

Multipole Moments of Kerr-Newman horizons

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 - Zero temperature states
- Relations between source multipoles and asymptotic, observable ones

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Kerr-Newman is the *most general way to reach extremality*, hence why we extend work done in [Gourgoulhon, Casals, Le Tiec, 2026].

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Defined for Axisymmetric horizons

[Ashtekar, 2004]

Generic $M_{l,m}$

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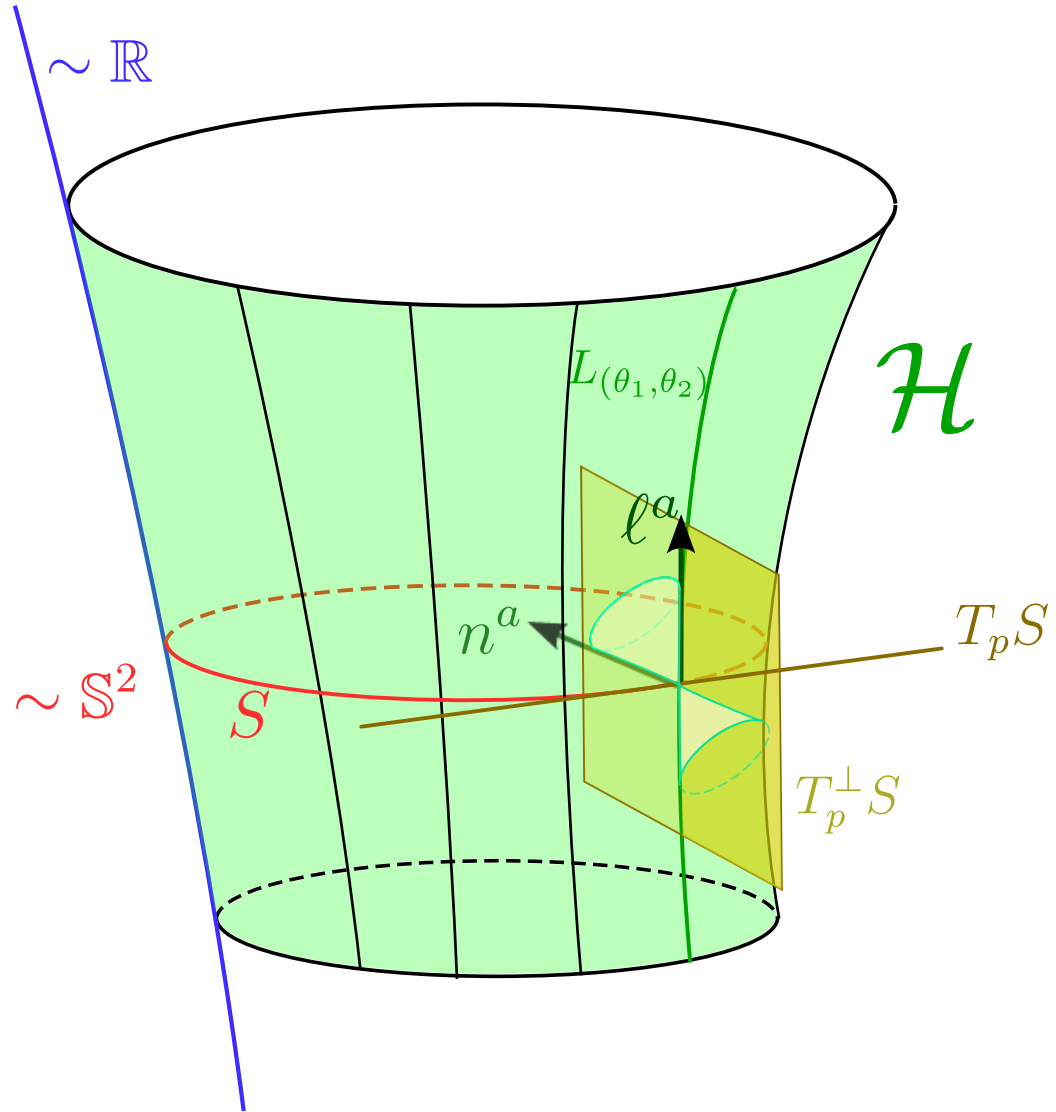
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[Ashtekar, 2022]

One formula:

$$I_{l,m} + iL_{l,m} = \oint_S \mathcal{K} \tilde{Y}_{l,m} dS \quad \text{with} \quad \mathcal{K} = -\frac{1}{4}\mathcal{R} + \frac{i}{2}*d\Omega$$

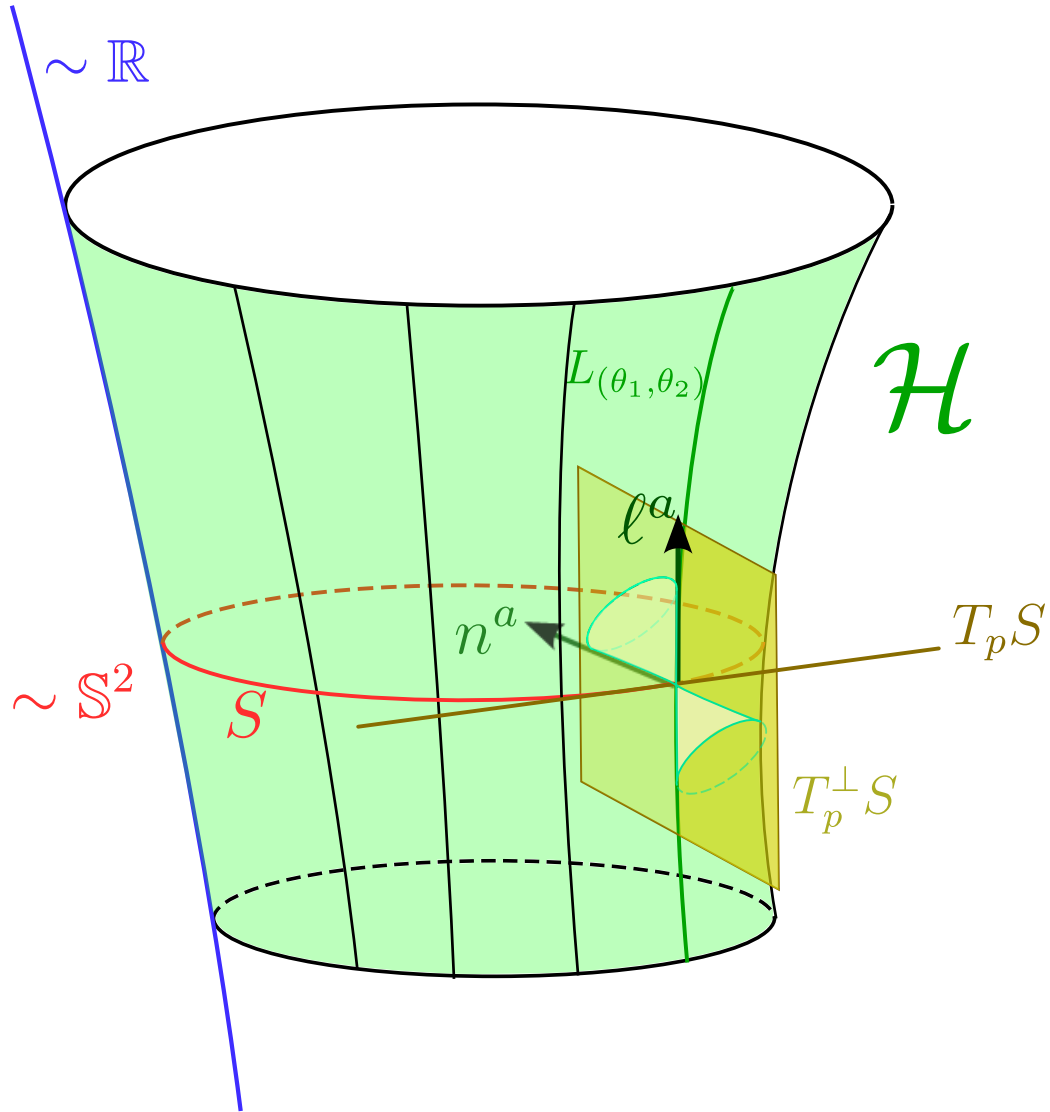
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Step 1: Foliate the horizon as

$$\mathcal{H} \sim \mathbb{R} \times \mathbb{S}^2.$$

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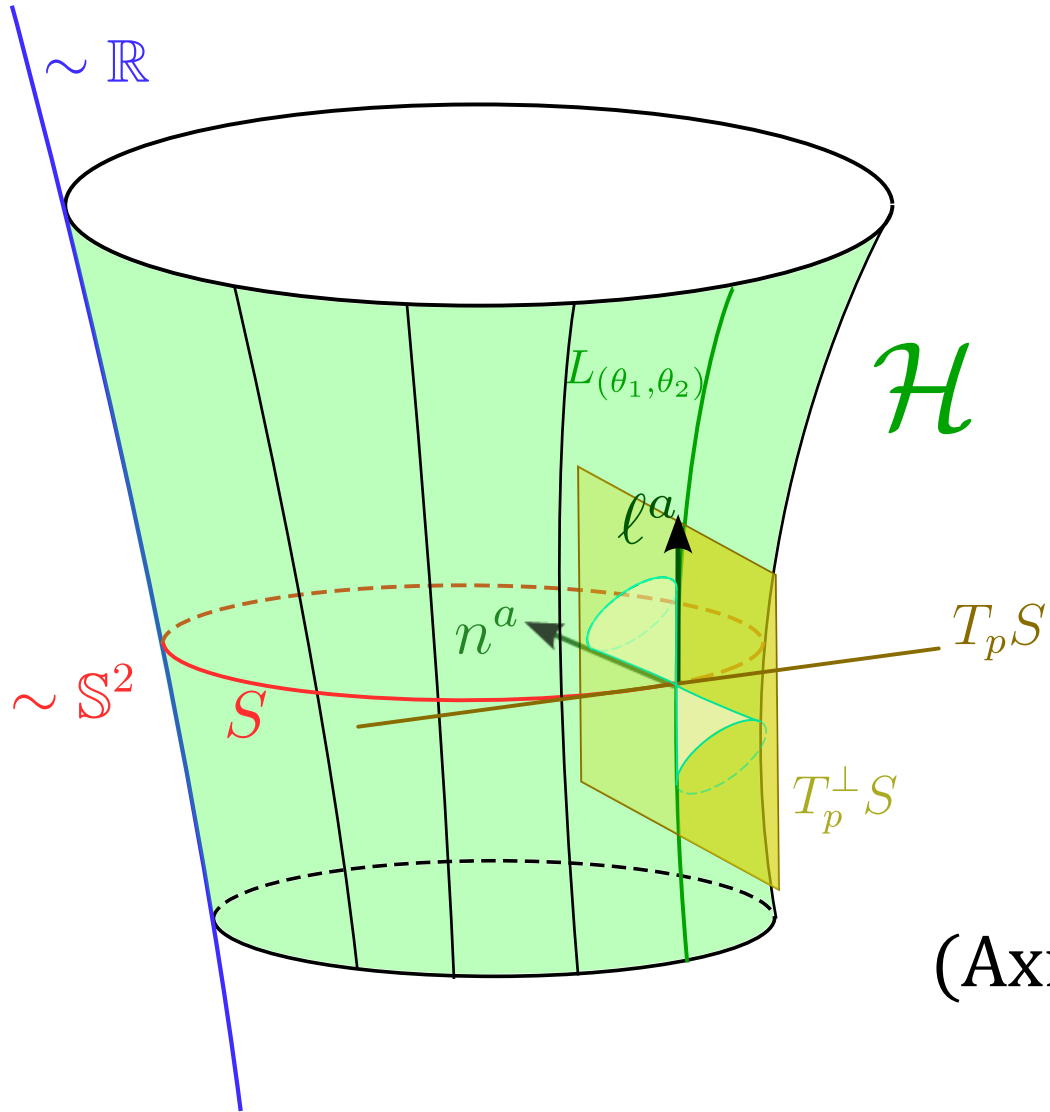


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Step 2: Introduce null normals ℓ^a, n^a

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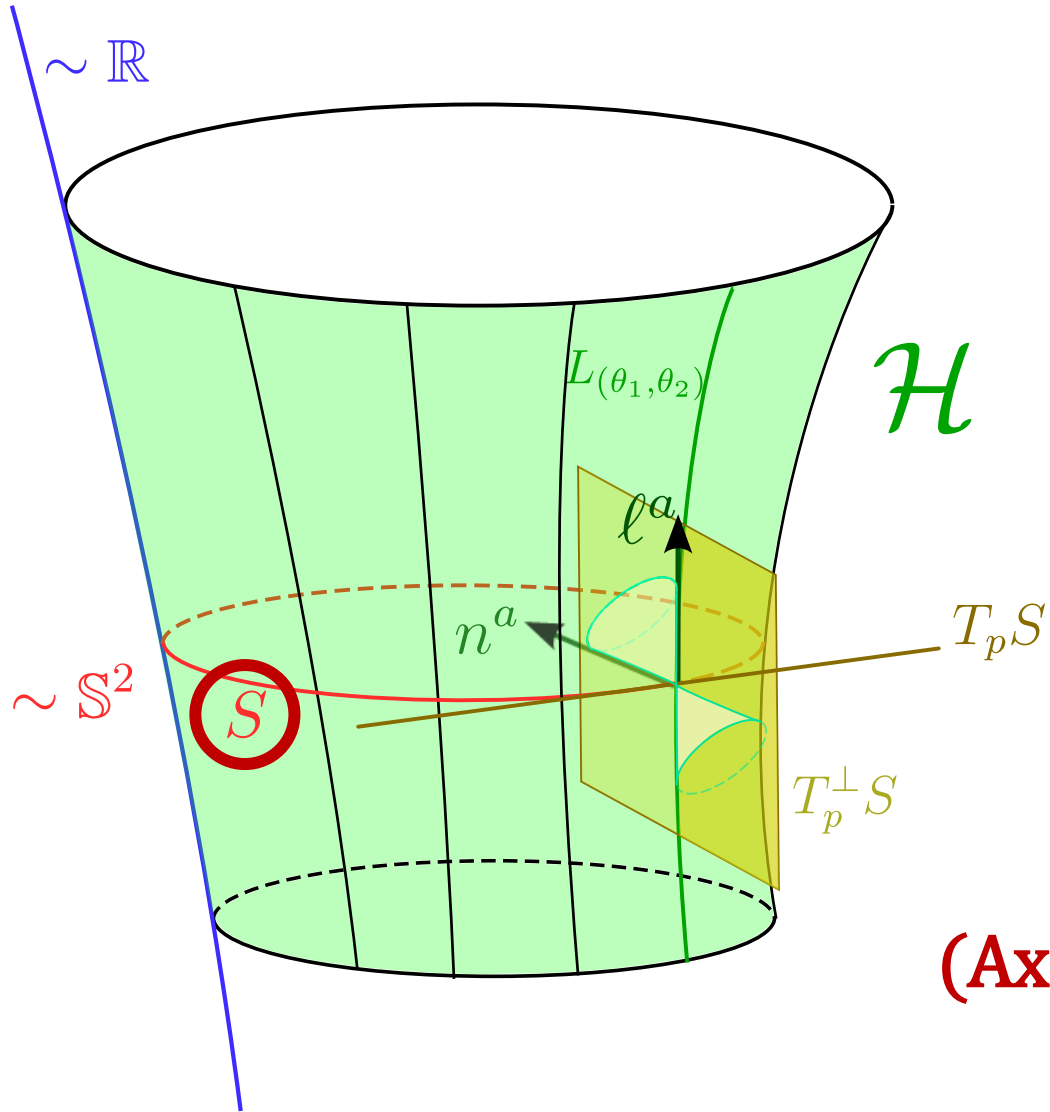
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Step 3: Rebuild 2-sphere metric on S , either:

Uniquely
(Axisymmetric horizon)

Conformally
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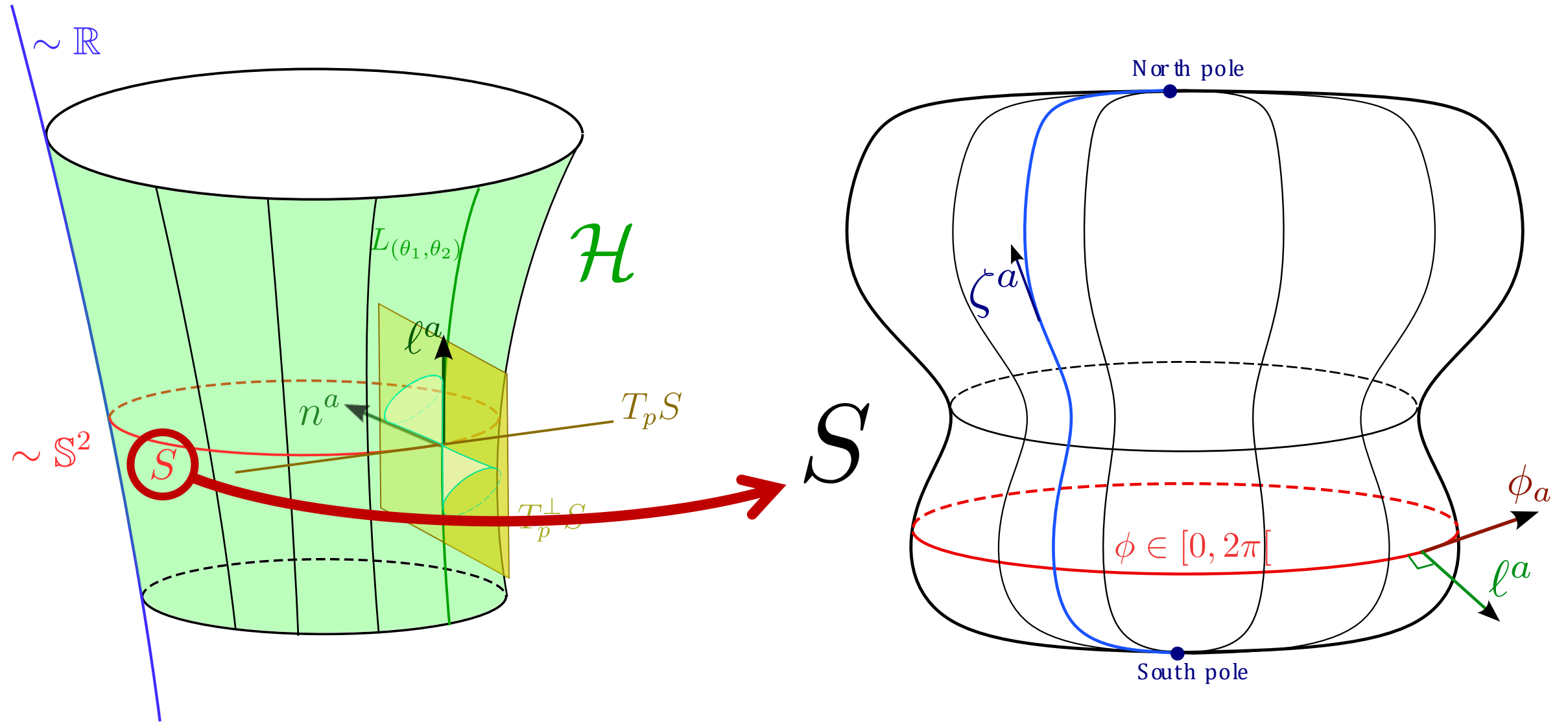
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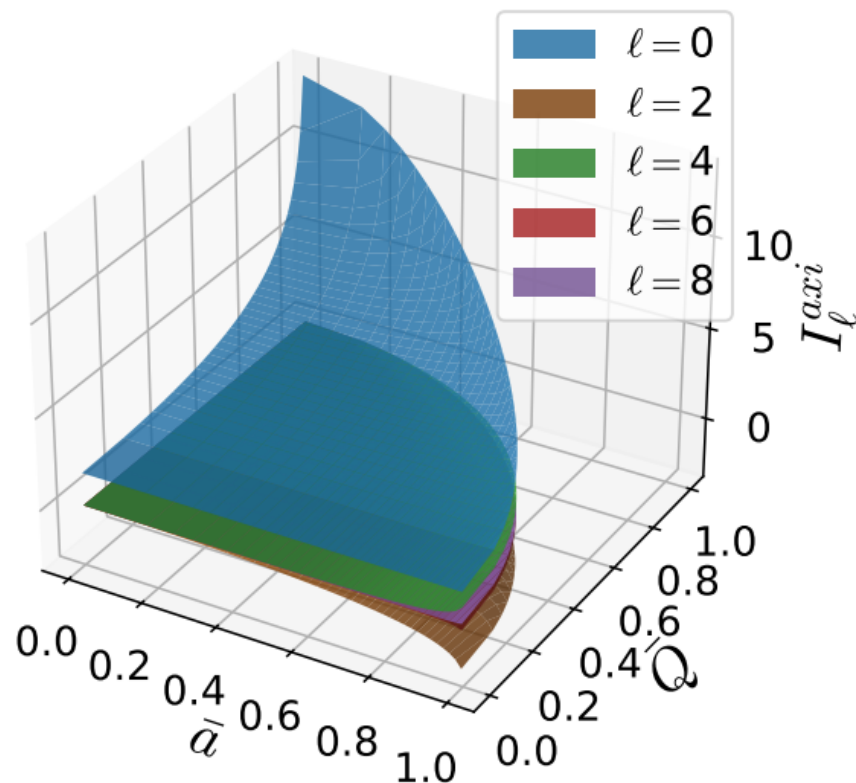
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Main results

Even Axisymmetric multipoles



Odd Axisymmetric multipoles

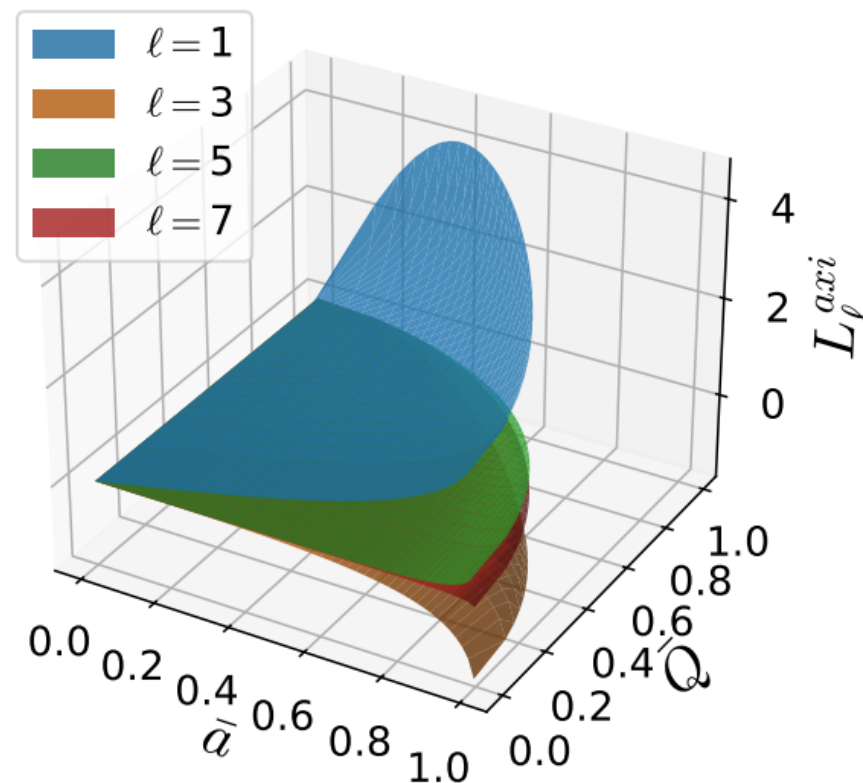


Figure 3: Horizon multipole moments $I_\ell^{axi} + iL_\ell^{axi}$ of a Kerr-Newman black-hole computed with axisymmetric definition.

Main results : unreadable yet very pretty plot

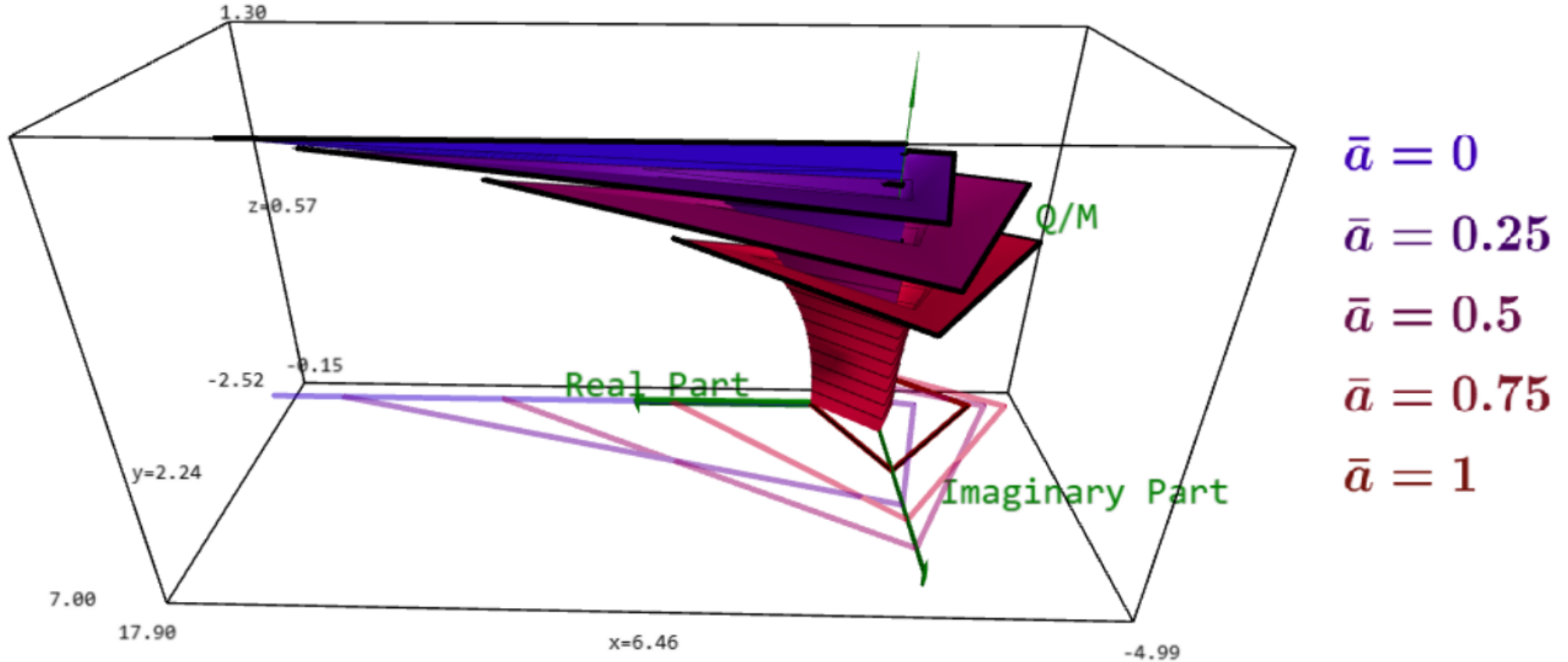
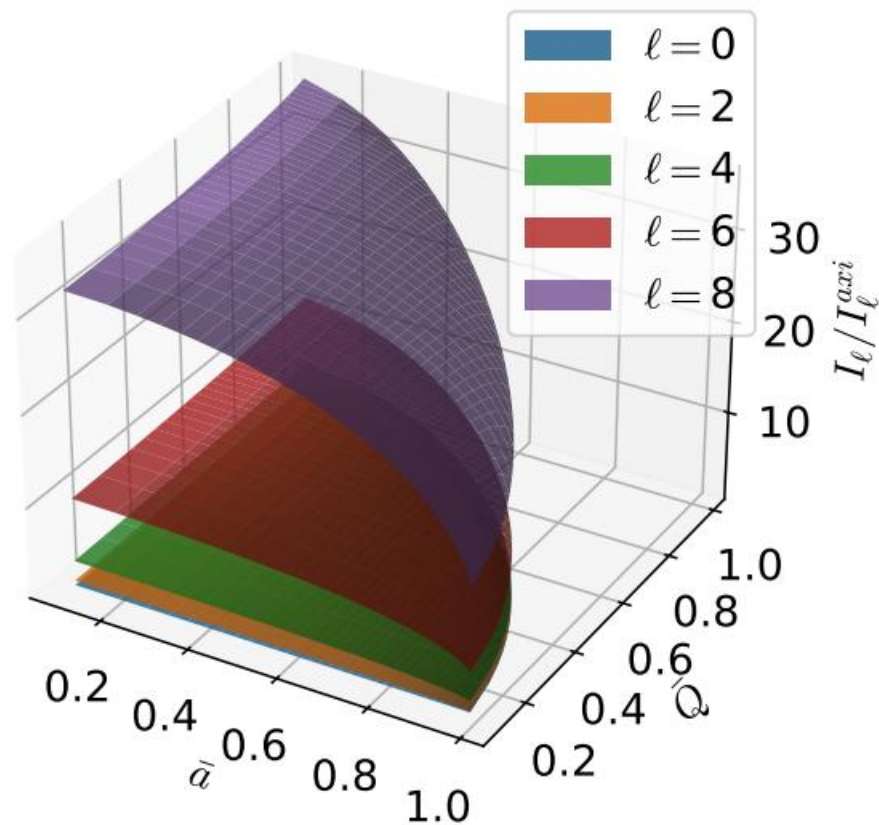


Figure 5: 'Flower' plot of $I_l^{axi} + iL_l^{axi}$: For a given $\bar{a}$ and $\bar{Q}$, multipoles spiral on the complex plane as ℓ increases and they reach 0, while their sign/axis changes from the i^ℓ term. The spiral of extremal multipoles are plotted at the structure's foot. Each of the conical structures do *not* cross, except for the trivial multipoles.

Main results

Even Gen. vs Axi. Multipoles



Odd Gen. vs Axi. Multipoles

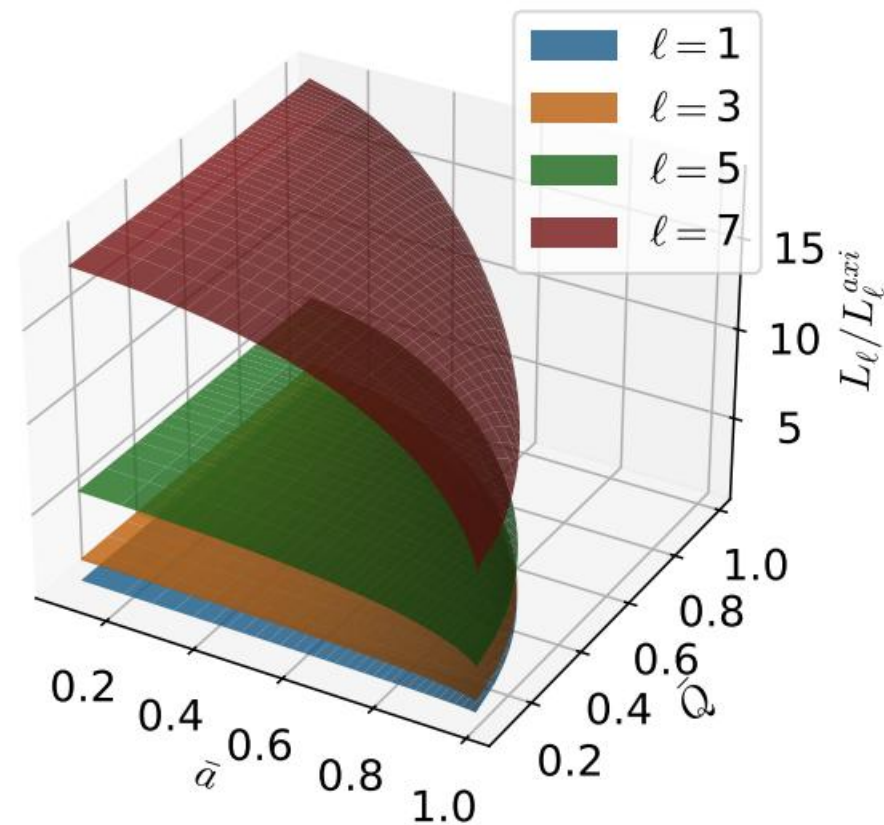


Figure 8: Comparison of horizon multipole moments M_l/M_l^{axi} .

Main results

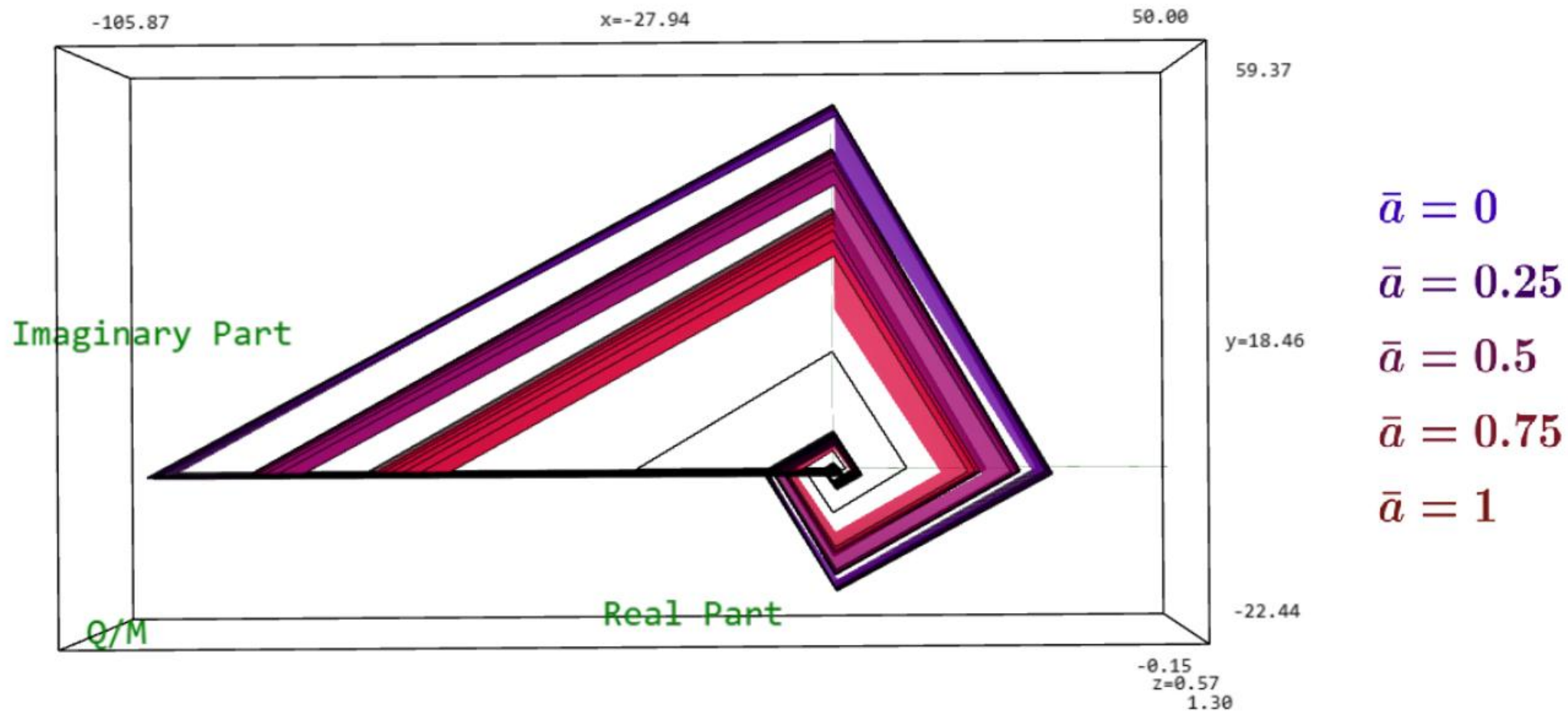


Figure 10: Bird's view of the Spiral Plot of M_l/M_l^{axi} , the $\bar{a} = 0$ and $l = 0$ mass multipoles are equal, hence are not visible due to figure scale.

Conclusion and outlooks

- No stark differences after adding charge.
- Extremal multipoles behave nicely, and have the least disparity between sets.

Extensions:

- Perform low $\bar{a}$, $\bar{Q}$ expansion.
- Compare to Hansen source multipoles
- Study tidal deformability.

Thank you!