Assessing the cosmological consistency of the most massive galaxy clusters using *gap statistics*

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ICC

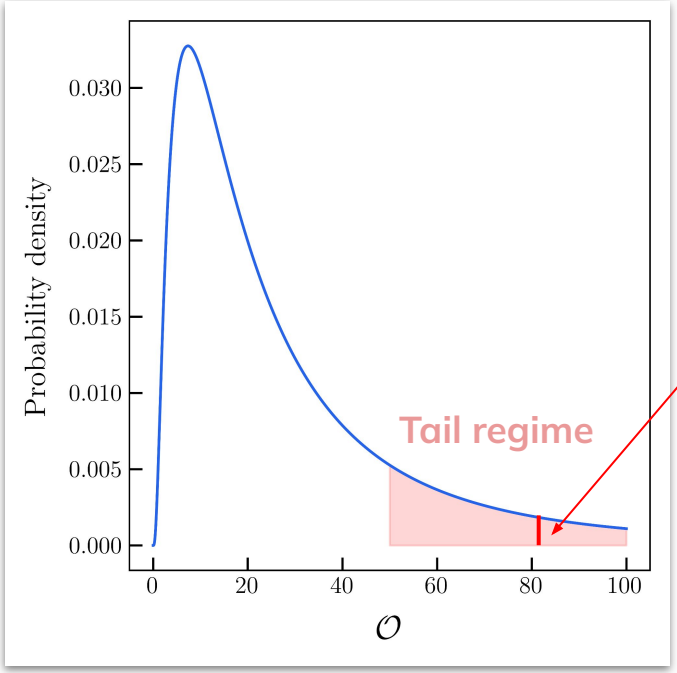
Motivation of this work

→ Test ΛCDM predictions with observed **extreme events**

Extreme events

$\mathcal{O}$: Physical quantity

→ Galaxy stellar mass
→ Void size
→ Galaxy cluster masses
→ etc..



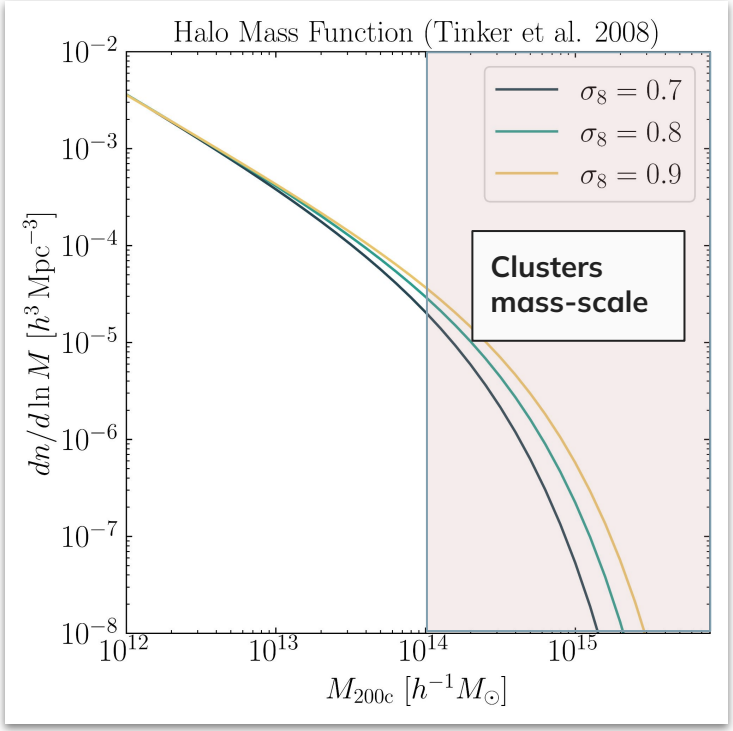
→ Highly sensitive to the cosmology

Galaxy clusters - Abundance

→ Distribution characterized by the Halo Mass Function (HMF)

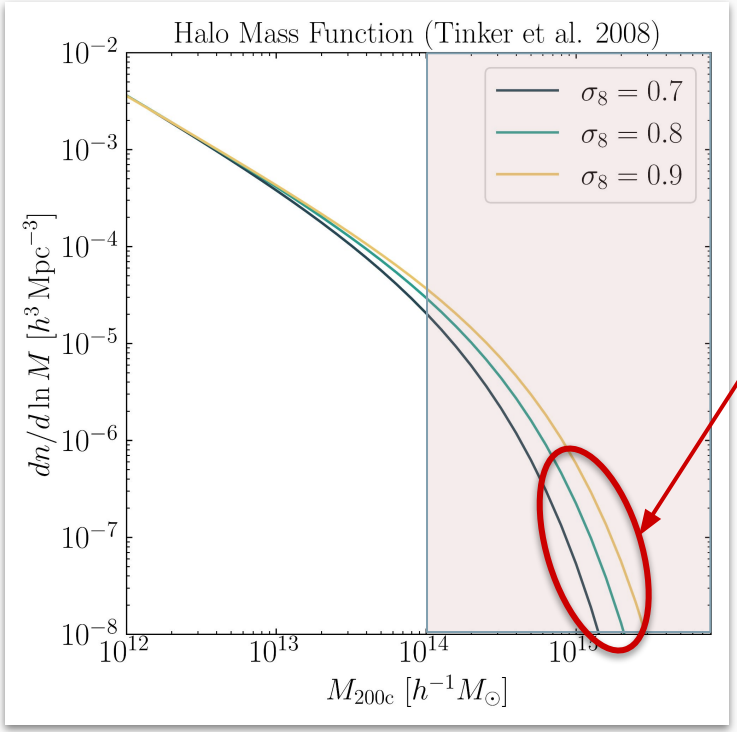
→ Comoving number density of haloes per mass bin at redshift *z*

→ Very sensitive to σ_8 and Ω_m, and the **dark energy (DE) equation of state**.



Higher mass ⇒ lower abundance ⇒ higher sensitivity

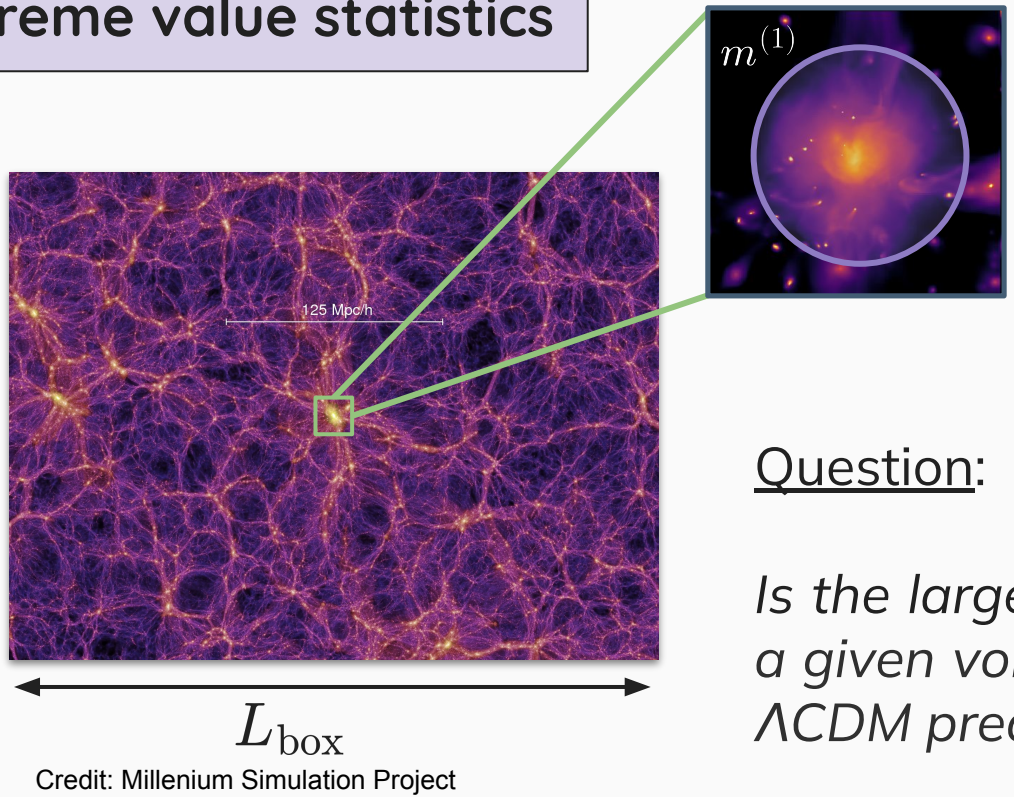
The most massive GC



Most massive GC
→ Very **scarce**
→ **High constraining power**
→ Prompt to **systematics**

How to model them ?

Extreme value statistics

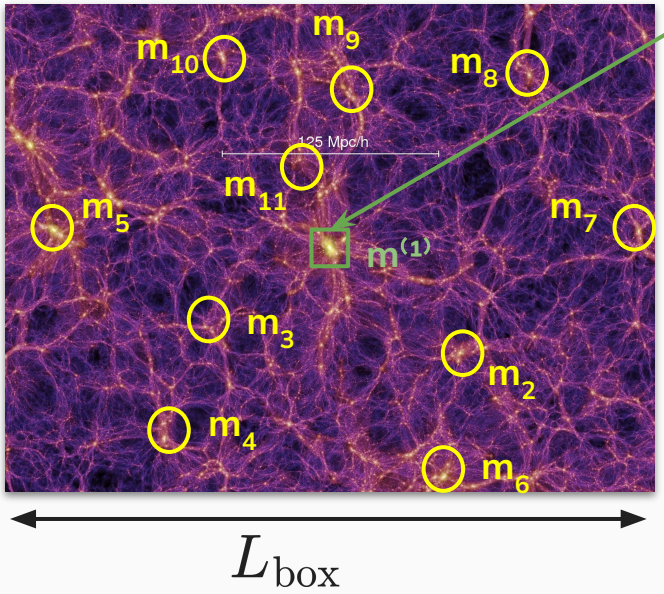


Credit: Millenium Simulation Project

Question:

Is the largest observed cluster within a given volume V consistent with the ΛCDM prediction?

Extreme value statistics



$$m^{(1)} = \max\{\mathrm{m}_1, \mathrm{m}_2, \dots, \mathrm{m}_\mathrm{N}\}$$

$$L_{\mathrm{box}} \gg 100\,\mathrm{Mpc\,h^{-1}}$$

EVS distribution

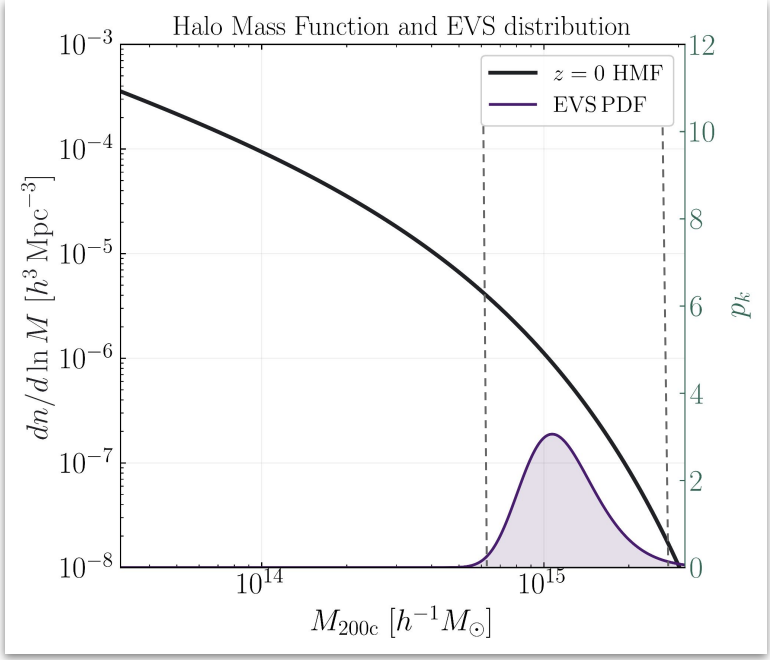
$$p_1(m) = -\mathcal{N}'(m)e^{-\mathcal{N}(m)}$$

Davis et al. 2011
Chongchitnan et al 2012

$\mathcal{N}(m)$: Number of clusters above *m*

→ Depends on the **HMF** and the **volume**

Extreme value statistics



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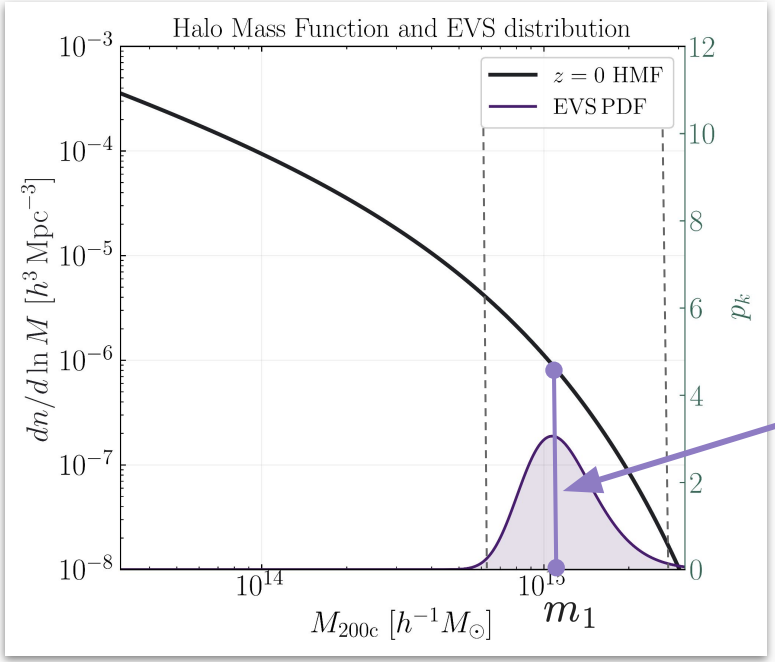

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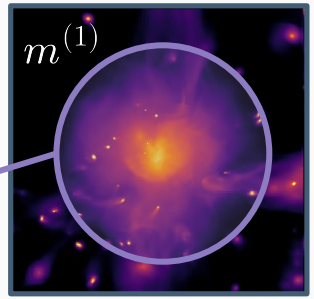

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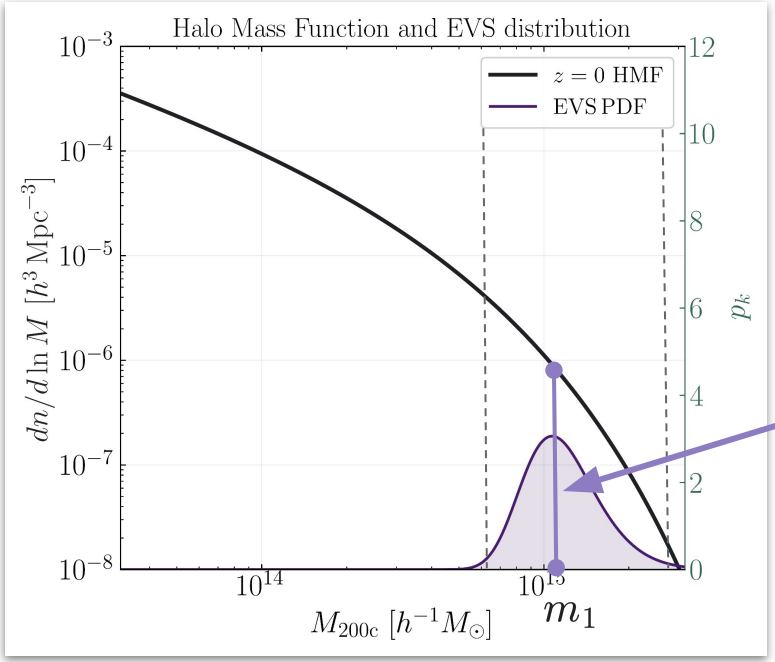

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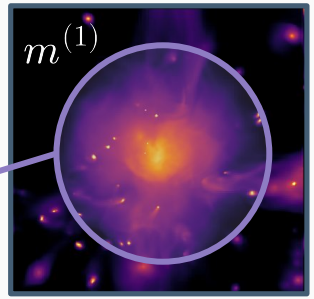
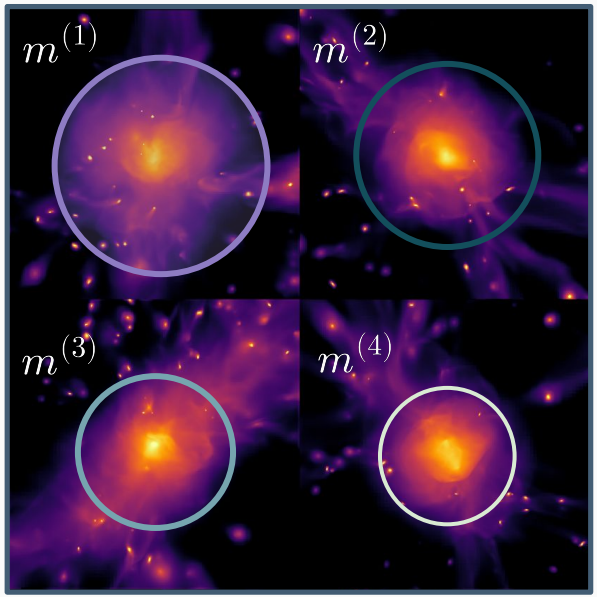


→ No significant tensions with LCDM

(e.g. Harrison et al. 2012
Waizmann et al. 2011)

Beyond EVS: r-Largest Order Statistics

r = 4



4 most massive galaxy clusters

Credit: F. Tornatore (SWANLAKE simulation suite)

m_1

Question:

Are the top-r most massive observed clusters consistent with ΛCDM ?

$$M^{(r)} < ... < M^{(1)}$$

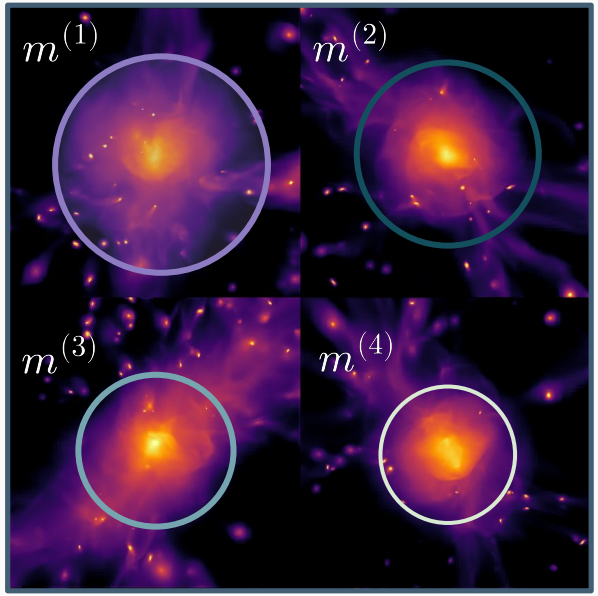
Introduced by Waizmann et al. 2012

→ **Order statistics framework**

Beyond EVS: r-Largest Order Statistics

$$M^{(4)} < M^{(3)} < M^{(2)} < M^{(1)}$$

r = 4



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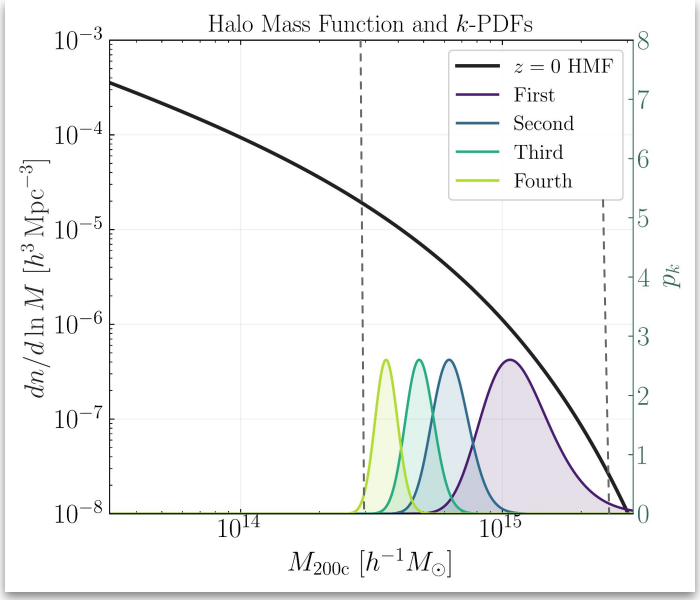

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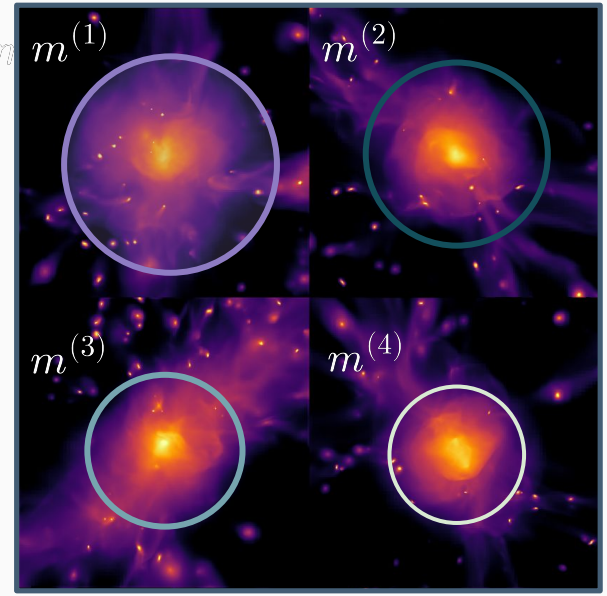

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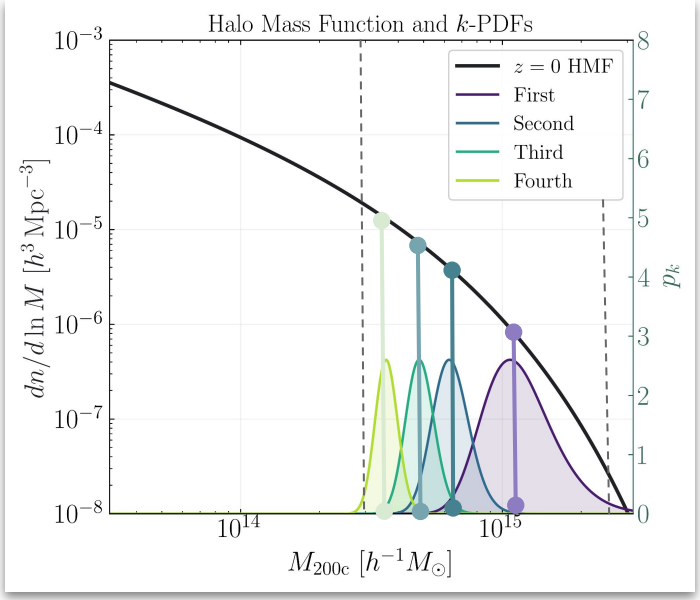

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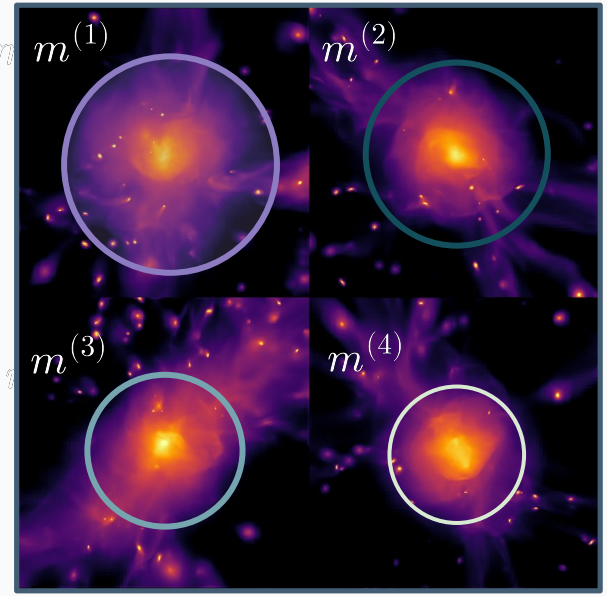

4 most massive galaxy clusters

Credit: F. Tornatore (SWANLAKE simulation suite)

Limitations

→ Focused on single orders

→ Used ~15 years ago, small sky-fraction surveys

(E.g. SPT Williamson et al. 2011, Piffaretti et al. 2011)

→ Need to make assumptions on cluster mass computation (e.g., hydrostatic mass bias)

My contribution to order statistics for cluster cosmology

Goals:

1) Move **beyond single order** consistency checks

2) Move to **wider surveys**

3) **Mitigate** the mass-measurements **systematics**

Gap statistics - concept

Starting from $M^{(r)} < ... < M^{(k)} < ... < M^{(1)}$

We can construct $D^{(ij)} = M^{(j)} - M^{(i)}$

Arnold et al. 2008, Ch. 5

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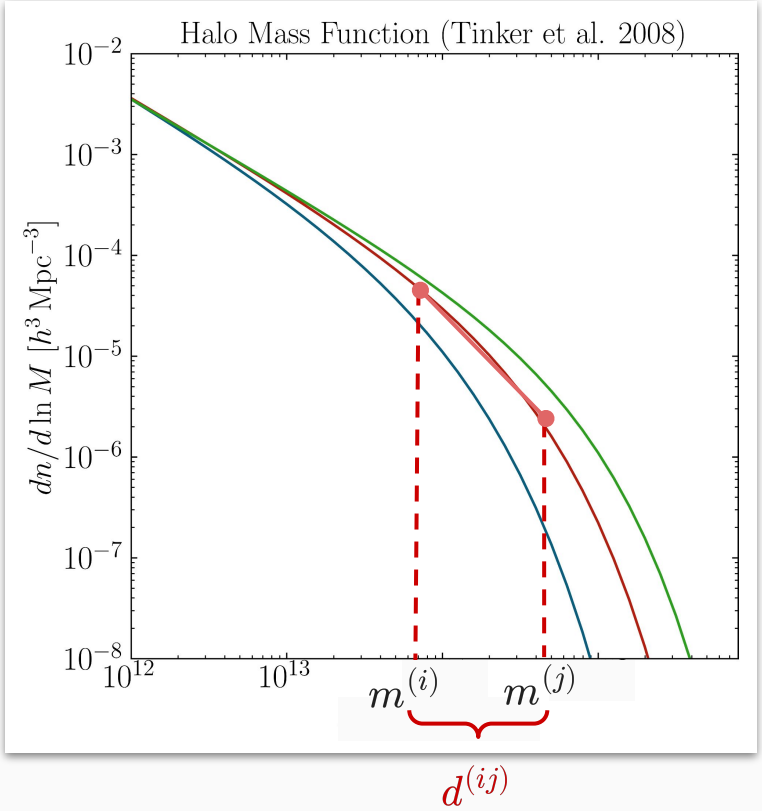
Physical meaning:

***Gap* ~ measure of the local number density**



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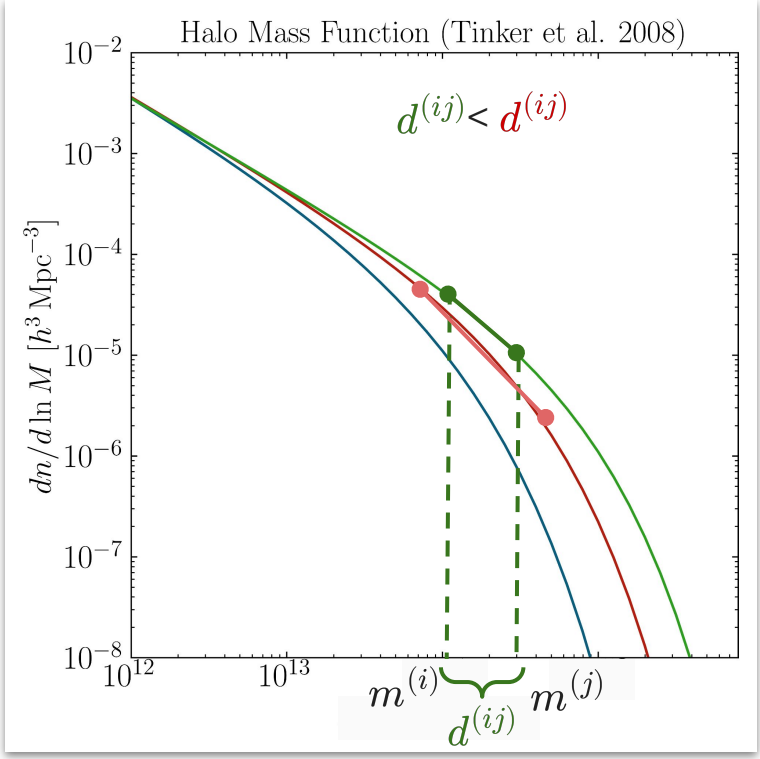
***Small gap* → large local density of clusters**



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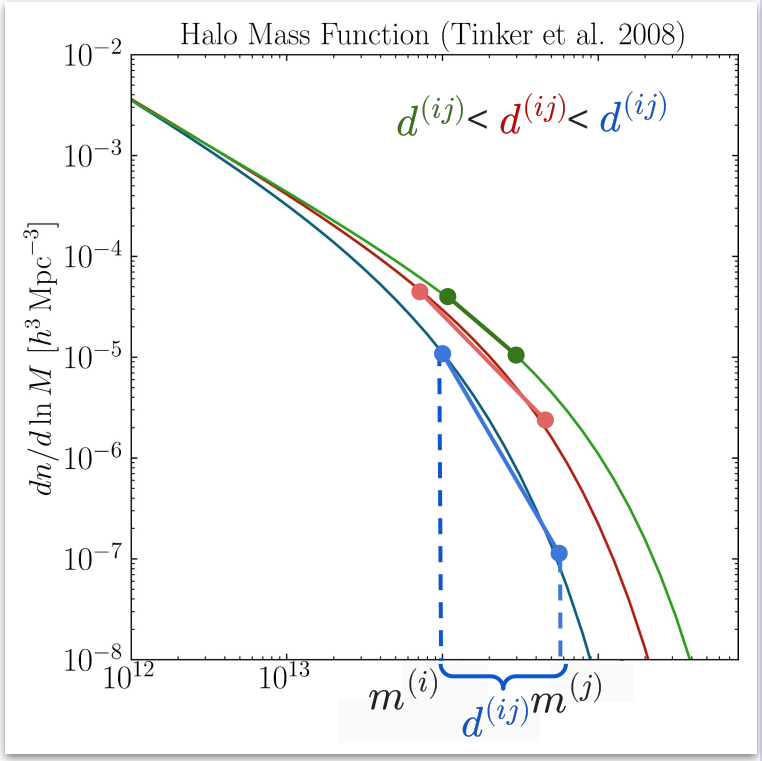
Physical meaning:

Gap **~ measure of the local number density**

Small gap **→ large local density of clusters**

Large gap **→ low local density of clusters**



Gap statistics - concept

$$D^{(ij)} = M^{(j)} - M^{(i)}$$

Probability distribution function

$$p_{ij}(d) = A^{(ij)} \int_0^\infty e^{-\mathcal{N}(m)} \left[\mathcal{N}(m) - \mathcal{N}(m+d)\right]^{i-j-1} \times \mathcal{N}(m+d)^{j-1} \mathcal{N}'(m) \mathcal{N}'(m+d) dm$$

Gayoux et al. 2026 (in prep.)

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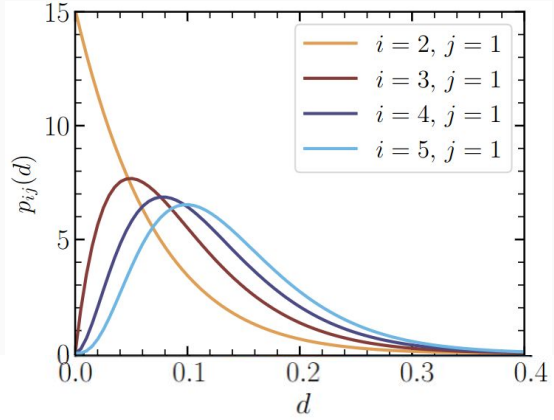

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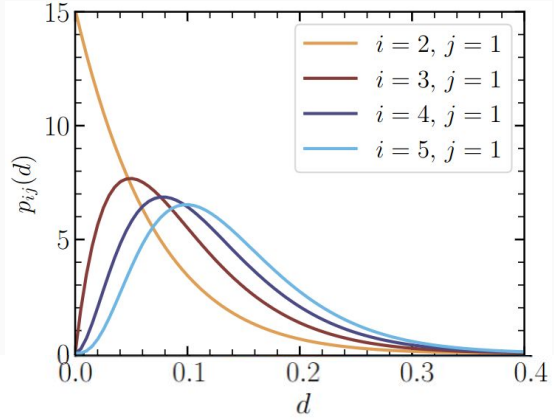

Goal:
To use it as an outlier detection tool!

Anomalous large or small measured gap → strong systematics and/or new physics

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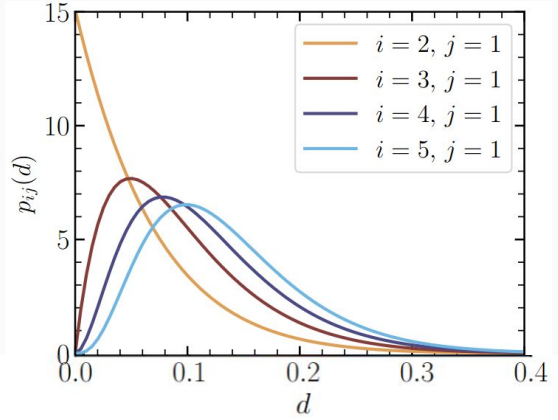

Goal:
To use it as an outlier detection tool!

Anomalous large or small measured gap → strong systematics and/or new physics

Trick: work in log-space to **cancel out multiplicative mass-biases**

$$\Longrightarrow \quad \mathcal{D}^{(ij)} = \log_{10}\left[\frac{M^{(j)}}{M^{(i)}}\right]$$

Gap statistics - ΛCDM prediction

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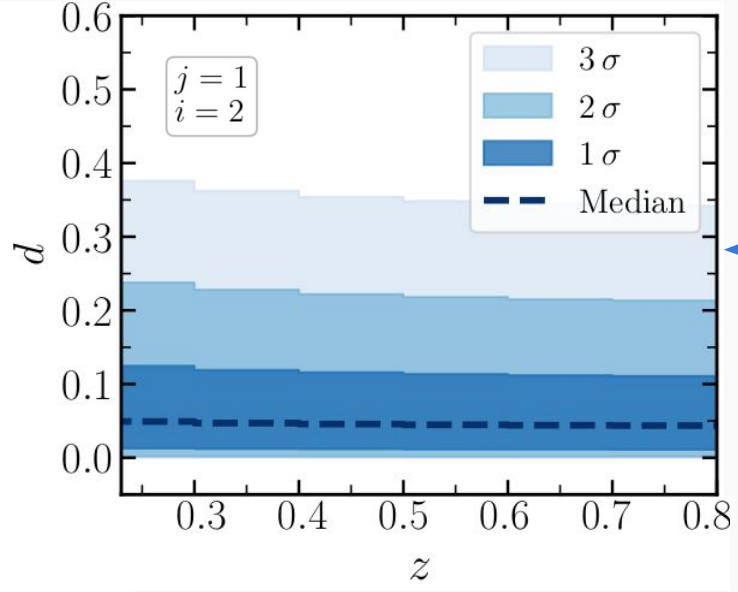
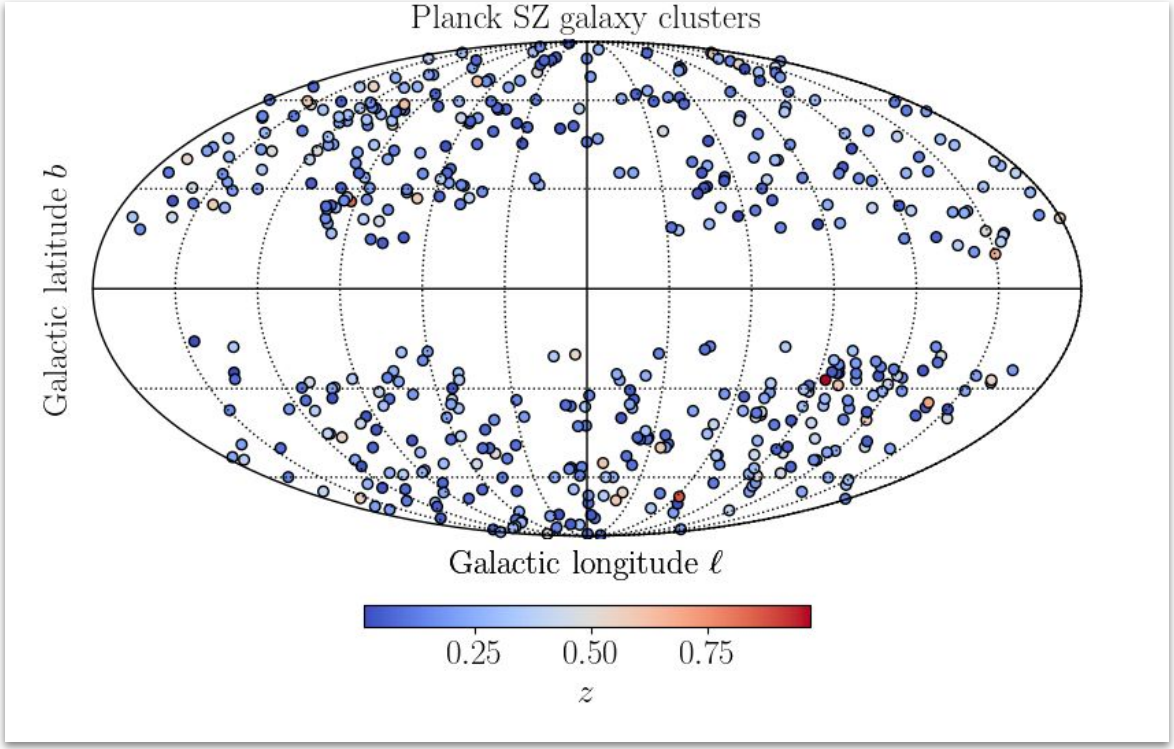


Application to Planck 2nd cluster catalogue



→ Large sky fraction

→ Most massive

→ HMF tail

→ M_{SZ} hydrostatic mass catalogue

Application to Planck 2nd cluster catalogue

Evaluation of the true Planck mass gaps

SZ Planck masses

$$M_{\rm SZ} = \underbrace{\beta(\tilde{M}_{500c}, z)}_{\text{Hydrostatic mass bias}} \tilde{M}_{500c}$$

Application to Planck 2nd cluster catalogue

Evaluation of the true Planck mass gaps

SZ Planck masses

$$M_{\rm SZ} = \underbrace{\beta(\tilde{M}_{500c}, z)}_{\text{Hydrostatic mass bias}} \tilde{M}_{500c} \Longrightarrow$$

Mass and redshift dependent bias

$$\beta(\tilde{M}_{500c}, z) = \beta_0 \left(\frac{\tilde{M}_{500c}}{\langle \tilde{M}_{500c} \rangle} \right)^{\alpha_b} \left(\frac{1+z}{\langle 1+z \rangle} \right)^{\beta_b}$$

Salvati et al. 2019

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Salvati et al. 2019

$\log_{10}$

$$d_{\rm true}^{(ij)} = \frac{d_{\rm SZ}^{(ij)}}{1+\alpha_b} - \frac{\beta_b}{1+\alpha_b} \log_{10} \left(\frac{1+z_j}{1+z_i} \right)$$

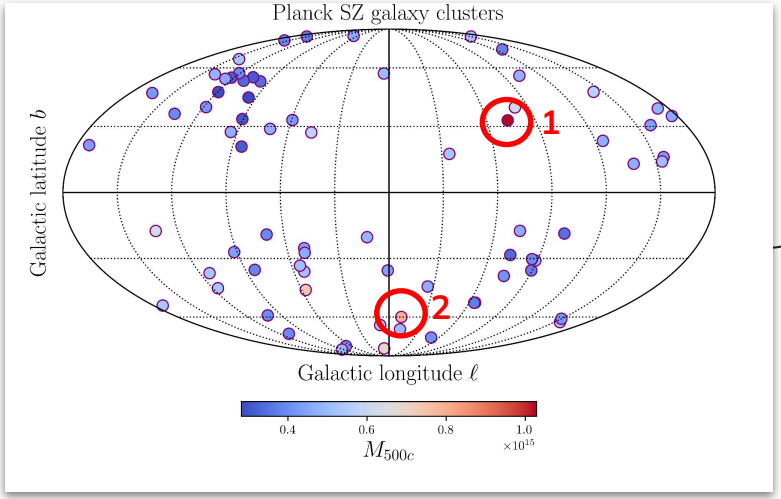
$$d_{\rm SZ}^{(ij)} = \log_{10} M_{\rm SZ}^{(j)} - \log_{10} M_{\rm SZ}^{(i)}$$

Application to Planck 2nd cluster catalogue

Evaluation of the true Planck mass gaps

→ 6 redshift bins from *z* = *0.2* to *z* = *0.8*

e.g. within: $0.3 < z < 0.4$



$$d_{\mathrm{SZ}}^{(2,1)} \Longrightarrow d_{\mathrm{true}}^{(2,1)}$$

Application to Planck 2nd cluster catalogue

Evaluation of the true Planck mass gaps

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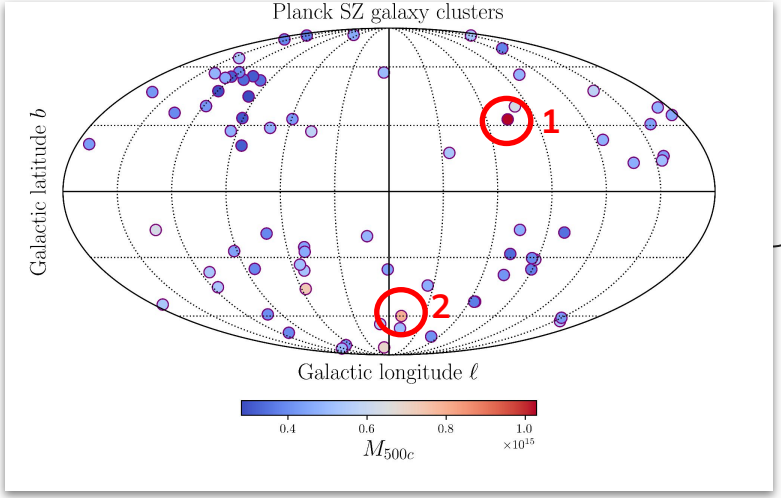

$$d_{\rm SZ}^{(2,1)} \Longrightarrow d_{\rm true}^{(2,1)}$$

Case 1: **constant** bias (*Cl*-tSZ + *NC-tSZ* + *CMB* data)

$$\beta_0 = 0.62,\ \alpha_b = 0.00,\ \beta_b = 0.00$$

$$d_{\rm true}^{(ij)} \simeq d_{\rm SZ}^{(ij)}$$ *Salvati et al. 2019*

Application to Planck 2nd cluster catalogue

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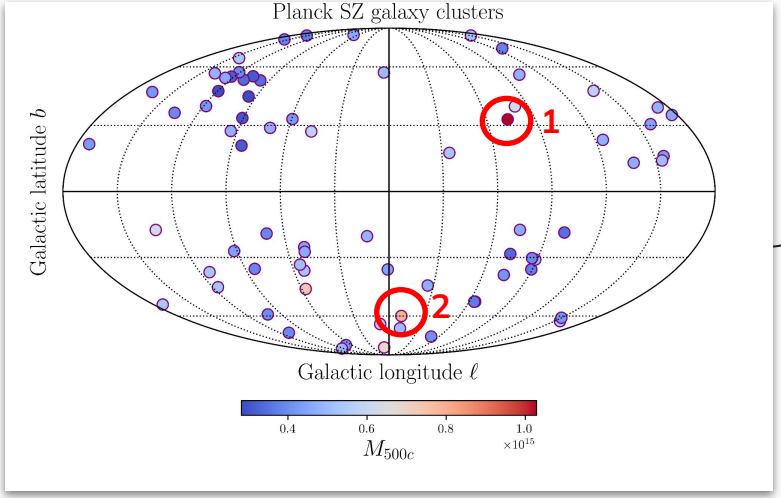

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Case 2: **mass and redshift dependent** bias (*NC-tSZ* + *BAO* + *WL* prior)

$$\beta_0 = 0.66,\ \alpha_b = 0.10,\ \beta_b = 0.24$$

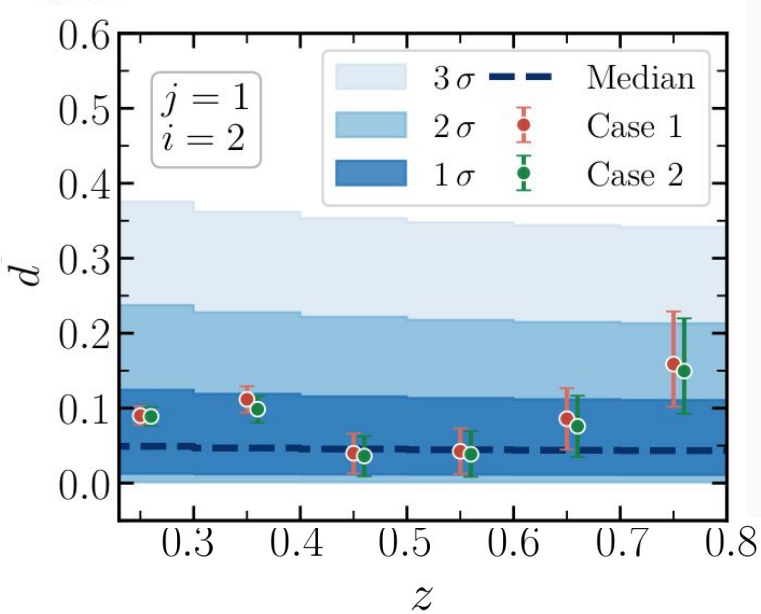
$$d_{\rm true}^{(ij)} \simeq \frac{d_{\rm SZ}^{(ij)}}{1+\alpha_b} - \frac{\beta_b}{1+\alpha_b}\log_{10}\left(\frac{1+z_j}{1+z_i}\right)$$

Application to Planck 2nd cluster catalogue

ΛCDM consistency test

Case 1:
constant bias

Case 2:
mass and redshift dependent bias



Most massive vs second most massive

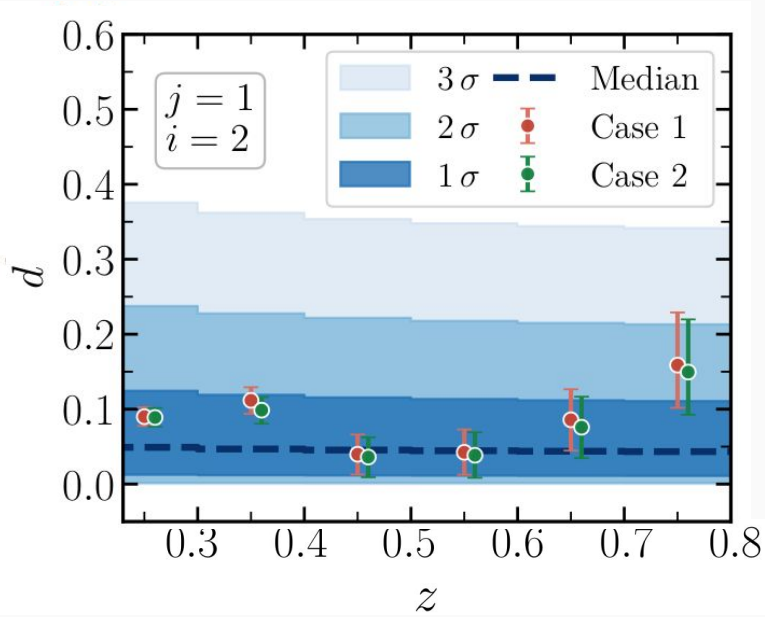
Application to Planck 2nd cluster catalogue

ΛCDM consistency test

Gaps in log-space → **Mildly sensitive** to the **hydrostatic bias** assumption

→ No need to assume a bias to be consistent with the ΛCDM model

→ **Novel way to test the consistency of cosmological models**



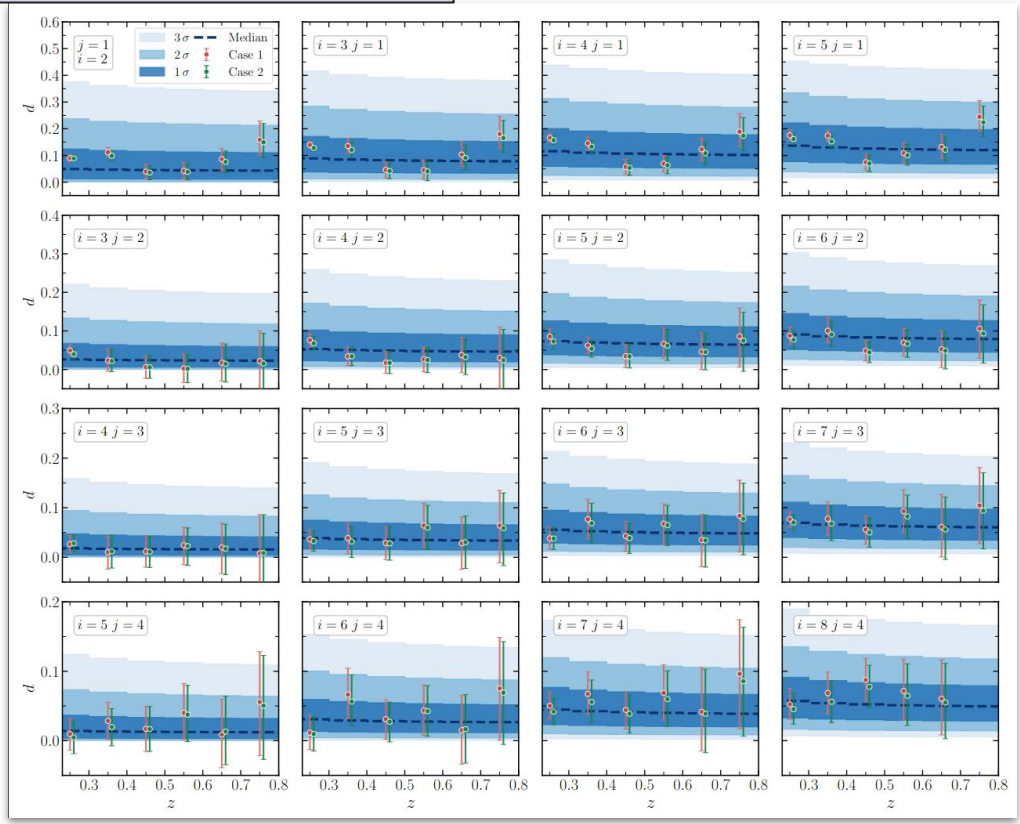
Most massive vs second most massive

Application to Planck 2nd cluster catalogue

ΛCDM consistency test

8 most massive per z-bin → **48 most massive** clusters in PSZ2 catalogue

All gap combination → 96 gap pairs → all consistent at the 1σ level with ΛCDM



Conclusion

→ **Novel framework** for consistency checks based on **mass gaps**

→ Mitigate **mass-measurement systematics**

→ Can be **extended** to models **beyond LCDM**

→ Can be **extended** to **the observable space** and to **other physical parameters**

Thank you for your attention :)