the FLI map intersecting the vertical line $\phi = \pi/3$.

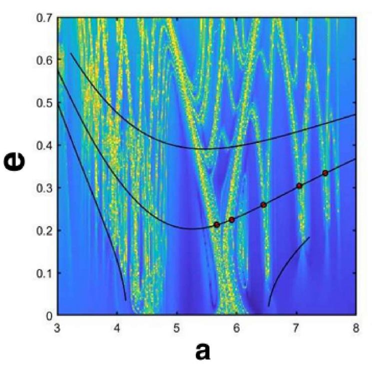

Figure 4. The entire FLI map for forward integrated trajectories with initial conditions in a grid in the (a, e) plane for the section $\phi = \pi/3$, for continous variation of the Jacobi energy E_J.

4.Fli maps

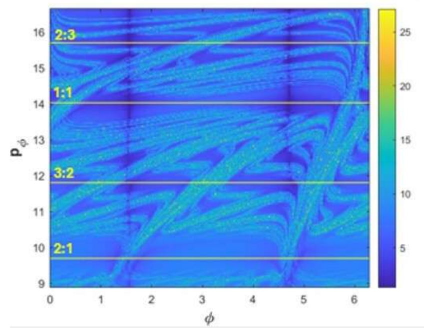

Figure 5. Color Representation of the chaos indicator **FLI** computed in an interval time $[0, 100]$ on a grid of polar initial conditions (φ, p_φ) on the phase-space; the value of $log(FLI)$ is represented using a color scale.

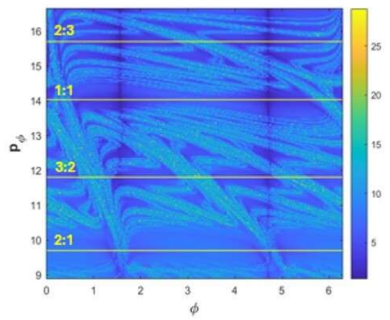

Figure 6. Color Representation of the chaos indicator **FLI** computed in an interval time $[0, -100]$.

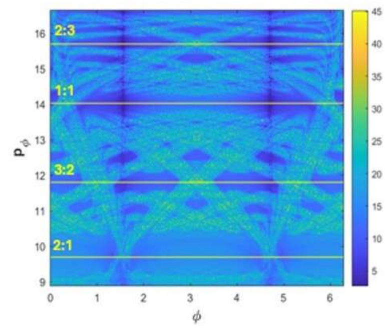

Figure 7. Overlap of the two plots (Figura 5.) and (Figura 6.).

Conclusions

We present numerical computations of stable and unstable invariant manifolds associated with the 2:1, 3:2, and 2:3 MMRs with Jupiter, and use short-time FLI maps to reveal the global Lagrangian Coherent Structure in the resonance overlap domain. We provide numerical evidence that heteroclinic connections between manifolds of different MMRs can drive the 'resonance hopping' observed in small Solar System bodies.

Working mainly in the planar circular RTBP, we further show that these manifold structures persist when switching to the elliptic problem, suggesting a broader applicability of results as the ones here reported. This subject is a natural next step for further study.

References

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- [4] Nataša Todorović et al., (2020), *The arches of chaos in the Solar System* in: Science Advances Vol 6, Issue 48.

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