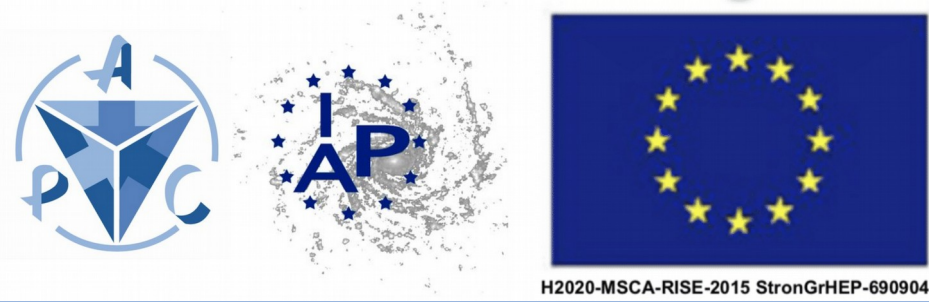


Alexandre Toubiana (APC/IAP)

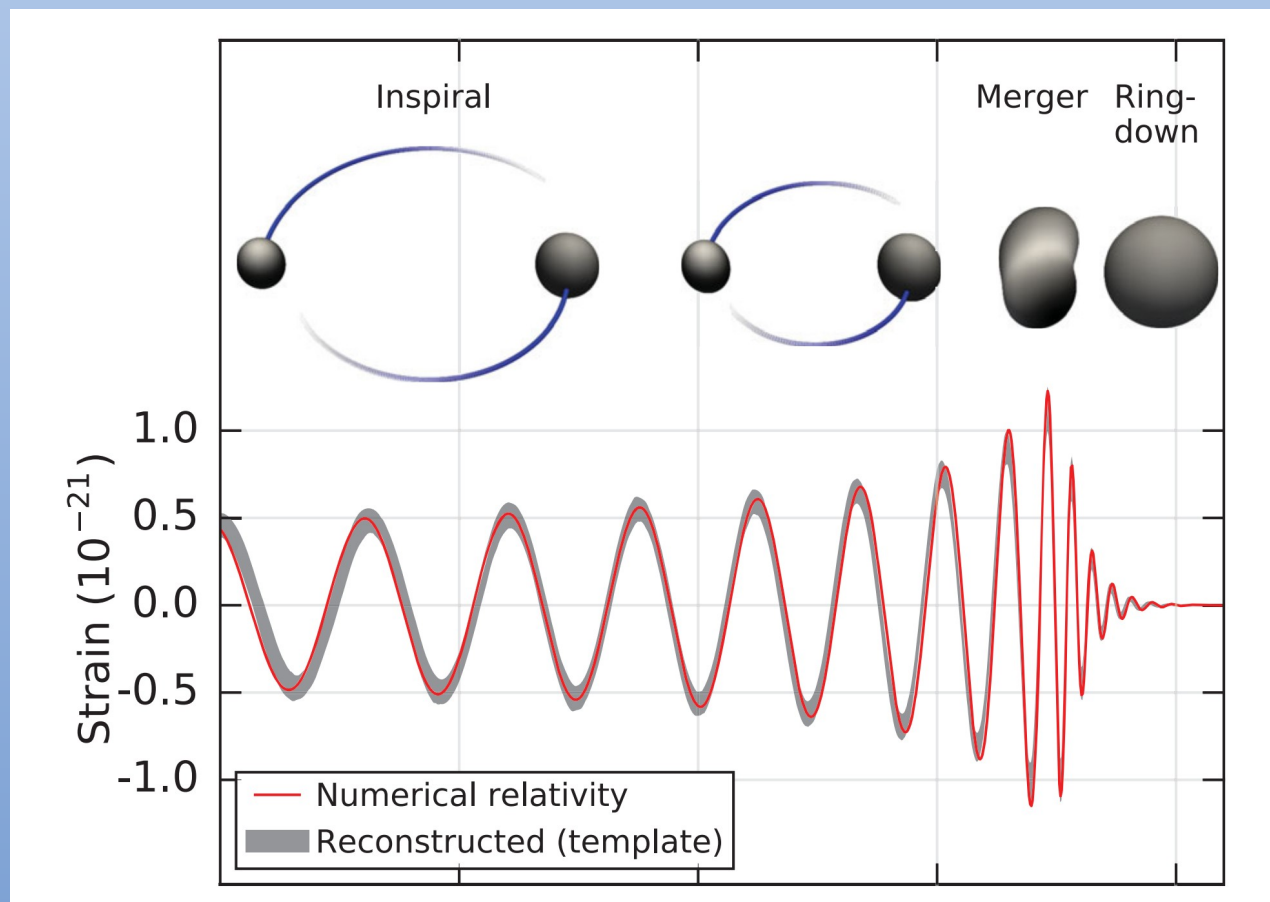


with:

S. Babak (APC), E. Barausse (SISSA), L. Lehner (PI)

Rencontre du GdR Tests de la Relativité Générale  
February 4<sup>th</sup> 2020:

**Mimicking Black Hole mimickers**



Black hole signal  
LIGO/VIRGO Collaboration  
PRL 2016

**Goal:**  
**Construct a waveform for black hole mimickers and assess if ground based detectors could distinguish it from a black hole**

# Black Hole Mimickers

- Compact Objects similar to black holes from the gravitational point of view:

$$C = \frac{M}{R} \gtrsim 0.1 \quad C_{BH} \geq \frac{1}{2}$$

- No horizon
- Merger can lead to a BH or object of same nature
- Example: Boson Stars

# Boson Stars

- Scalar field solution of Einstein-Klein Gordon equation
- Example: Massive Boson Stars:

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 |\phi|^2 - \frac{1}{4} \lambda |\phi|^4 \right]$$

$$M_{max} \simeq \left( \frac{0.10 \text{ GeV}}{m_\phi} \right)^2 \lambda^{1/2} M_\odot$$

Colpi, Shapiro,  
Wasserman (PRL 1986):

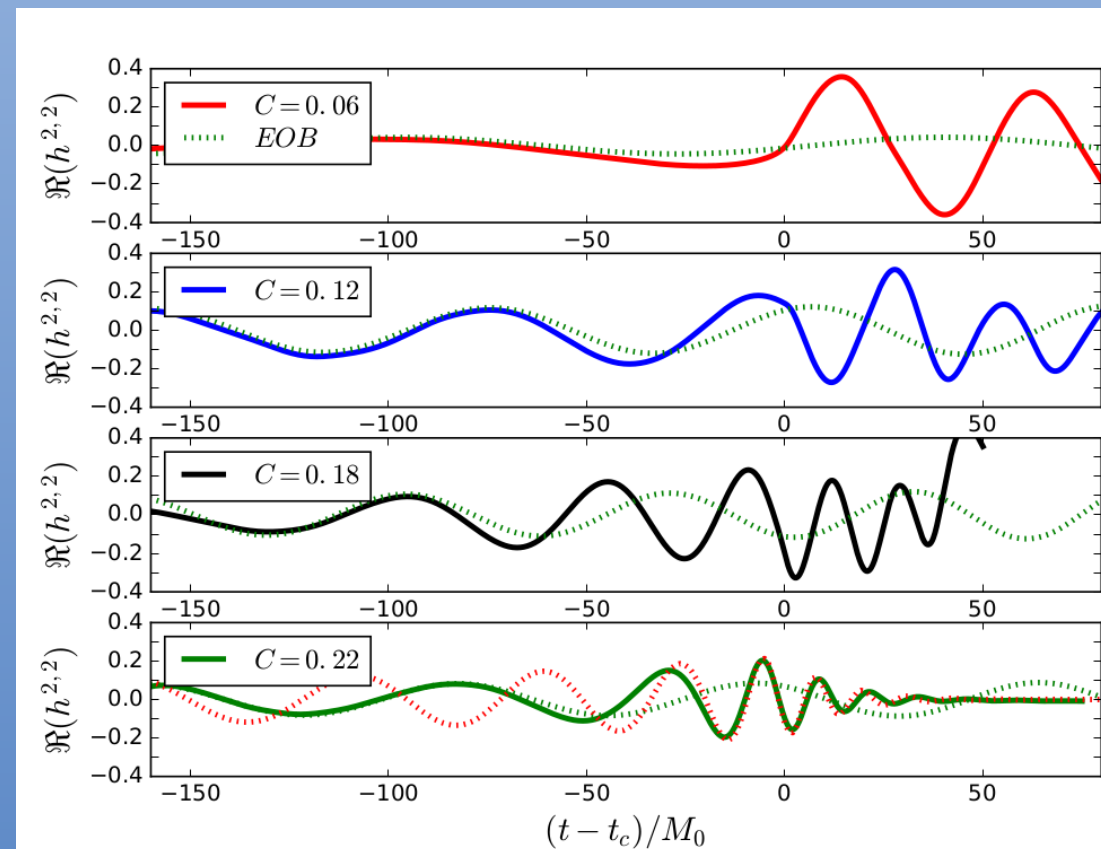
- Self interaction increases mass and compactness

$$C \lesssim 0.3$$

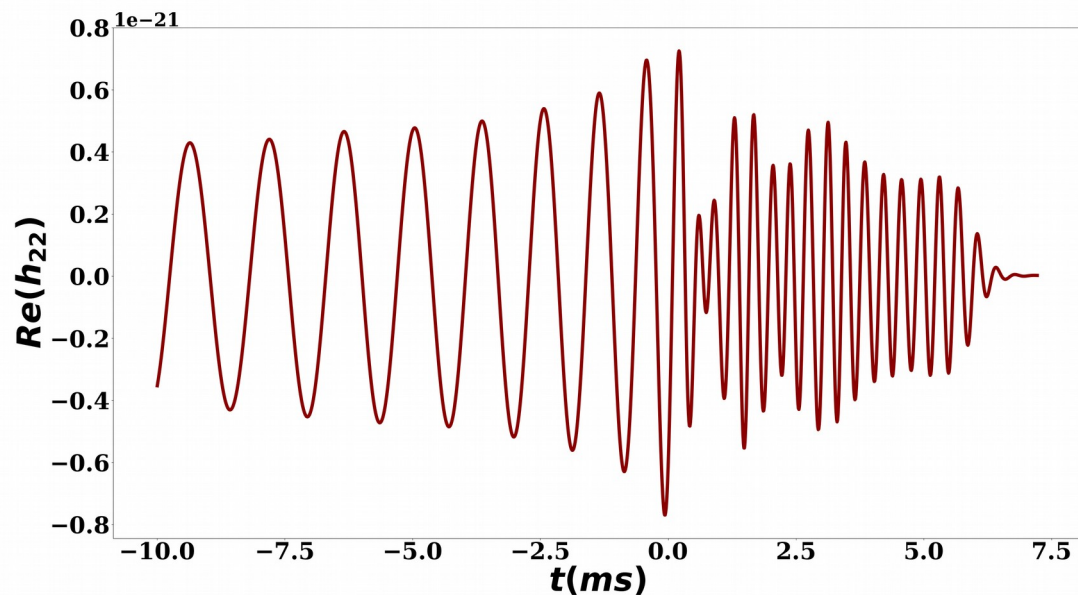
# Numerical simulations of Boson Stars

- If formed from collapse or coalescence seemingly do not support rotation (Sanchis-Gual et. al PRL 2019, Bezares et. al Phys. Rev. D 2017, Palenzuela et al. Phys Rev D 2017)
- Radial oscillations at characteristic frequency (Palenzuela et al. Phys Rev D 2017):

$$M_r \omega_r = -0.064 + 1.72 M_0 \omega_c$$



# Post-merger signal of Neutron Star from Numerical Simulations

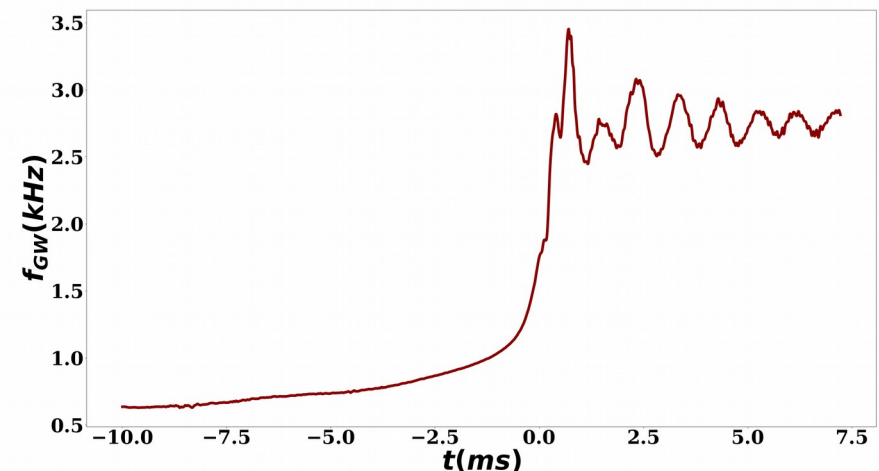
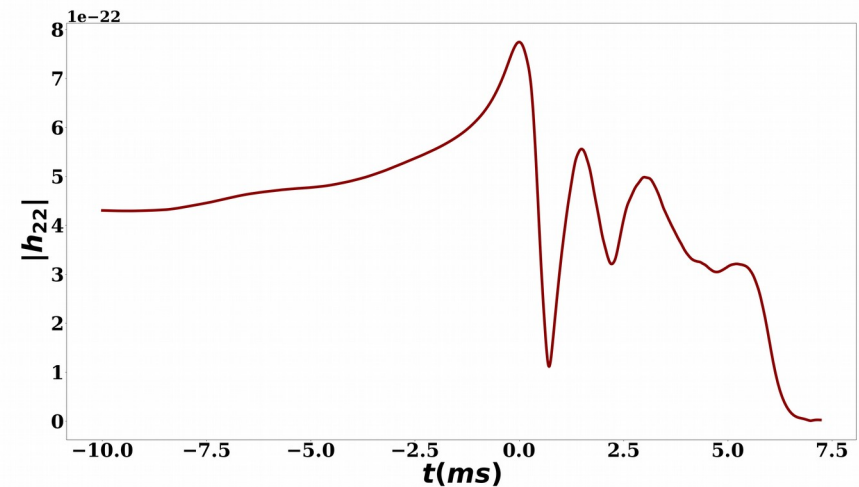


Equation of state: ALF2

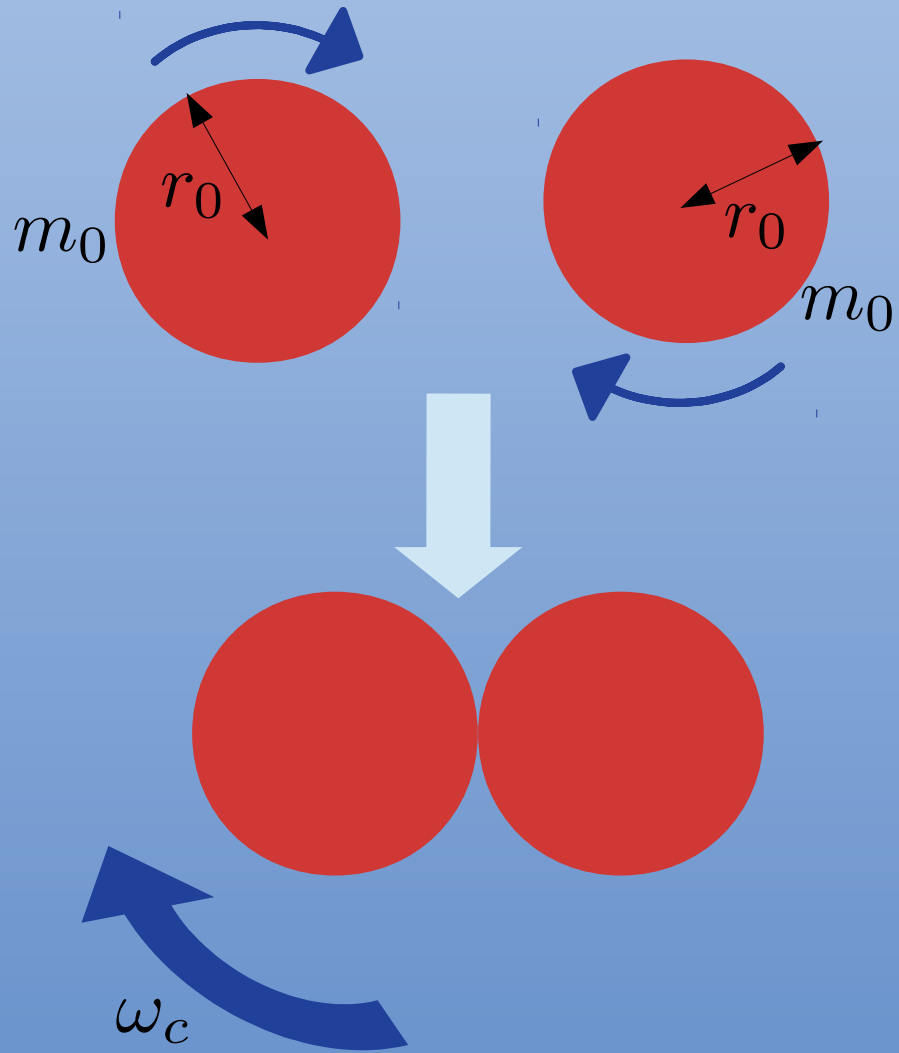
$$m_1 = m_2 = 1.35 M_{\odot}$$

Taken from:

<http://www.computational-relativity.org>



# Toy Model



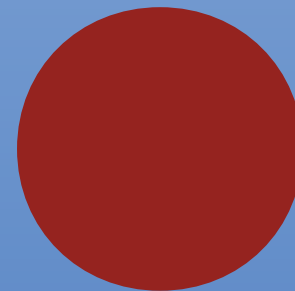
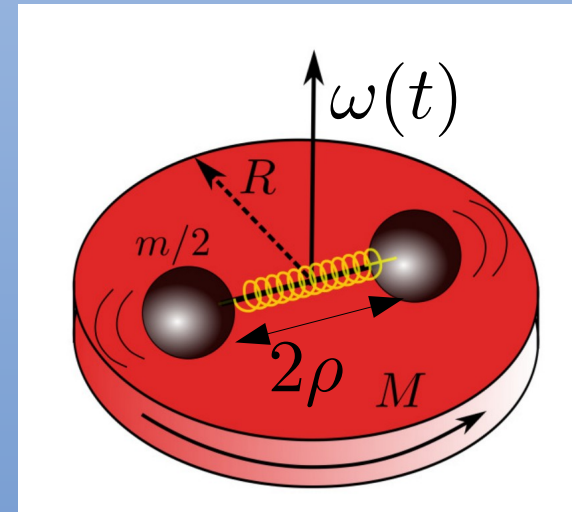
$$C_0 = \frac{m_0}{r_0}$$

$$a_0 = 0$$

$$\Delta t \simeq \frac{1}{2} \frac{2\pi}{\omega_c}$$



Toy Model for the intermediary phase  
(Takami, Rezzolla, Baiotti  
Phys. Rev. D 2015)



# Settings

- 4 free parameters related to the equation of state of the initial bodies:

- $m$  ( $M = 2m_0 - m$  assuming mass conservation)

- $R$

- Spring constant:  $k$

- Length at rest:  $2\rho_0$



- Initial conditions:

$$\rho(0) = R - r_0$$

$$\varphi(0) = \varphi_0$$

$$E(0) = E_c$$

$$J(0) = \alpha J_c$$



$\alpha = 0$  : "Boson Star"

$\alpha = 1$  : "Neutron Star"



# Evolution of the system

- Effective particle in an effective potential:

$$V_{\text{eff}} = V_{\text{centrifugal}} + V_{\text{gravitational}} + V_{\text{spring}}$$

- Adiabatic evolution over one period:

$$\dot{\rho} = \sqrt{\frac{2}{m}} \sqrt{E - V_{\text{eff}}(\rho)}$$

$$\dot{\varphi} = \omega = \frac{J}{I}$$

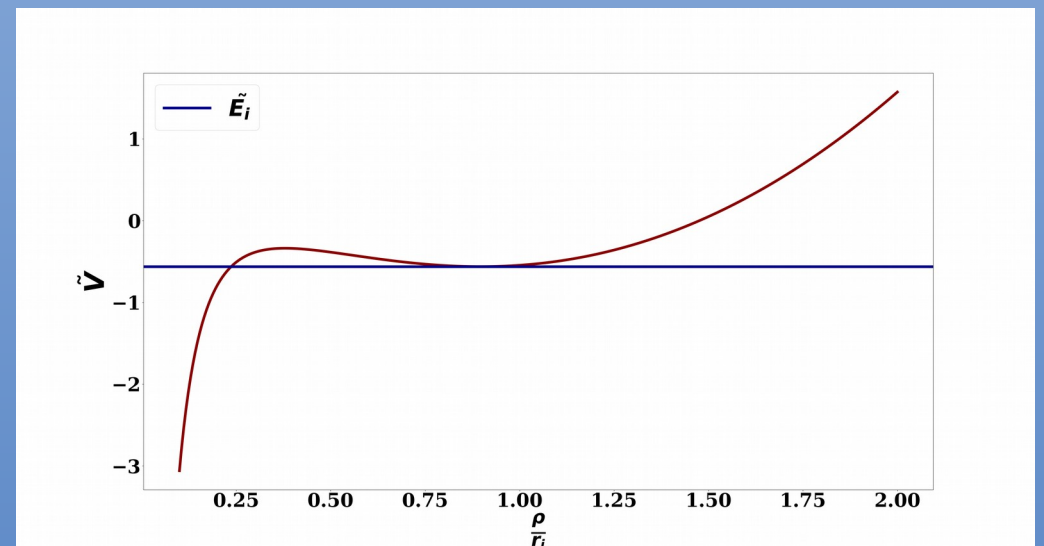
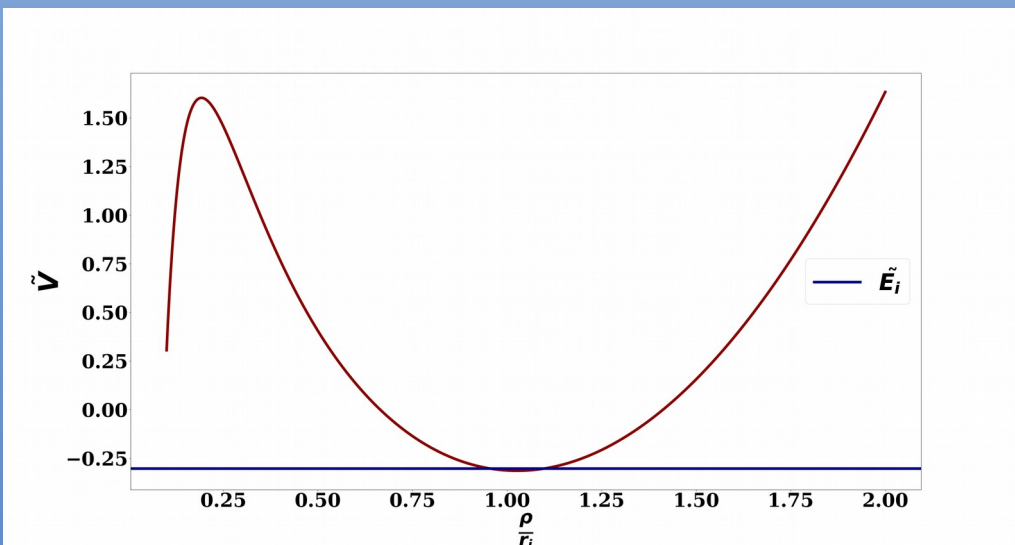
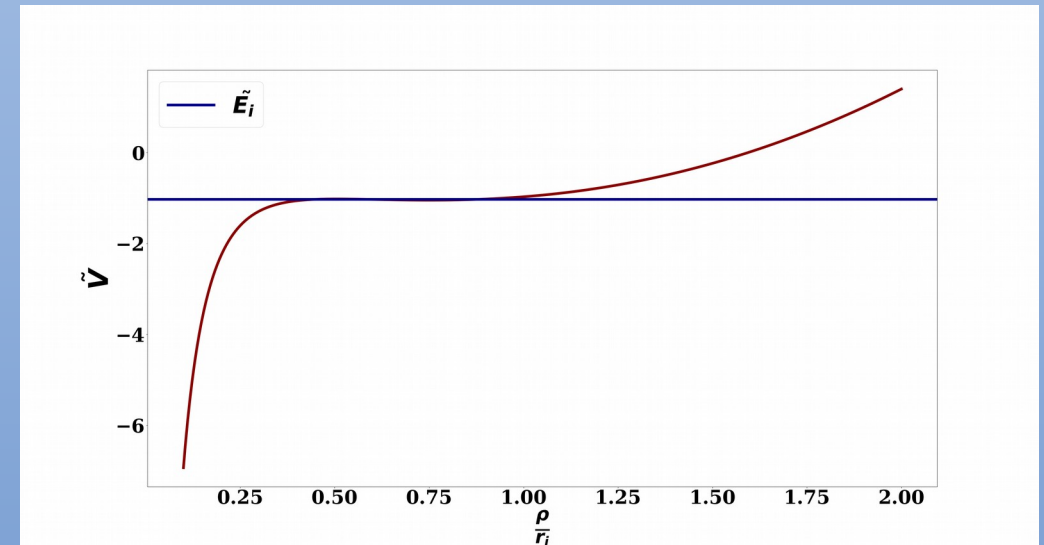
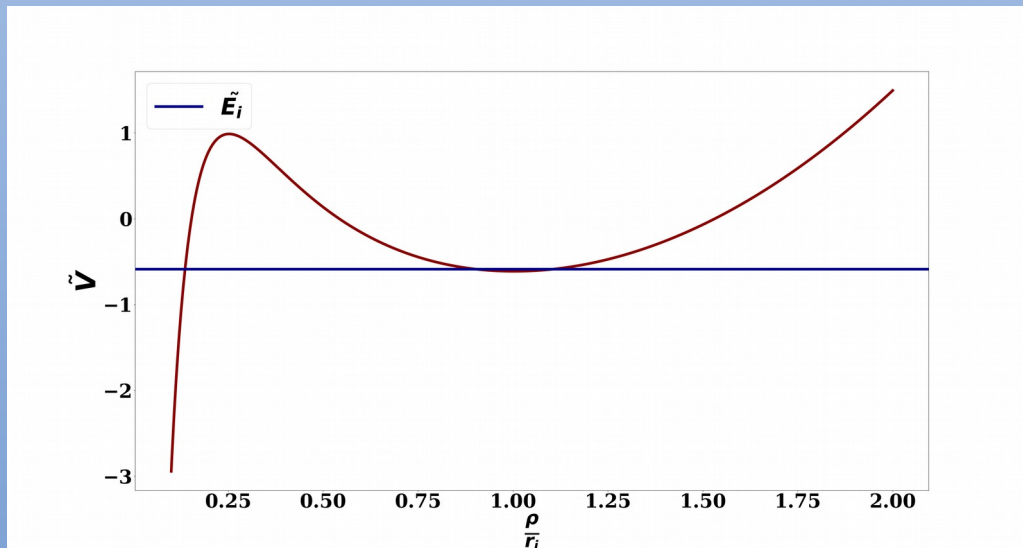
$$\dot{E} = - \langle P_{\text{rad}} \rangle$$

$$\dot{J} = - \langle J_{\text{rad}} \rangle$$



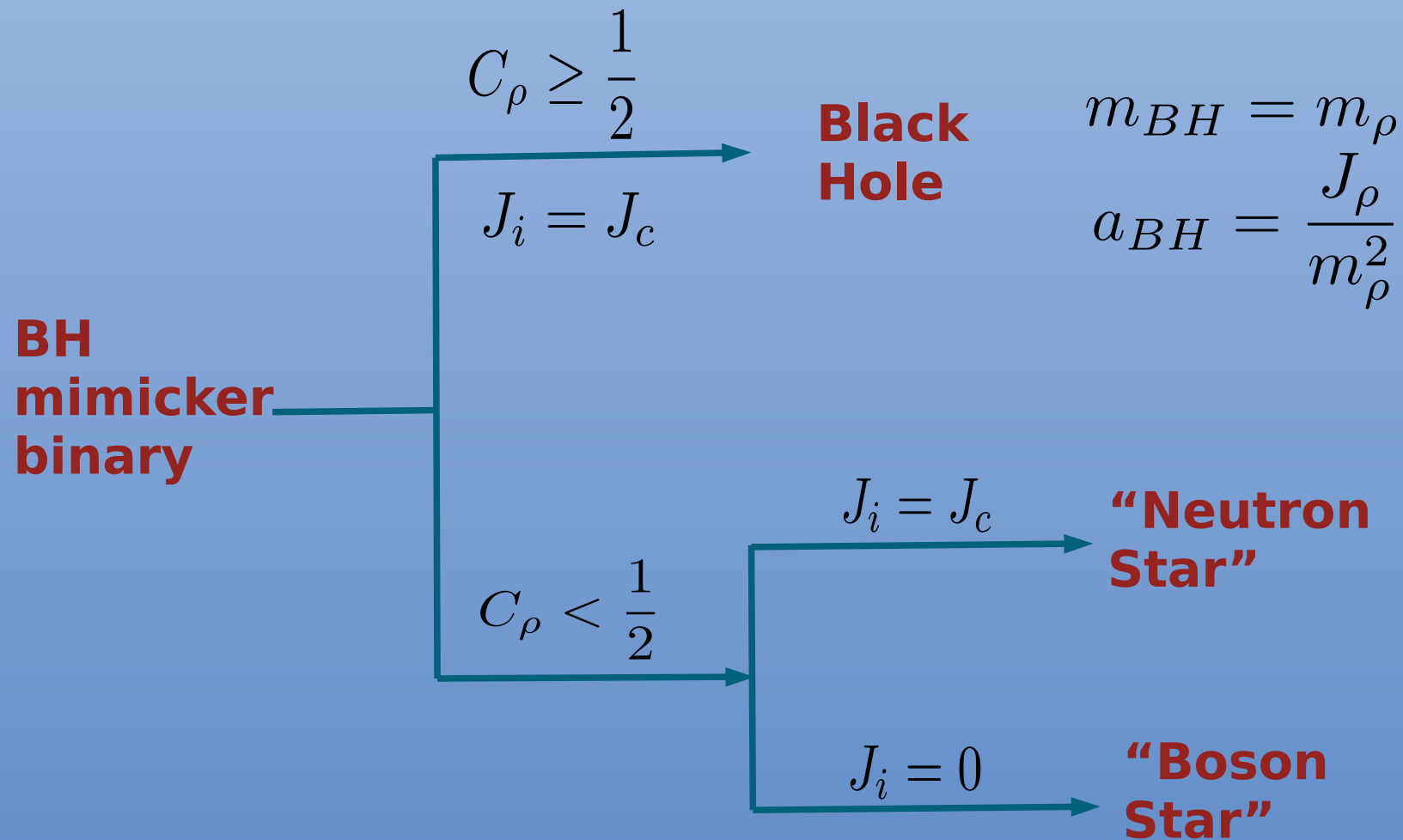
**Computed with  
quadrupole formula  
assuming non perturbed  
equations of motion**

# Evolution of the system

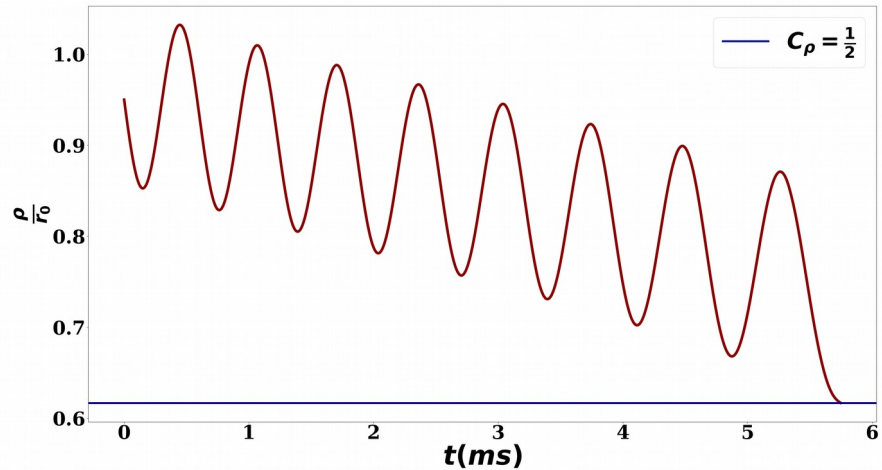


# End scenarios:

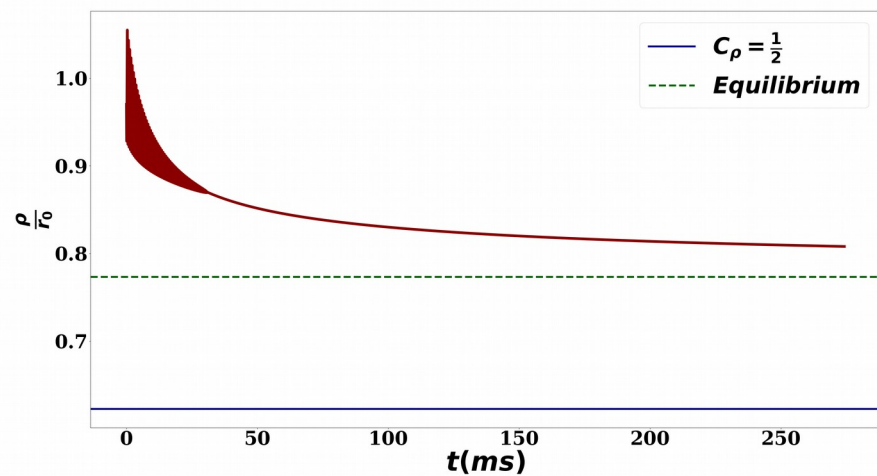
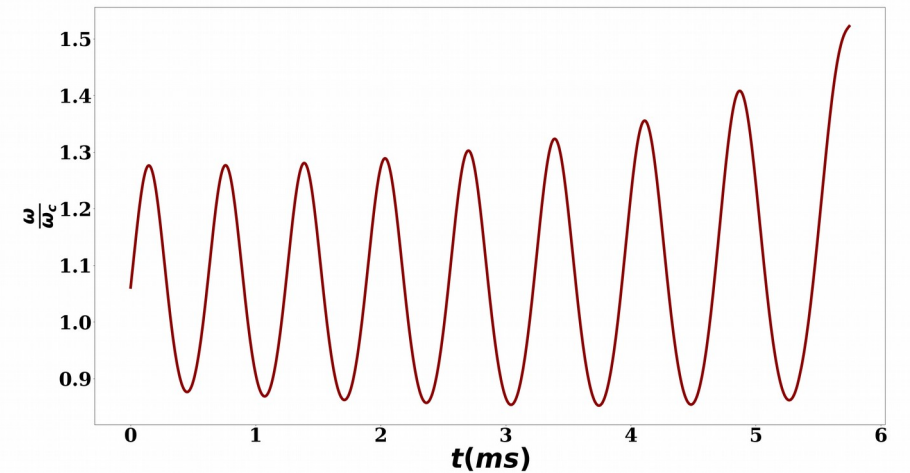
- Compactness:  $C_\rho = \frac{m_\rho}{\rho} = \frac{m + M \left(\frac{\rho}{R}\right)^2}{\rho}$



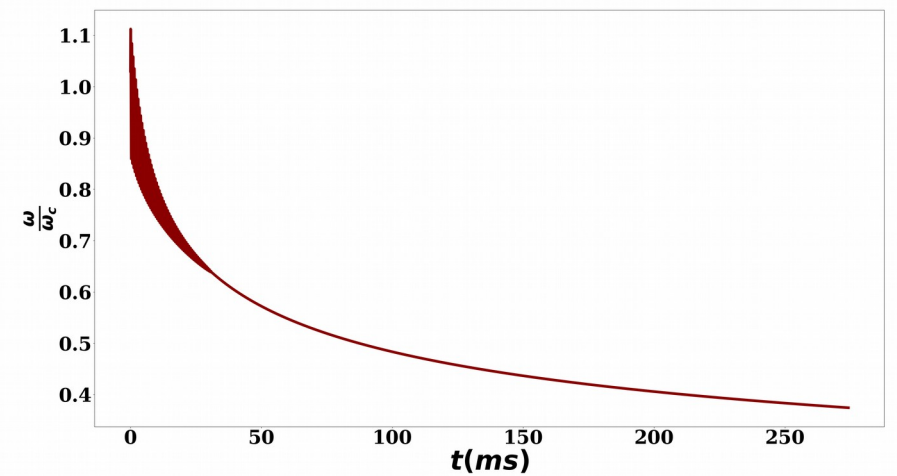
# Dynamics



End state:  
BH



End state:  
"NS"



# Full signal

- Inspiral: IMRPhenomD\_NRTidal until  $f_{GW} = 2f_c$
- 
- Toy model computed with quadrupole formula

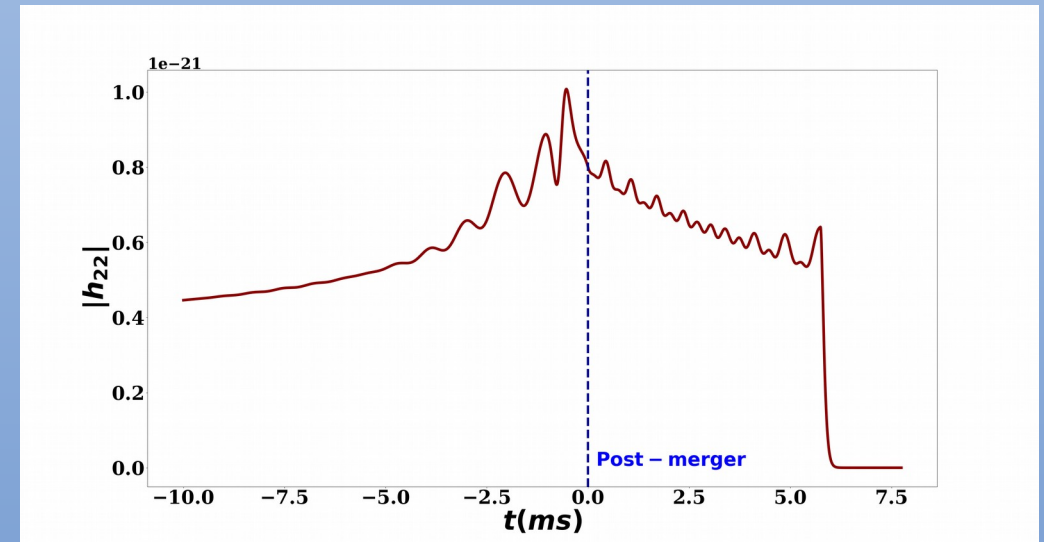
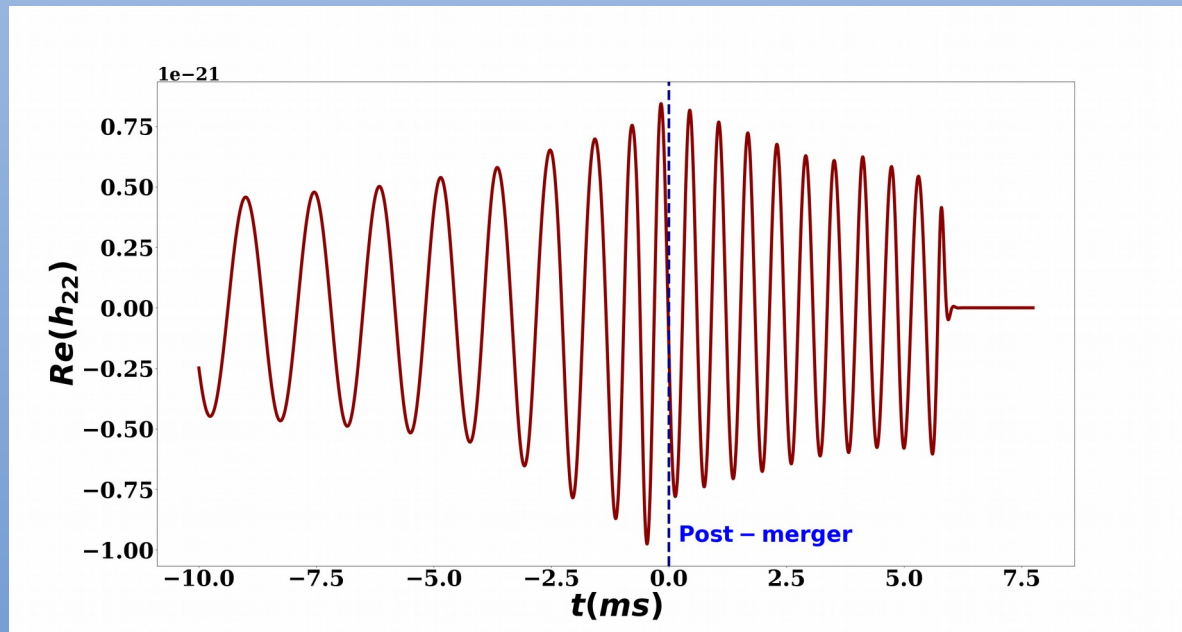
# Full signal

- Inspiral: IMRPhenomD\_NRTidal until  $f_{GW} = 2f_c$
- Matching in amplitude and phase for  $\Delta t = \frac{1}{2} \frac{1}{f_c}$
- Toy model computed with quadrupole formula

# Full signal

- Inspiral: IMRPhenomD\_NRTidal until  $f_{GW} = 2f_c$
- Matching in amplitude and phase for  $\Delta t = \frac{1}{2} \frac{1}{f_c}$
- Toy model computed with quadrupole formula
- If final state is BH, attach ringdown as  
in Damour and Nagar (Phys. Rev. D 2014) with  
QNMs from Berti et.al (Class.Quant.Grav 2009)
- Flexibility for different end behaviours

# Time domain: collapse to a BH

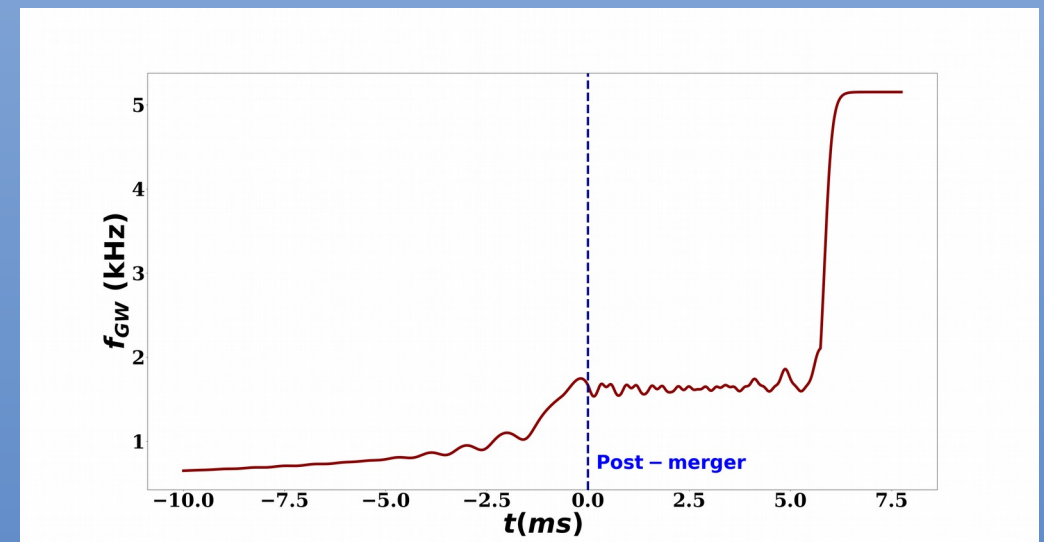


$$m_1 = m_2 = 1.35M_{\odot}$$

$$C_0 = 0.16$$

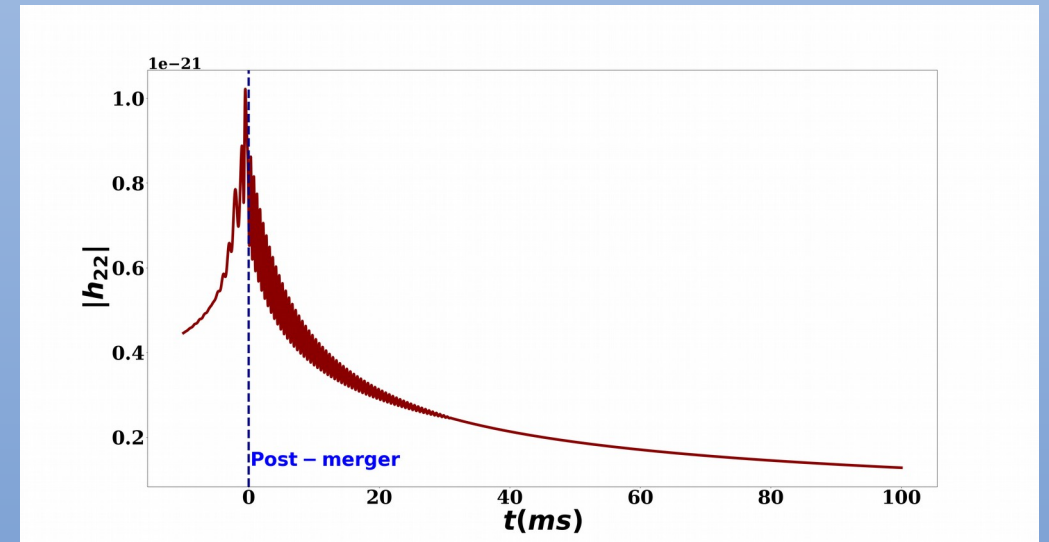
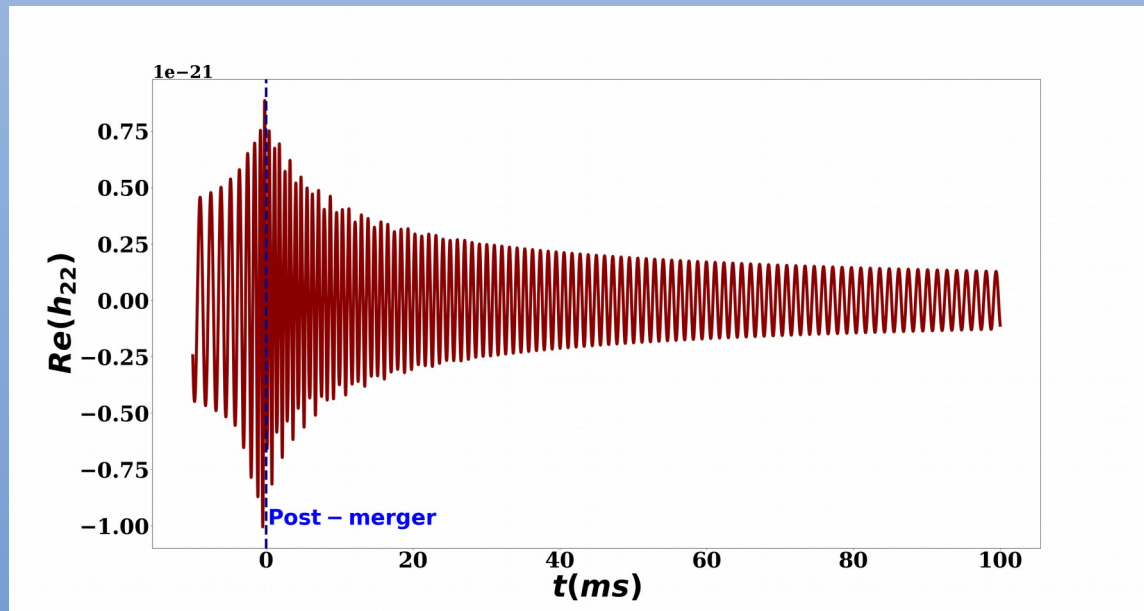
$$m_{BH} = 2.60M_{\odot}$$

$$a_{BH} = 0.37$$





# Time domain: “NS” remnant

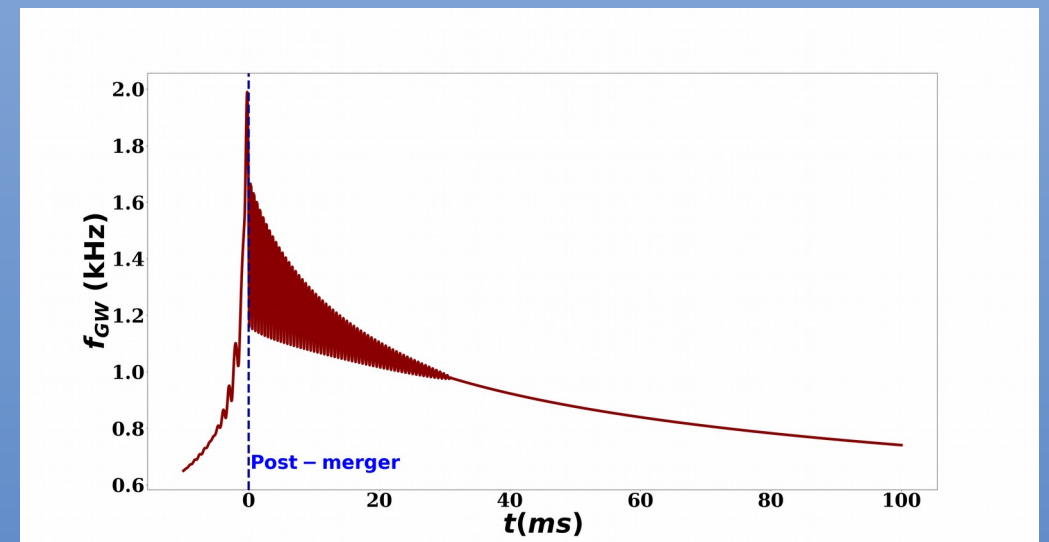


$$m_1 = m_2 = 1.35M_{\odot}$$

$$C_0 = 0.16$$

$$m_{BH} = 2.60M_{\odot}$$

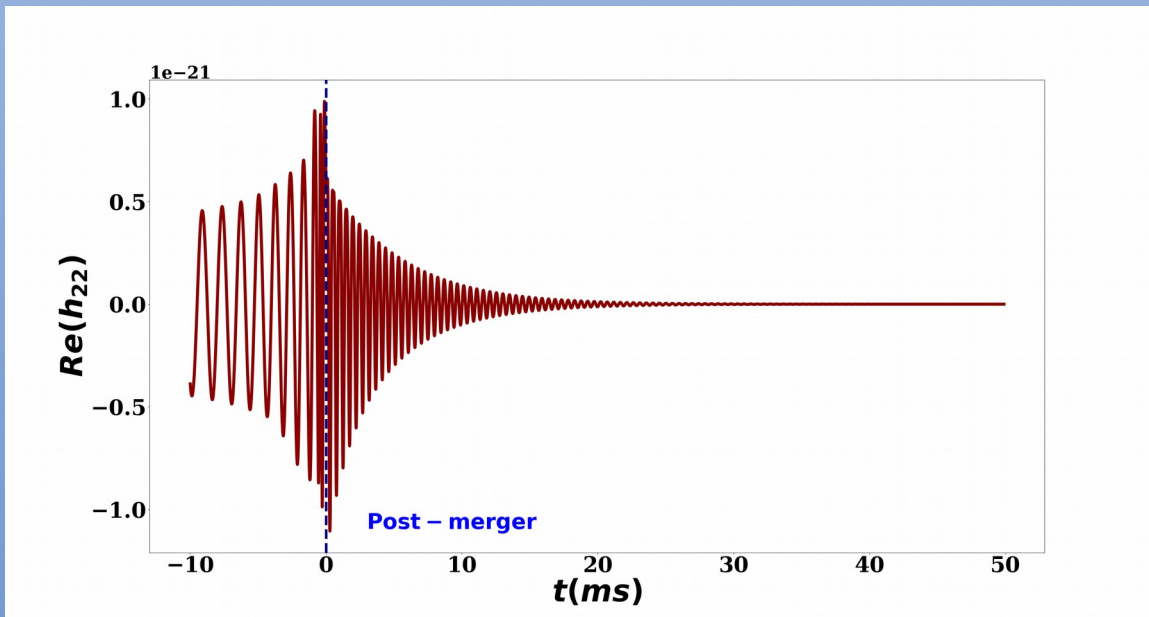
$$a_{BH} = 0.37$$



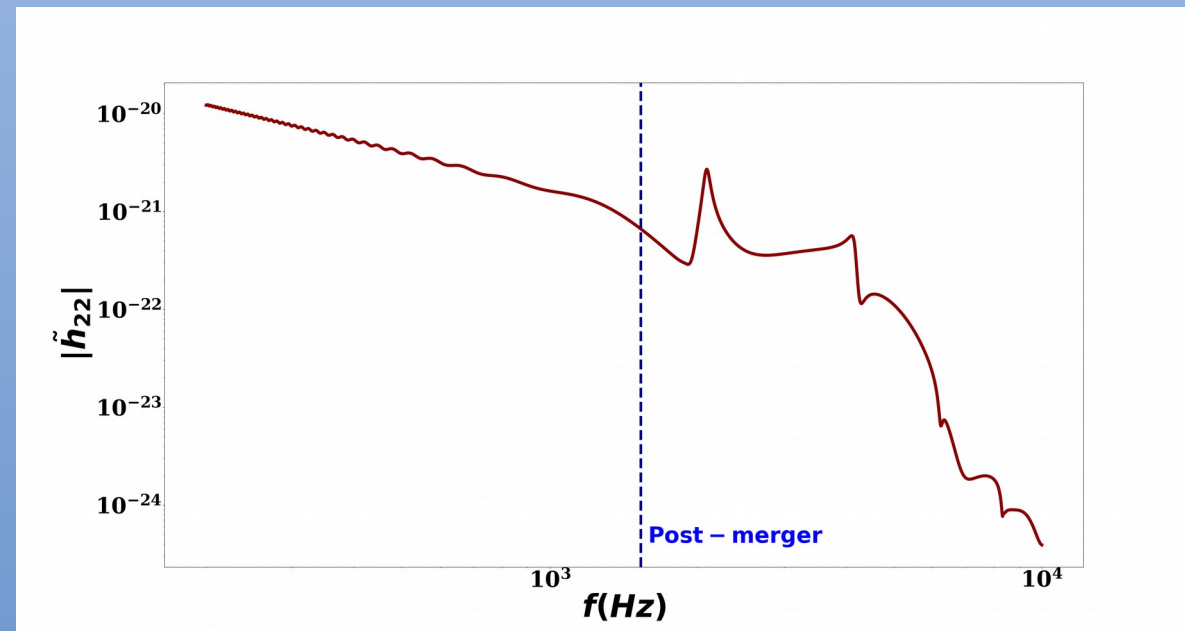
# “BS” remnant

$$m_1 = m_2 = 1.35M_{\odot}$$

$$C_0 = 0.16$$



Time domain



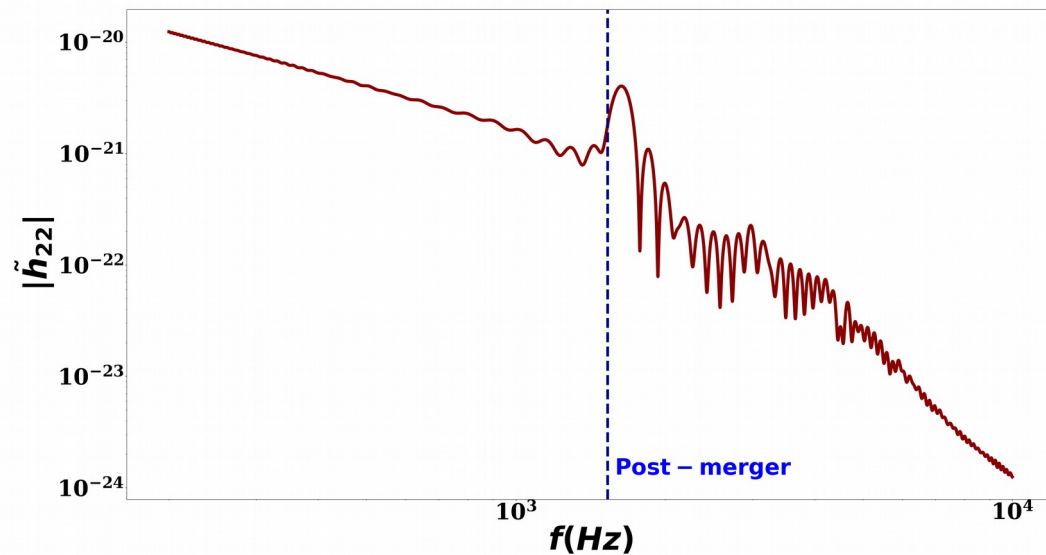
Frequency domain

$$J_i = 0$$

$$\omega_{car} = 2.1 \text{ kHz}$$

# Frequency domain

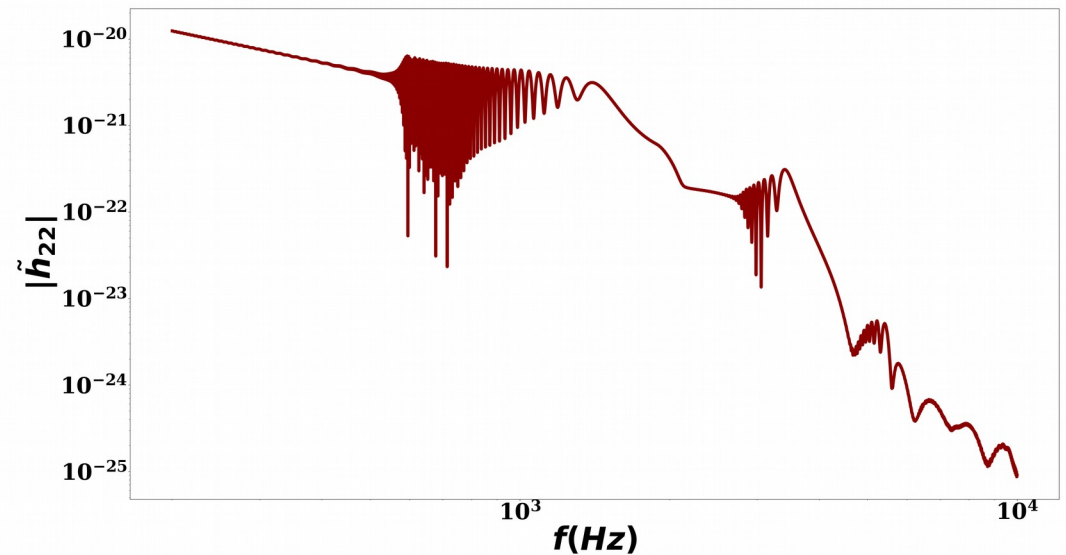
$$m_1 = m_2 = 1.35M_{\odot} \quad C_0 = 0.16$$



Collapse to a BH

$$m_{BH} = 2.60M_{\odot}$$

$$a_{BH} = 0.37$$



“Neutron Star”

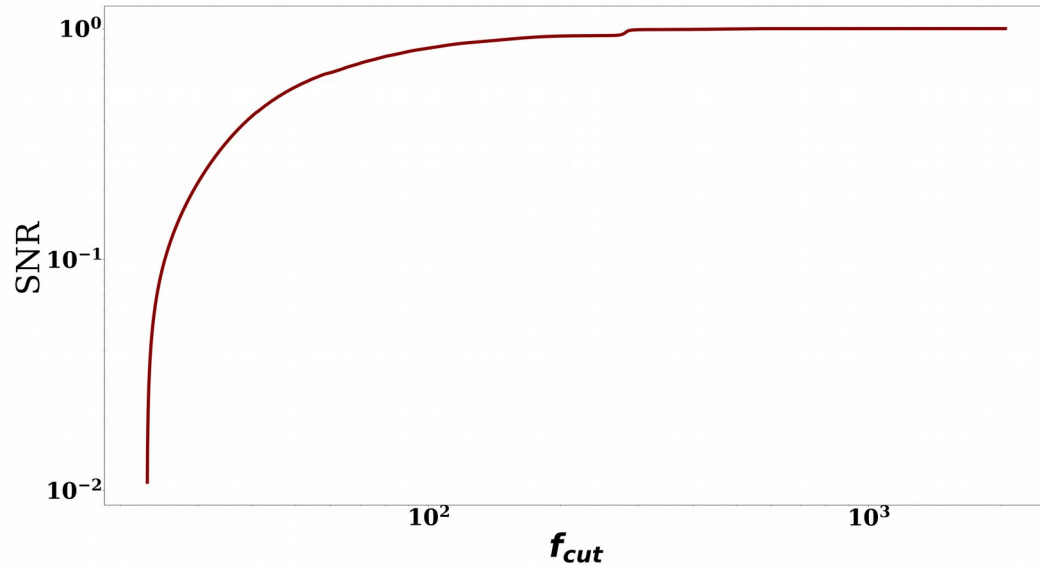
# Data analysis

- Inner product:  $(d|h) = 4\mathcal{R} \left( \int \frac{\tilde{d}(f)\tilde{h}^*(f)}{S_n(f)} df \right)$
- Signal to Noise Ratio (SNR):  $\sqrt{(h|h)}$
- Is the post merger signal detectable?
- Is the signal distinguishable from a GR BH?

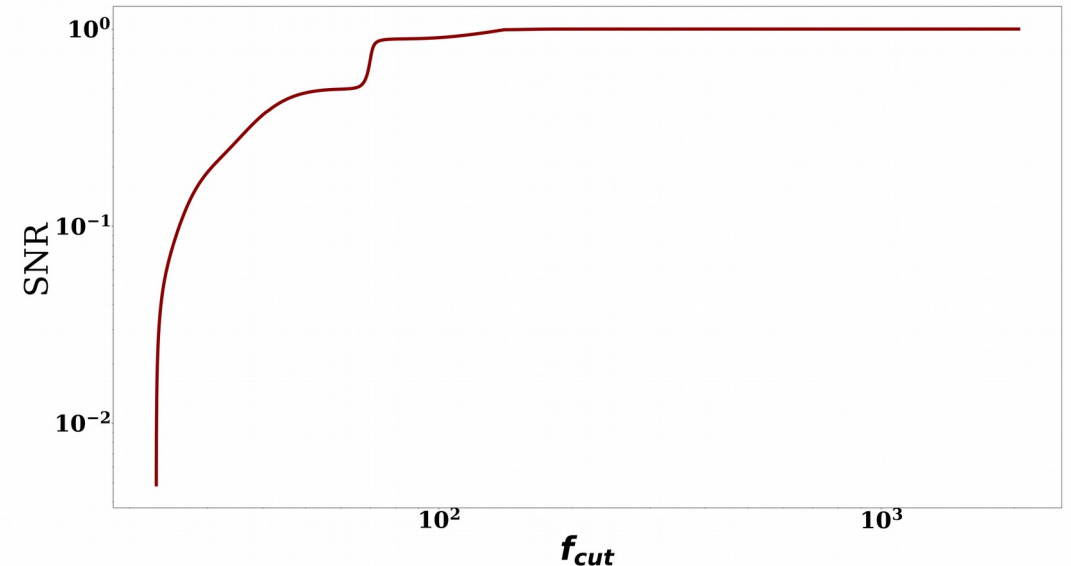
# Impact on the SNR

Optimally oriented system:

“Boson Star” in Advanced Ligo:



$$m_{tot} = 20M_{\odot}$$
$$SNR_{pm} = 0.37 SNR_{tot}$$



$$m_{tot} = 80M_{\odot}$$
$$SNR_{pm} = 0.84 SNR_{tot}$$

# Detectability

- **Threshold:**  $SNR_{pm} > 8$

| Distance | End state      |                 |                 |
|----------|----------------|-----------------|-----------------|
|          | Black Hole     | "Boson Star"    | "Neutron Star"  |
| 40 Mpc   | $5.8M_{\odot}$ | $7.5M_{\odot}$  | $3.8M_{\odot}$  |
| 400 Mpc  | $18M_{\odot}$  | $20.5M_{\odot}$ | $12.5M_{\odot}$ |

Minimum total mass for detectability of post-merger signal with **Advanced Ligo**

| Distance | End state      |                |                |
|----------|----------------|----------------|----------------|
|          | Black Hole     | "Boson Star"   | "Neutron Star" |
| 40 Mpc   | $2.1M_{\odot}$ | $2M_{\odot}$   | $1.1M_{\odot}$ |
| 400 Mpc  | $3.6M_{\odot}$ | $4.2M_{\odot}$ | $2.3M_{\odot}$ |

Minimum total mass for detectability of post-merger signal with **Einstein Telescope**

# Detectability in O1/O2 events

| Event    | $\mathcal{M}_c (M_\odot)$ | $\text{SNR}_{\text{obs}}$ | $C = 0.16$ |      |      |
|----------|---------------------------|---------------------------|------------|------|------|
|          |                           |                           | BH         | BS   | NS   |
| GW150914 | 28.6                      | 24.4                      | 20.1       | 19.5 | 21.5 |
| GW150112 | 15.2                      | 10.0                      | 6.4        | 5.8  | 7.9  |
| GW151226 | 8.9                       | 13.1                      | 5.9        | 5.0  | 8.7  |
| GW170104 | 21.4                      | 13.0                      | 9.8        | 9.2  | 11.0 |
| GW170608 | 7.9                       | 14.9                      | 6.1        | 5.0  | 9.4  |
| GW170729 | 35.4                      | 10.8                      | 9.4        | 9.1  | 9.7  |
| GW170809 | 24.9                      | 12.4                      | 9.8        | 9.4  | 10.7 |
| GW170814 | 24.1                      | 15.9                      | 12.5       | 12.0 | 13.7 |
| GW170818 | 26.5                      | 11.3                      | 9.1        | 8.8  | 9.8  |
| GW170823 | 29.2                      | 11.5                      | 9.5        | 9.2  | 10.2 |

SNR for O1/O2 events if those were some BH mimicker binary assuming equal mass ratio. The SNR have been rescaled so that the total SNR matches the one measured by the detectors.

# Distinguishability

- **Fitting factor:**  $FF = \max_h \frac{(d|h)}{\sqrt{(d|d)(h|h)}}$   $d$  : BH mimicker signal  
 $h$  : GR BH template
- $m_1 = m_2 = 15M_\odot$  **in Advanced Ligo**  $SNR_{pm} \simeq SNR_{inspiral}$
- Maximimizing over time phase and intrinsic parameters:

Black Hole

$$FF = 0.85$$

$$m_1 = 32M_\odot \quad a_1 = 0.08$$

$$m_2 = 7M_\odot \quad a_2 = 0.18$$

“Boson Star”

$$FF = 0.8$$

$$m_1 = 29M_\odot \quad a_1 = -0.17$$

$$m_2 = 8M_\odot \quad a_2 = 0.36$$

“Neutron Star”

$$FF = 0.63$$

$$m_1 = 29M_\odot \quad a_1 = -0.13$$

$$m_2 = 8M_\odot \quad a_2 = 0.10$$



# Summary:

- Phenomenological model for BH mimickers waveforms
- Main difference is post merger signal
- Could already be seen in current detectors
- **Next steps:**
  - Consider different initial angular momentum:
$$J(0) = \alpha J_c \quad 0 \leq \alpha \leq 1$$
  - More rigorous data analysis
  - Analysis of residuals

# Numerical integration

- **Define:**  $\rho = \frac{p}{1 + e \cos(\chi)}$        $e = \frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}$   
 $p = 2 \frac{\rho_+ \rho_-}{\rho_+ + \rho_-}$
- $\rho_+, \rho_-, \rho_3, \rho_4, \rho_5$  **are the roots of**  $E - V_{\text{eff}} = 0$

- **So that:**  $\dot{\chi} = 2 \sqrt{\frac{k}{m} \frac{(1 + e \cos(\chi))}{\sqrt{1 - e^2}}} \sqrt{\frac{(\rho - \rho_3)(\rho - \rho_4)(\rho - \rho_5)}{\rho(\frac{MR^2}{2m} + \rho^2)}}$

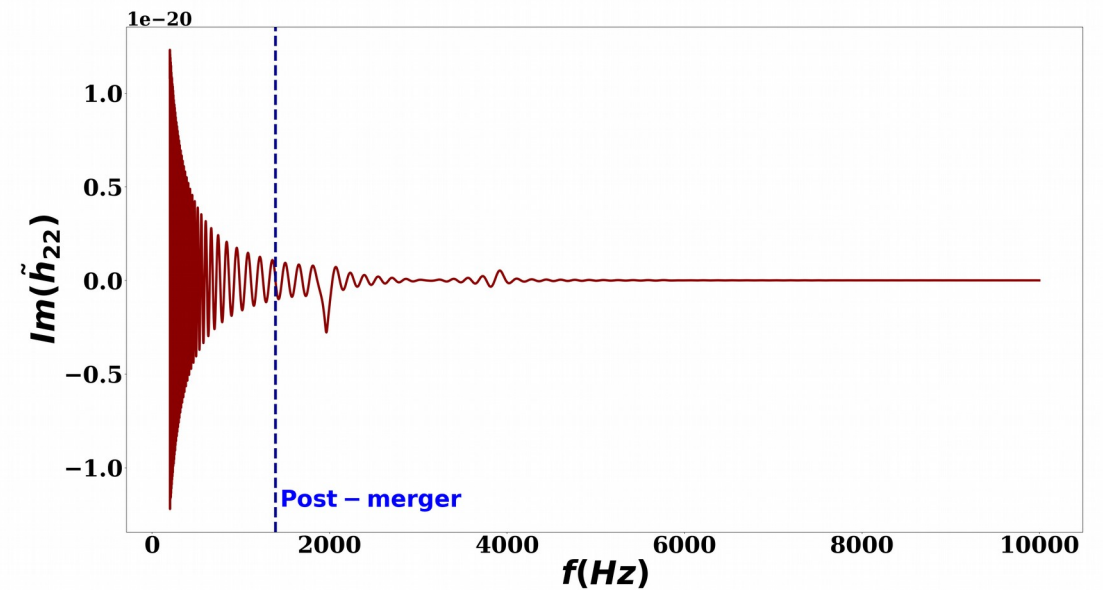
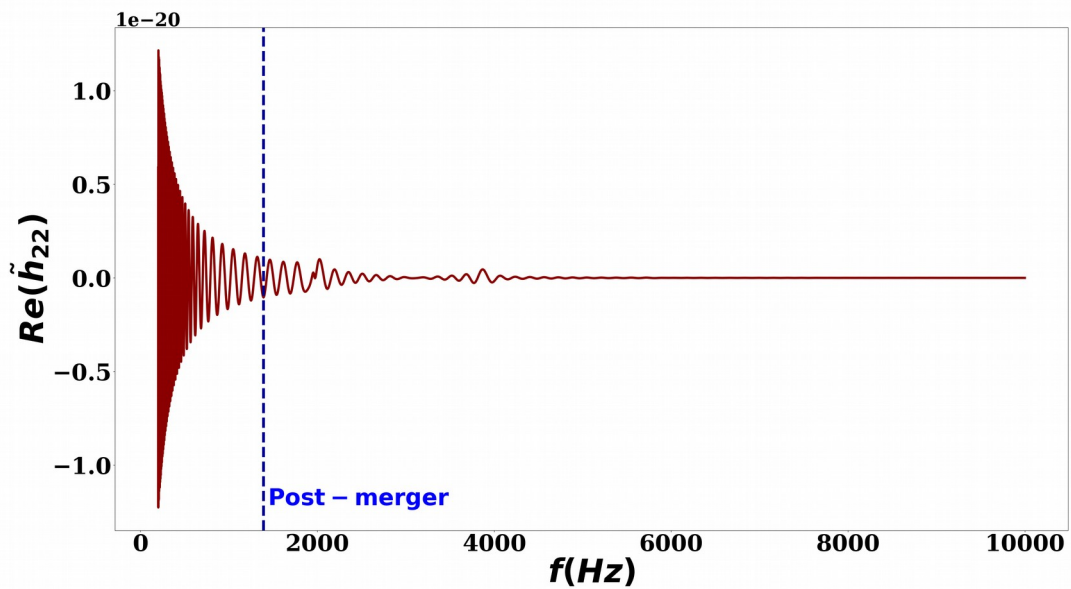
# Massive Boson Stars

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 |\phi|^2 - \frac{1}{4} \lambda |\phi|^4 \right]$$

- Colpi, Shapiro, Wasserman (PRL 1986):

$$M_{max} \simeq \left( \frac{0.10 \text{ GeV}}{m_\phi} \right)^2 \lambda^{1/2} M_\odot$$

# “Boson Star” waveform (FD)



# Black hole waveform (FD)

