

Dark energy after gravitational wave observations

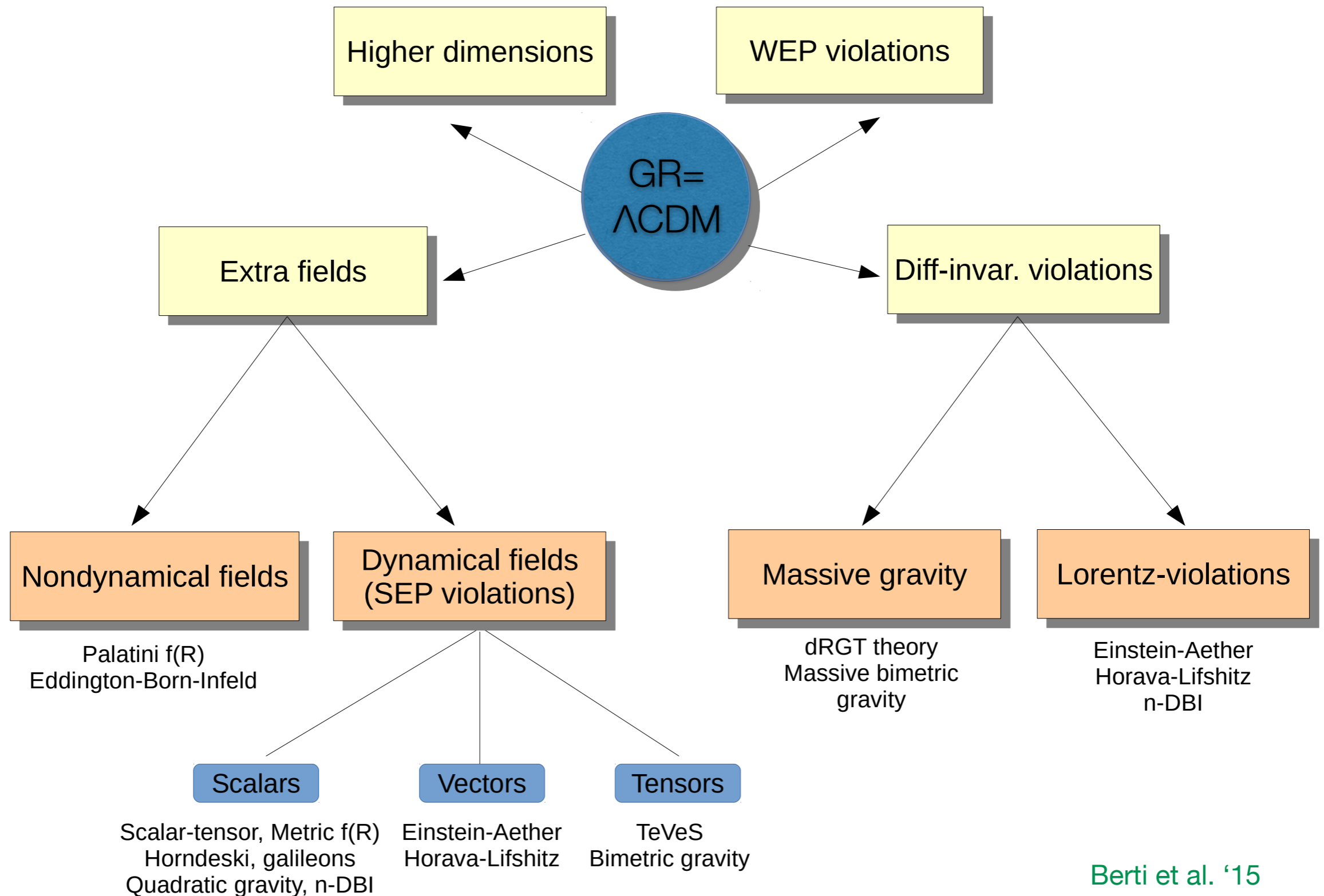
Filippo Vernizzi
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with Paolo Creminelli 1710.05877,
+ Matthew Lewandowski and Giovanni Tambalo, 1809.03484
+ Vicharit Yingcharoenrat, 1906.07015, 1910.14035

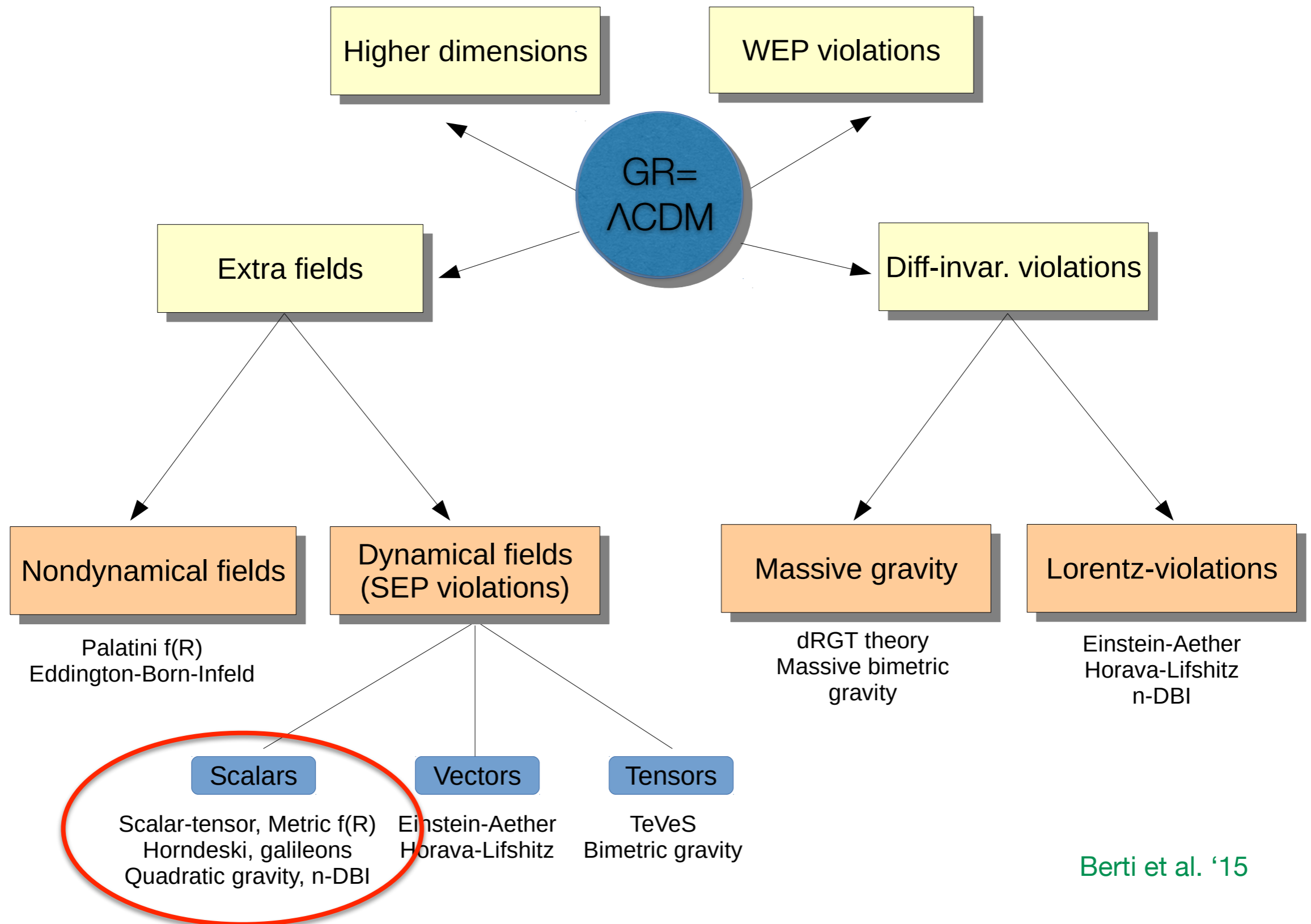
4 January 2020, Meudon

Rencontres des groupes de travaux “Formes d’ondes” et “Tests de la relativité générale et théories alternatives”, GdR Ondes Gravitationnelles

Dark energy and modified gravity



Dark energy and modified gravity



Generalized scalar-tensor theories

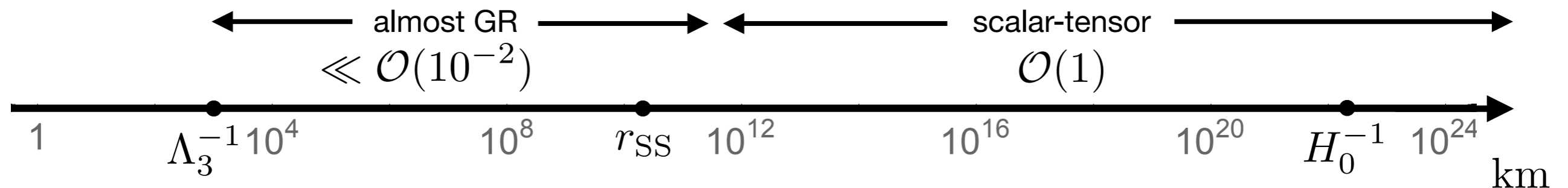
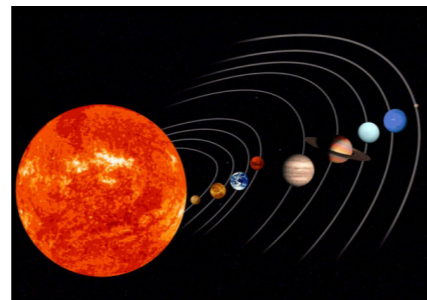
$$\begin{aligned}
 \mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi & \square\phi \equiv \phi_{;\mu}^{;\mu} & X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\
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 & - F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'} \\
 & - F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}
 \end{aligned}$$

Generalized scalar-tensor theories

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 \end{aligned}$$

Self-acceleration and **screening**: large classical scalar field nonlinearities

$$\begin{aligned}
 \Lambda_3 &\equiv (H_0^2 M_{\text{Pl}})^{1/3} \\
 &\sim (1000 \text{ km})^{-1}
 \end{aligned}$$



Generalized scalar-tensor theories

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \quad \square\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

Quintessence, k-essence, Brans-Dicke, f(R),... **Cubic Galileon, kinetic braiding, DGP...**

$$- 2G_{4,X}(\phi, X) \left[(\square\phi)^2 - (\phi_{;\mu\nu})^2 \right]$$

$$+ G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X) \left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3 \right]$$

$$- F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}$$

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• **Braiding:** scalar mixes with gravity $\mathcal{L} \supset \dot{\Psi}\dot{\pi}, \partial\Psi\partial\pi$

Deffayet et al. 2010

- Different than usual Brans-Dicke or f(R), e.g., $\Phi = \Psi$

Generalized scalar-tensor theories

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Galileons, Horndeski

Horndeski 73

Deffayet et al. 11

- **Braiding:** scalar mixes with gravity $\mathcal{L} \supset \dot{\Psi}\dot{\pi}$, $\partial\Psi\partial\pi$
 - Different than usual Brans-Dicke or f(R), e.g., $\Phi = \Psi$
- **Horndeski:** most generic theory with 2nd order EOM

Generalized scalar-tensor theories

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Beyond Horndeski

Gleyzes, Langlois, Piazza, FV '14
(See also Zumalacarregui, Garcia-Bellido '13)

- **Braiding:** scalar mixes with gravity $\mathcal{L} \supset \dot{\Psi}\dot{\pi}, \partial\Psi\partial\pi$
 - Different than usual Brans-Dicke or f(R), e.g., $\Phi = \Psi$

- **Horndeski:** most generic theory with 2nd order EOM

- **Beyond Horndeski:** more than 2 derivatives in EOM but degenerate with no ghosts

- Vainshtein screening violated inside matter
Kobayashi, Watanabe, Yamaguchi et al. 2018

$$\frac{d\Phi}{dr} = G \left[\frac{M(r)}{r^2} - \epsilon \frac{d^2 M(r)}{dr^2} \right]$$

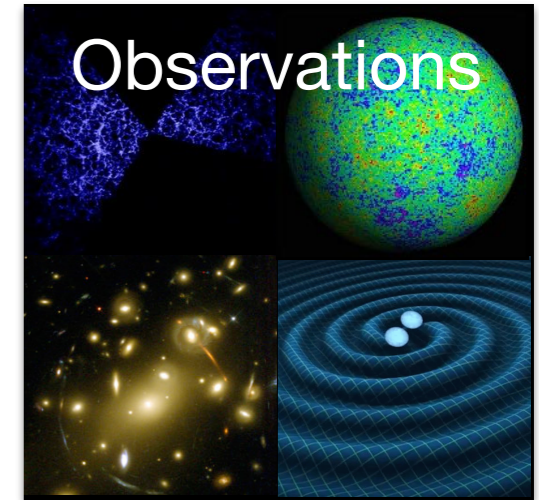
EFT of Dark Energy

Bridge models and observations
in a minimal and systematic way

Space of theories

$$G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi - 2G_{4,X}(\phi, X)[(\square\phi)^2 - (\phi_{;\mu\nu})^2] + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X) \times [(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3] + \dots$$

EFT of DE
 $\alpha_1(t), \alpha_2(t), \dots$

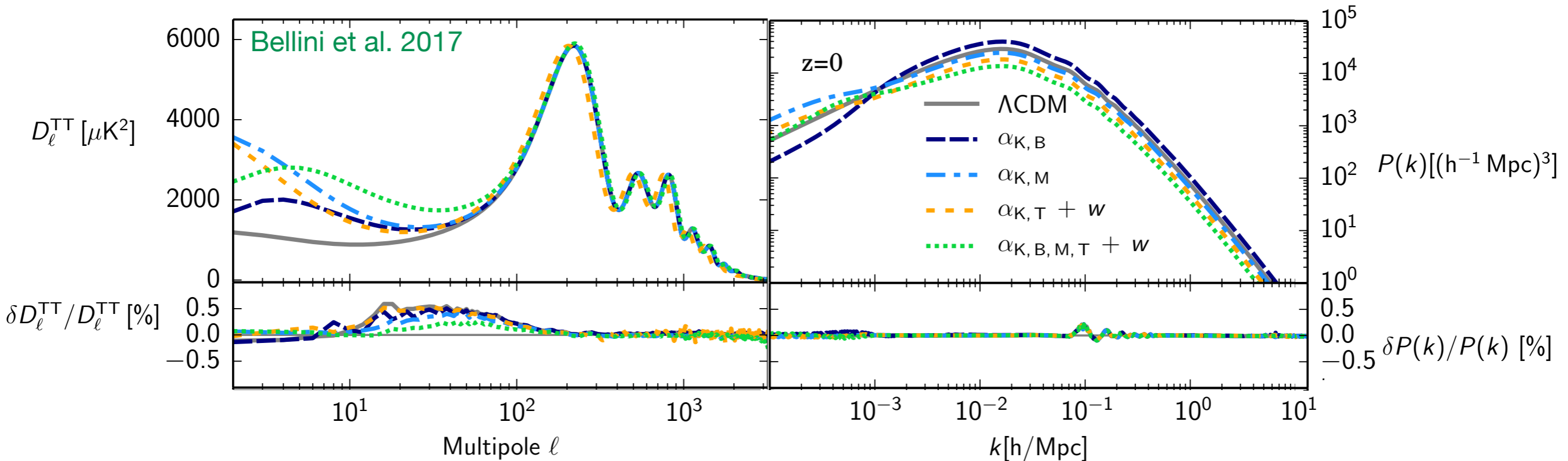


$$\mu = \mu(k; \alpha_1(t), \alpha_2(t), \dots)$$

$$\Sigma = \Sigma(k; \alpha_1(t), \alpha_2(t), \dots)$$

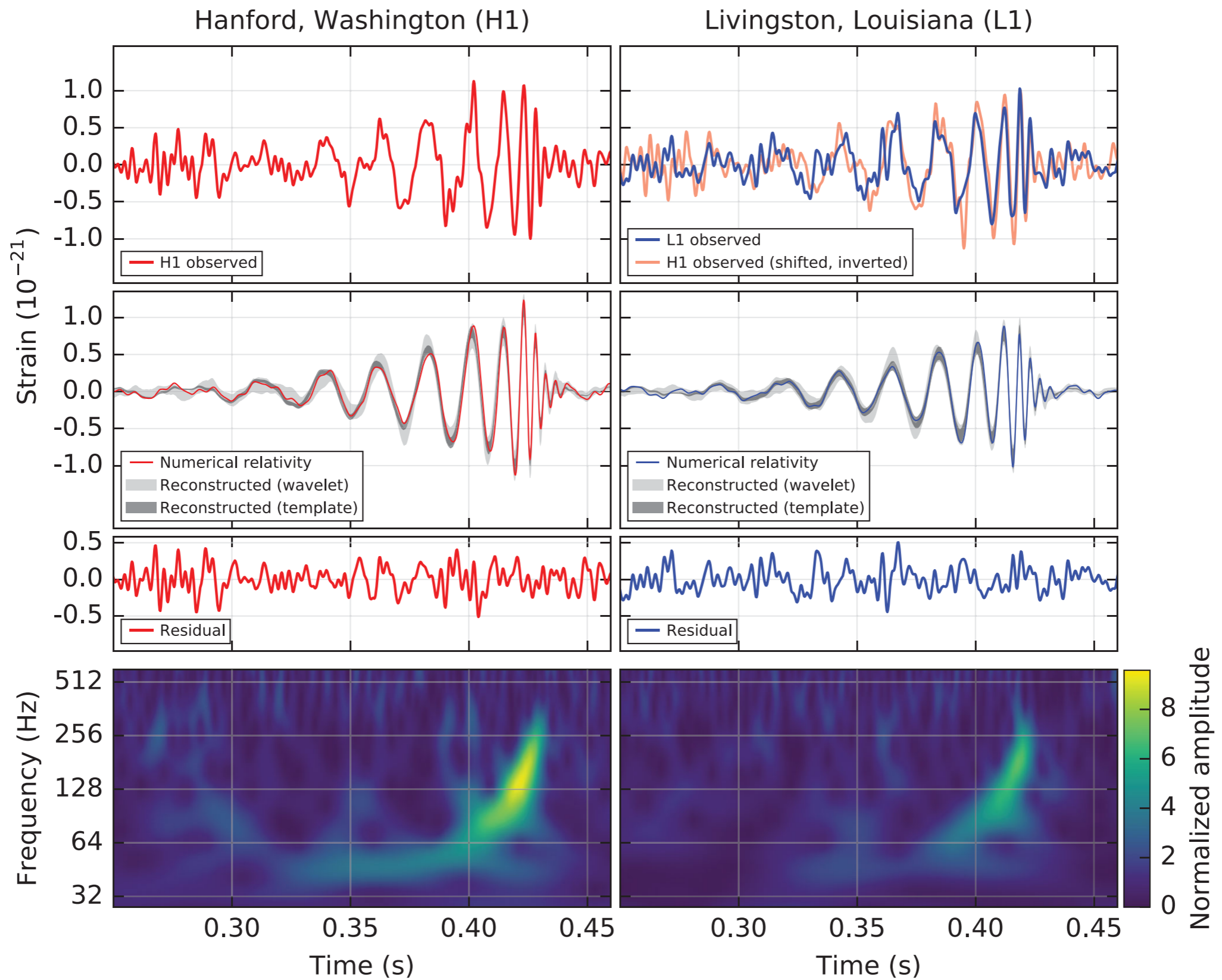
$$\nabla^2\Phi = 4\pi G \mu \delta\rho_m$$

$$\nabla^2(\Phi + \Psi) = 8\pi G \Sigma \delta\rho_m$$



GW150914: Gravitational Waves

Abbott et al. '16
first detection: 09/14, 2015



Modified gravitational wave propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are **absorbed** and **dispersed**. Effects accumulate on long time-scale.

$$\ddot{\gamma}_{ij} + [3H + \Gamma(k)] \dot{\gamma}_{ij} + [c_T^2 k^2 + f(k)] \gamma_{ij} = 0$$

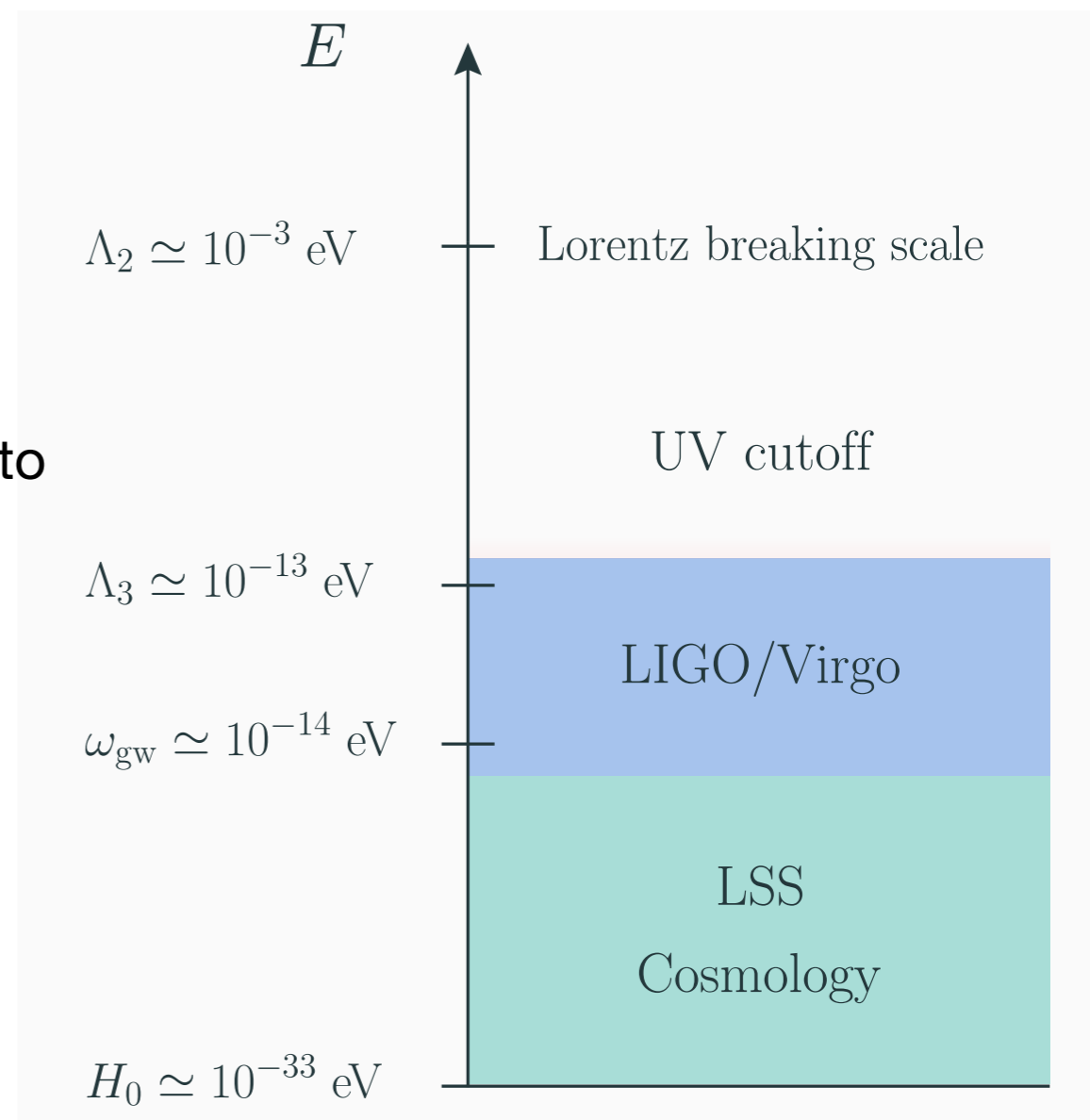


Modifications in the wave equation are related to modifications of gravity in the LSS:

$$\mu = \mu(\dots), \quad \Sigma = \Sigma(\dots)$$

$$\nabla^2 \Phi = 4\pi G \mu \delta\rho_m$$

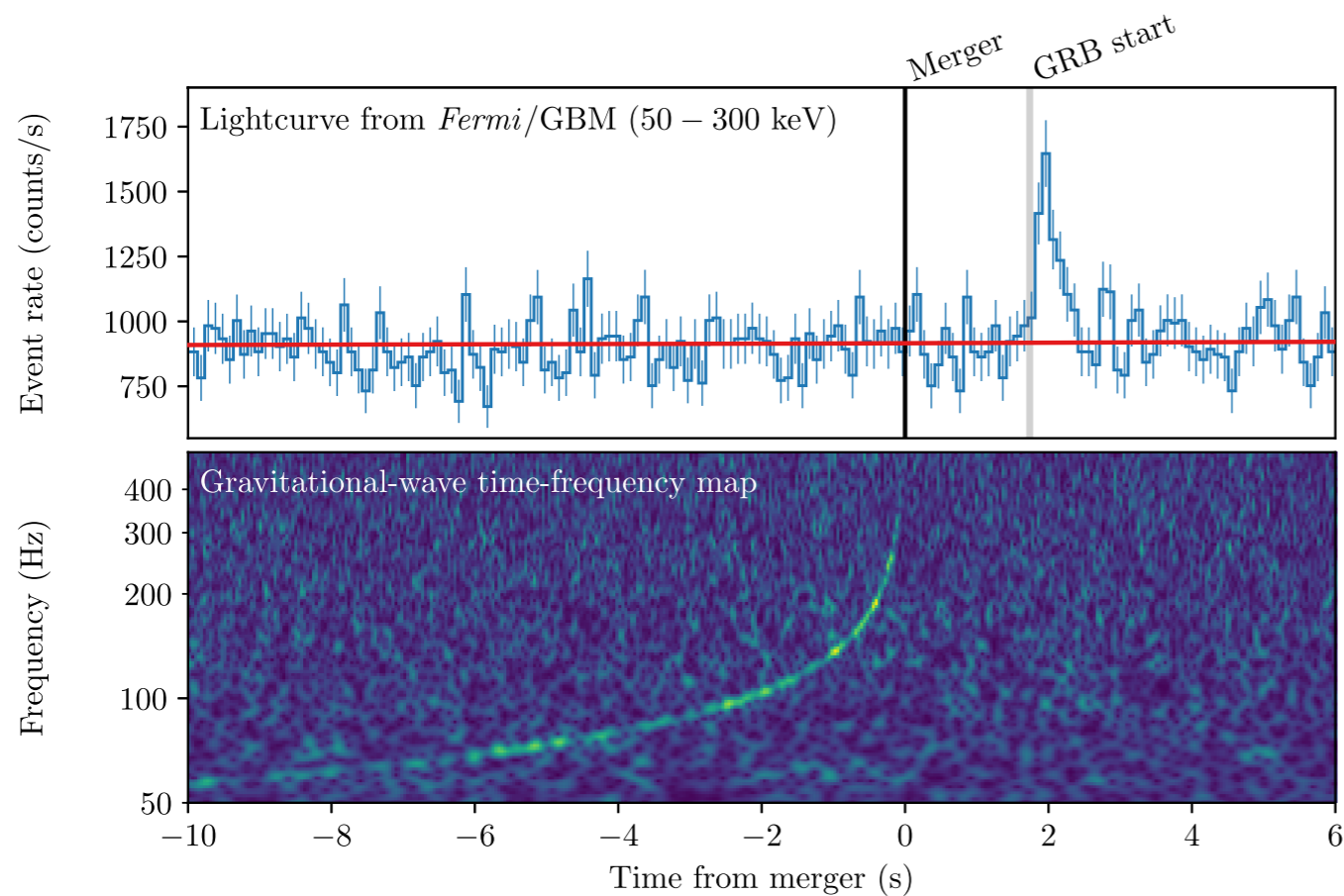
$$\nabla^2 (\Phi + \Psi) = 4\pi G \Sigma \delta\rho_m$$



Modified gravitational wave propagation

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$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + c_T^2 k^2 \gamma_{ij} = 0$$



$$-3 \times 10^{-15} \leq \frac{c_T - c}{c} \leq 7 \times 10^{-16}$$

$c_T=1$ implications

$$\begin{aligned}
 \mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi & \square\phi \equiv \phi_{;\mu}^{;\mu} & X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\
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$c_T=1$ implications

$$\dot{\gamma}_{ij}^2 - (\partial_k \gamma_{ij})^2$$

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\end{aligned}$$

$$G_5 = F_5 = 0, \quad XF_4 = 2G_{4,X}$$

Creminelli, FV '17; Sakstein, Jain '17 ; Ezquiaga, Zumalacarregui '17 ; Baker+ '17

$$\delta c_T \sim (\Lambda_3/\Lambda_2)^4 \sim 10^{-40} \ll 10^{-15}$$

$c_T=1$ tuning is stable
(approx. Galileon symmetry)

Can we rule out more?

$$\begin{aligned}\mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \\ & - 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & - F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\end{aligned}$$

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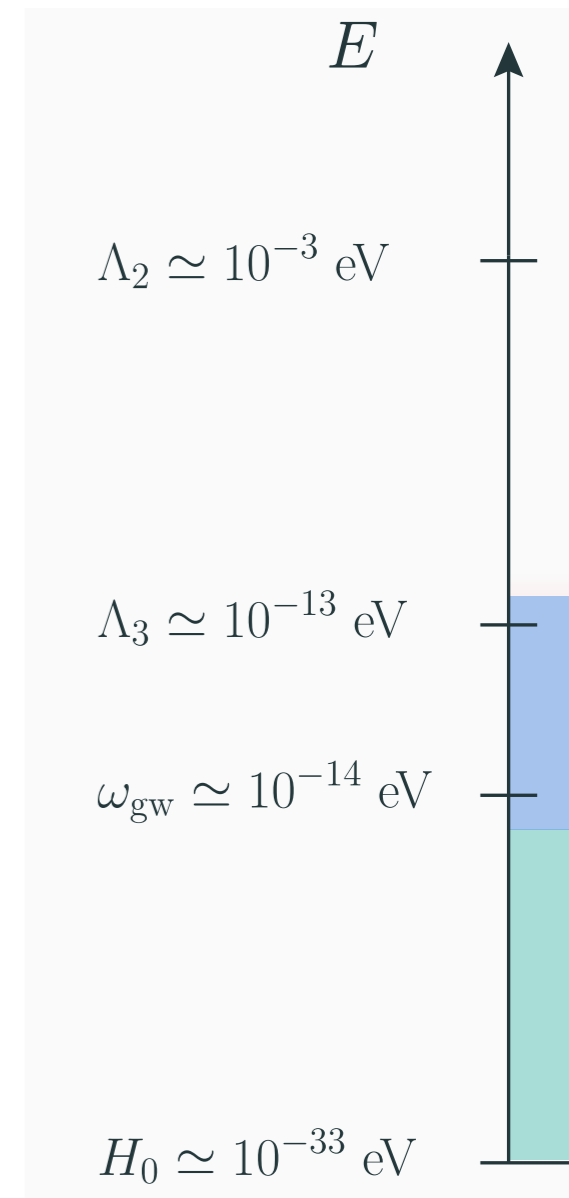
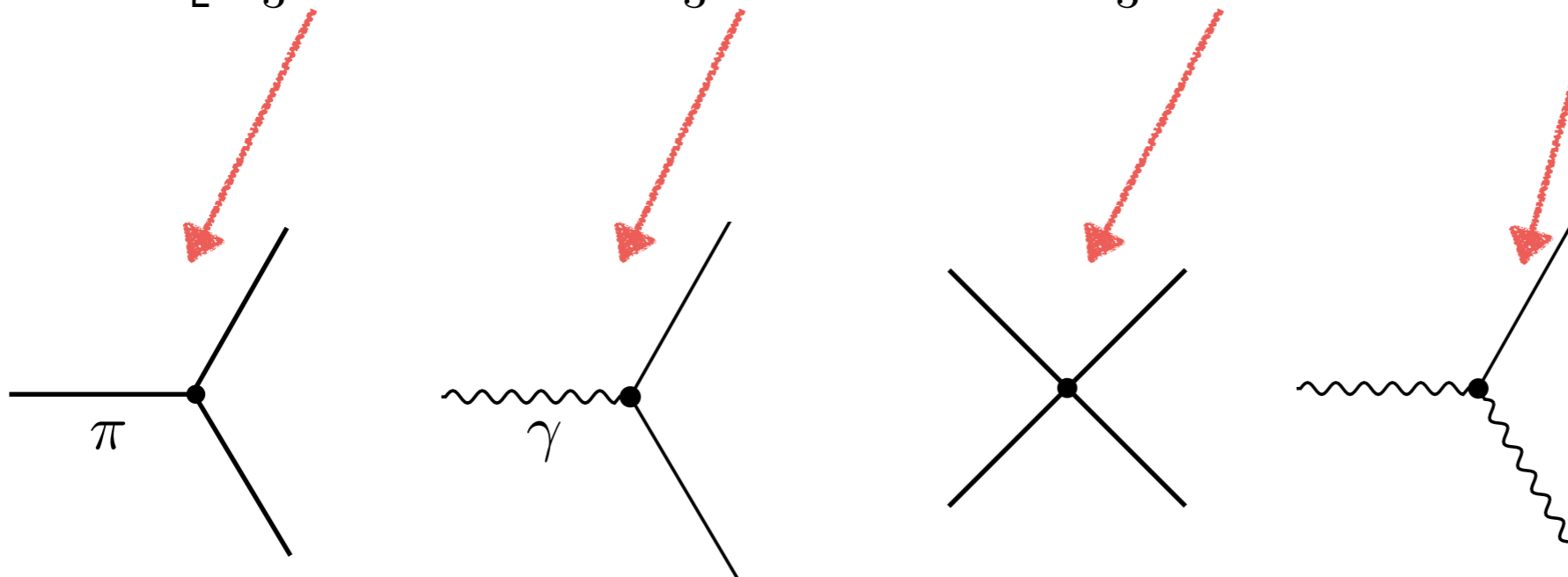
$$XF_4 = 2G_{4,X}$$

- **Beyond Horndeski:** $\alpha_H \equiv -\frac{X^2 F_4}{G_4}$

Expanded action for α_H

$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2) \quad \pi \equiv \delta\phi/\phi_0$$

$$+ \alpha_H \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial\pi)^2 + \frac{1}{\Lambda_3^3} \ddot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{\Lambda_3^6} (\square\pi)^2 (\partial\pi)^2 + \frac{1}{\Lambda_2^2} \dot{\pi} \dot{\gamma}_{ij}^2 \right]$$

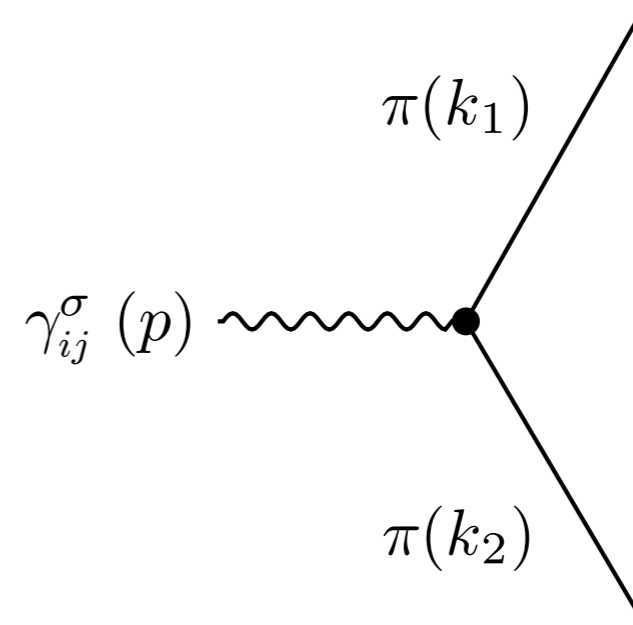


Graviton decay into dark energy

Creminelli, Lewandowski, Tambalo, FV '18

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3} \ddot{\gamma}_{ij}^c \partial_i \pi_c \partial_j \pi_c$$

$$\Lambda_3 \equiv (M_{\text{Pl}} H_0^2)^{1/3}$$



Beyond Horndeski interactions imply GW decay into scalar fluctuations π . Analogous to light absorption into a material

Decay allowed for $c_s < 1$ (c_s = sound speed of π fluctuations; assume $c_T=1$)

$$\Gamma \simeq \left(\frac{\alpha_H}{\Lambda_3^3} \right)^2 \frac{\omega_{\text{gw}}^7 (1 - c_s^2)^2}{c_s^7} \quad \text{decay rate}$$

$$d_S \Gamma < 1 \quad \Rightarrow \quad \alpha_H < 10^{-10} \quad \text{irrelevant for LSS observations } \alpha_H \lesssim 10^{-2}$$

(unless $c_s=1$ with great precision)

Coherent decay

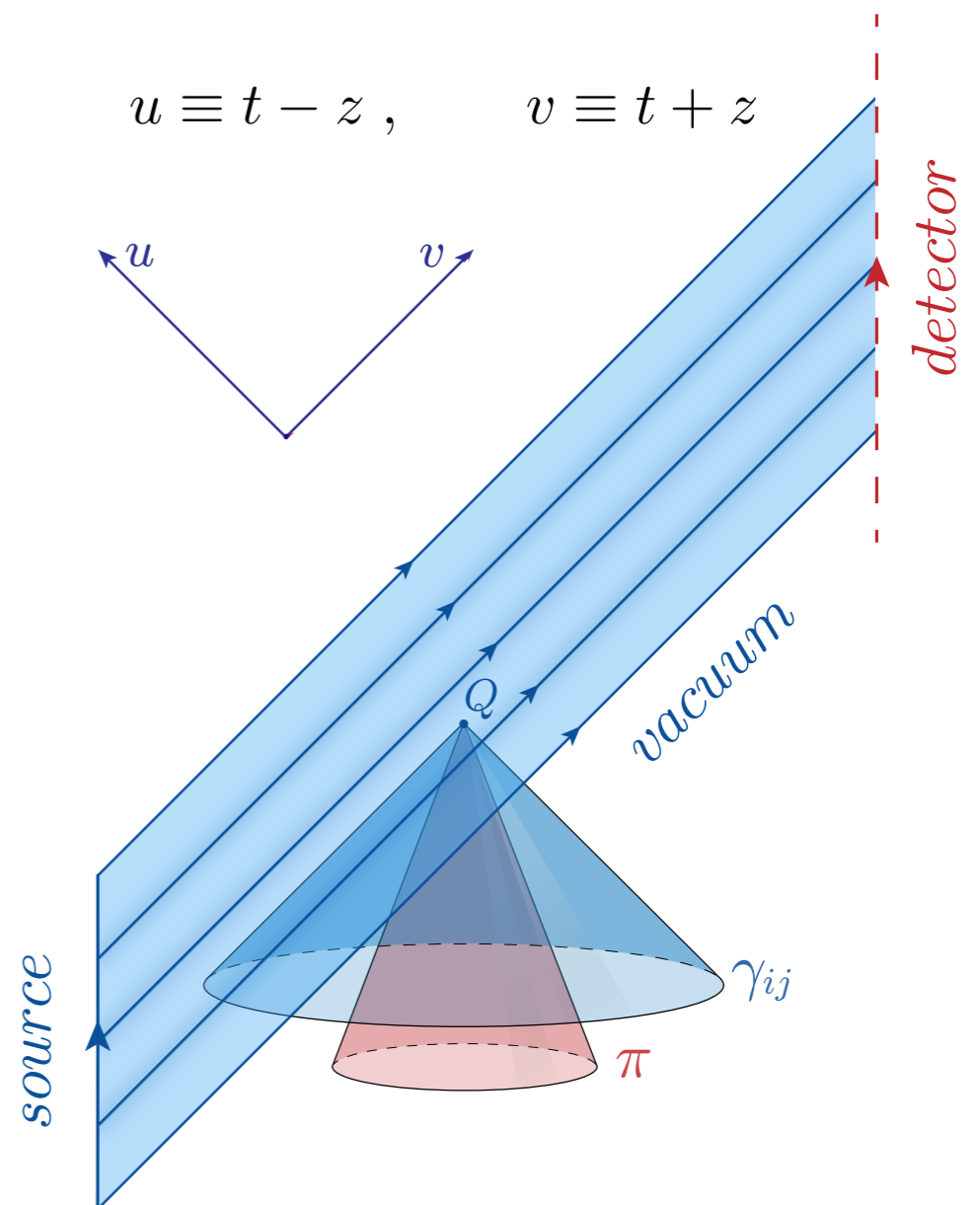
Creminelli, Tambalo, FV, Yingcharoenrat, '19

Decay enhanced by the large occupation number of the GWs ~ preheating

Classical wave: $\gamma_{ij} = M_{\text{Pl}} h_0^+ \cos(\omega u) \epsilon_{ij}^+$, $\beta = \frac{|\alpha_H|}{\alpha c_s^2} \left(\frac{\omega}{H}\right)^2 h_0^+$

Oscillator with changing frequency:

$$\ddot{\pi} - c_s^2 \left[\nabla^2 + \beta \cos(\omega u) \epsilon_{ij}^+ \partial_i \partial_j \right] \pi = 0$$



Coherent decay

Creminelli, Tambalo, FV, Yingcharoenrat, '19

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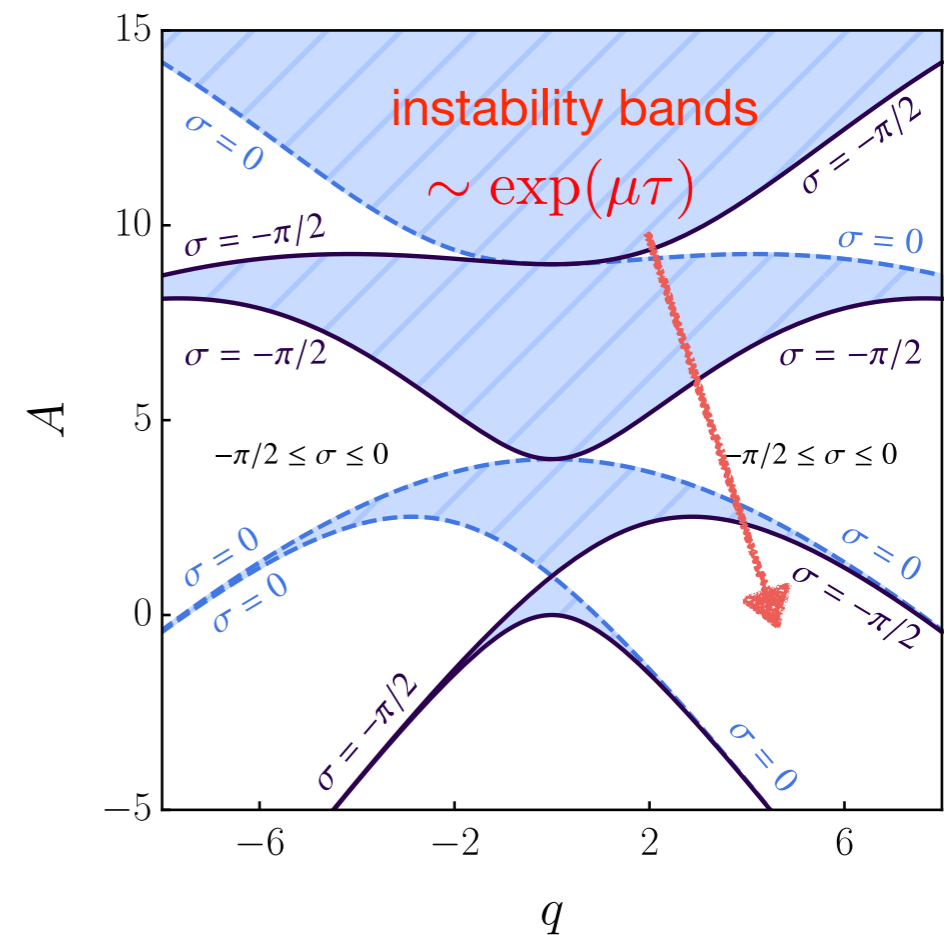
Oscillator with changing frequency:

$$\ddot{\pi} - c_s^2 \left[\nabla^2 + \beta \cos(\omega u) \epsilon_{ij}^+ \partial_i \partial_j \right] \pi = 0$$

Each Fourier mode satisfies a Mathieu equation
 \Rightarrow **parametric resonance**.

$$\frac{d^2 \pi_{\vec{k}}}{d\tau^2} + (A_{\vec{k}} - 2q_{\vec{k}} \cos(2\tau)) \pi_{\vec{k}} = 0$$

Resonant modes grow exponentially: $\pi_{\vec{k}} \sim e^{\mu_{\vec{k}} \tau}$

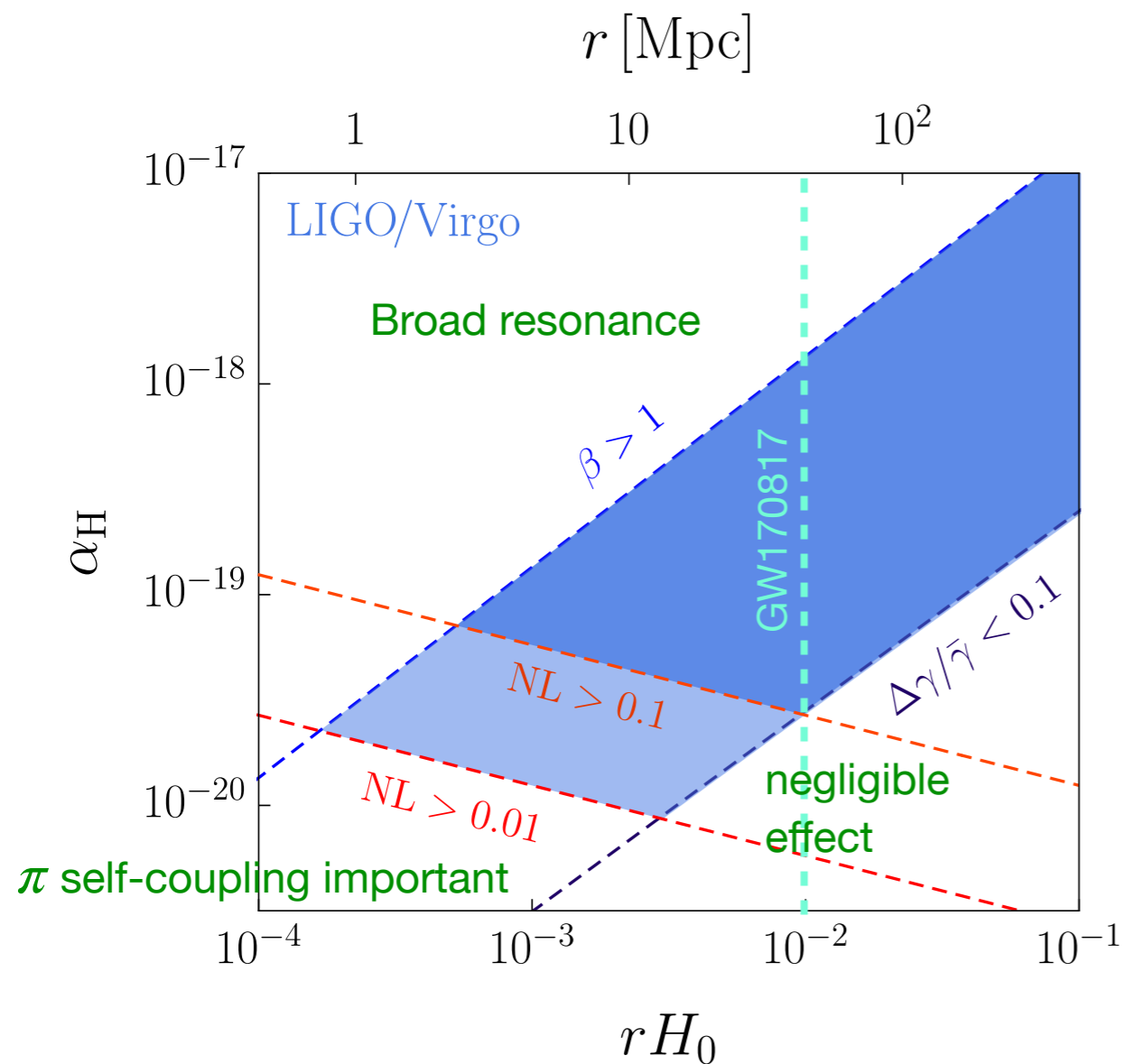


Narrow resonance $\beta \ll 1$: $\mu \sim \beta/4 \Rightarrow \rho_\pi \propto e^{\beta \omega u/4} \Rightarrow \Delta \gamma_{ij} \propto v \gamma_0 e^{\beta \omega u/4} \epsilon_{ij}^+$

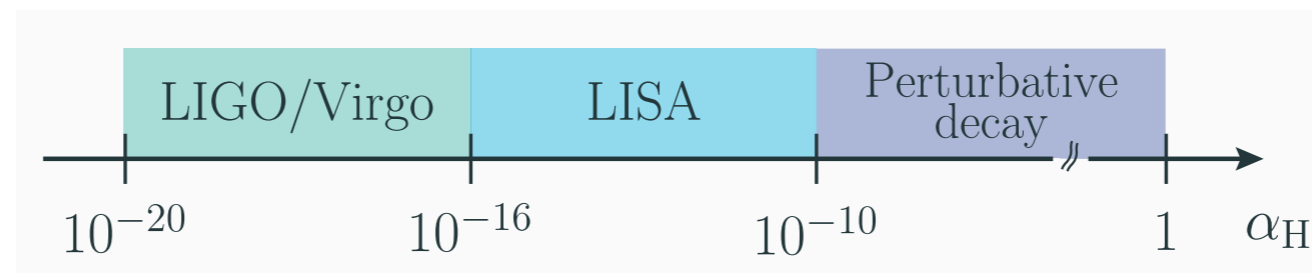
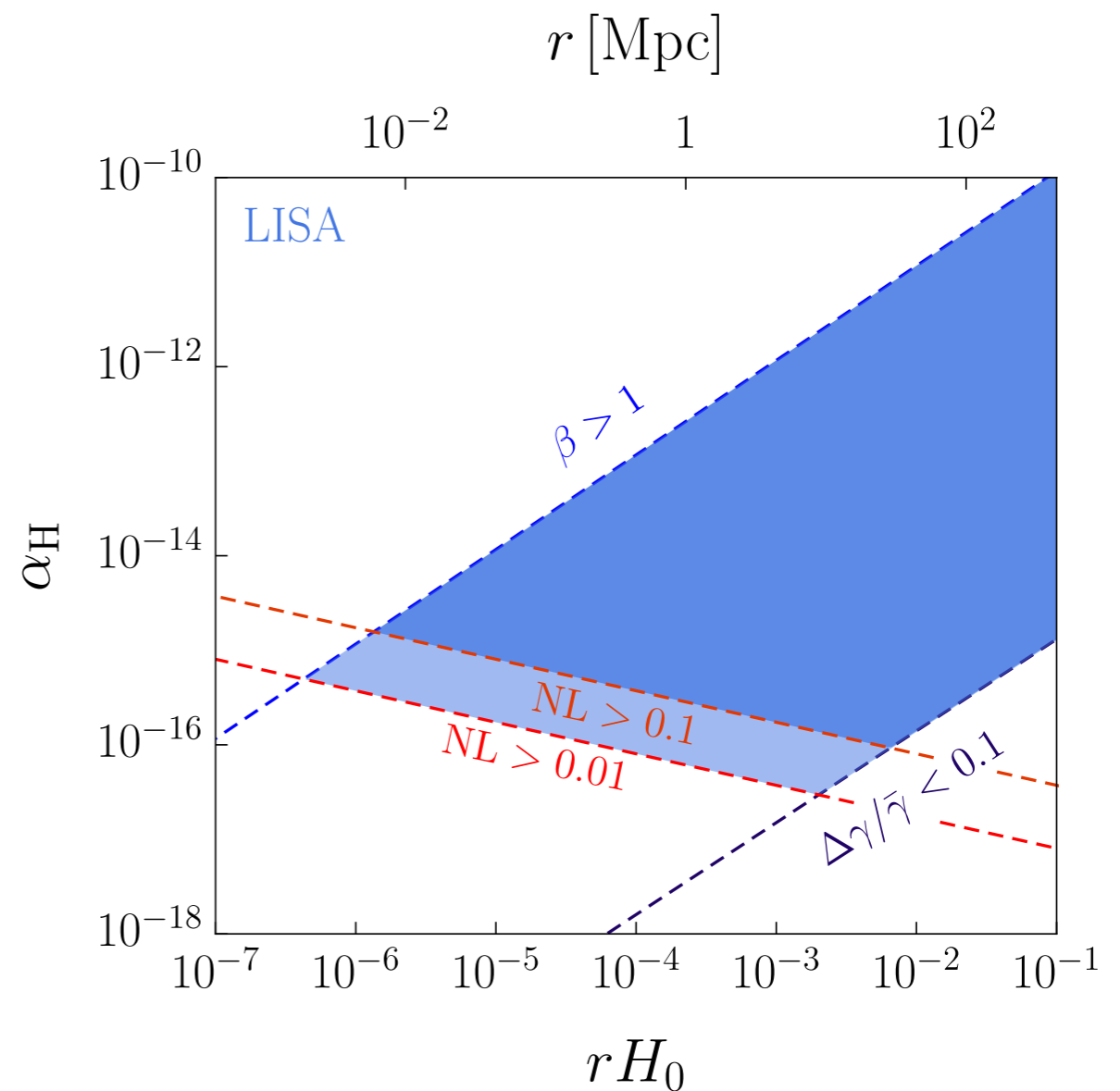
Same direction and polarization. Same frequency + higher harmonics (precursors)

GW modification

$$f = 30 \text{ Hz}, \quad M_c = 1.2 M_\odot$$



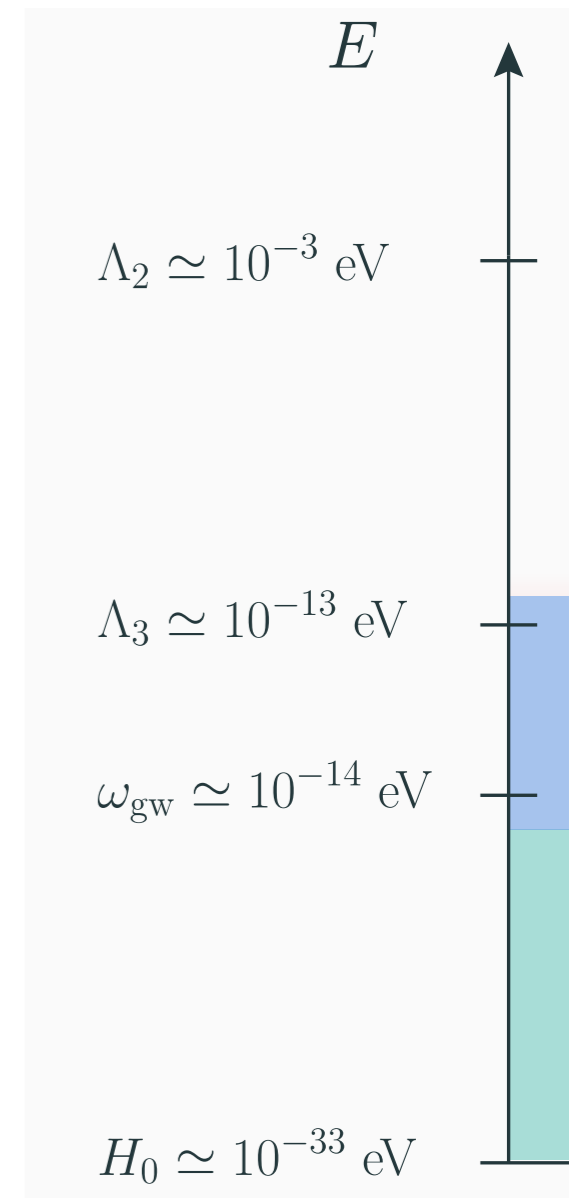
$$f = 10^{-2} \text{ Hz}, \quad M_c = 30 M_\odot$$



Expanded action for α_H

$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2) \quad \alpha_H \equiv -\frac{X^2 F_4}{G_4}$$

$$+ \alpha_H \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial \pi)^2 + \frac{1}{\Lambda_3^3} \ddot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{\Lambda_3^6} (\square \pi)^2 (\partial \pi)^2 + \frac{1}{\Lambda_2^2} \dot{\pi} \dot{\gamma}_{ij}^2 \right]$$



Theory after no decay

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi$$
~~$$- 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right]$$~~
~~$$- F_4(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}$$~~

$$\square\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

$$XF_4 = 2G_{4,X}$$

~~• *Beyond Horndeski*: $\alpha_H \equiv -\frac{X^2 F_4}{G_4}$~~

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$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi$$

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~~$$- F_4(\phi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma} \phi_{;\mu} \phi_{;\mu'} \phi_{;\nu\nu'} \phi_{;\rho\rho'}$$~~

$$\square\phi \equiv \phi_{;\mu}^{\mu} \quad X \equiv g^{\mu\nu} \phi_{;\mu} \phi_{;\nu}$$

$$XF_4 = 2G_{4,X}$$

- **Braiding:** $\alpha_B = \frac{\dot{\phi} X G_{3,X}}{H G_4}$

Expanded action for α_B

$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2)$$

$$\alpha_B = \frac{\dot{\phi} X G_{3,X}}{H G_4}$$

$$+ \alpha_B \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial \pi)^2 + \frac{1}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{M_{\text{Pl}}} \pi \dot{\gamma}_{ij}^2 \right]$$

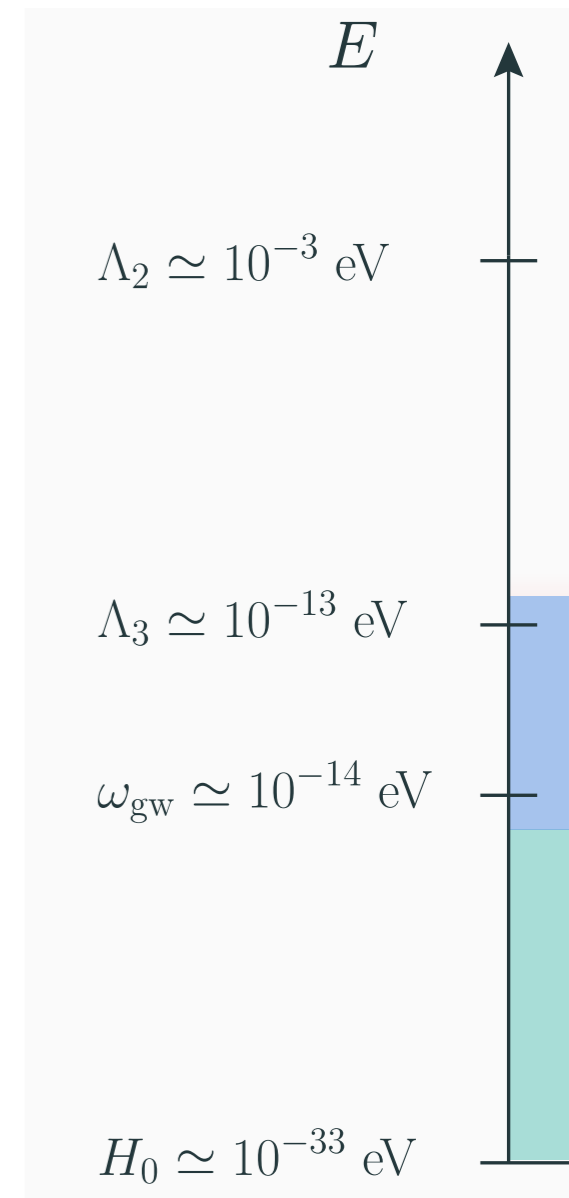
$$\Lambda_2 \equiv (H_0 M_{\text{Pl}})^{1/2}$$

Same calculation but with $\beta = \frac{|\alpha_B|}{\alpha c_s^2} \frac{\omega}{H} h_0^+$

Exponential growth **quenched** by large self-couplings of π .

Kills the effect? Simulations ~ preheating

No clear constraints on α_B ...



Expanded action for α_B

Creminelli, Tambalo, FV, Yingcharoenrat, '19

$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2)$$

$$\alpha_B = \frac{\dot{\phi} X G_{3,X}}{H G_4}$$

$$+ \alpha_B \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial \pi)^2 + \frac{1}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{M_{\text{Pl}}} \pi \dot{\gamma}_{ij}^2 \right]$$

The regime $\beta > 1$ seems problematic:

gradient instability < 0

$$\ddot{\pi} + c_s^2 [k^2 + \beta \cos(\omega u) \epsilon_{ij}^+ k^i k^j] \pi = 0$$

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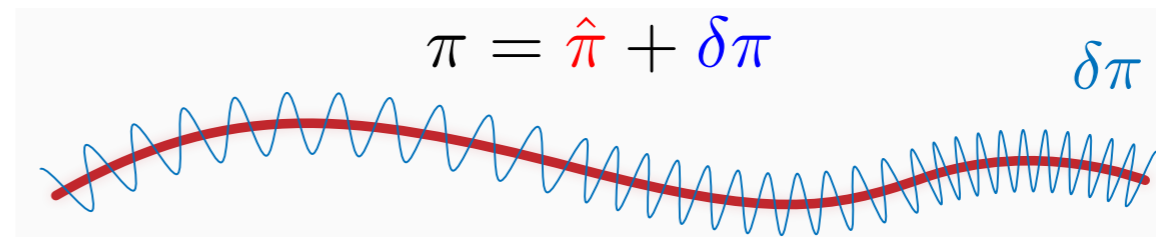
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Better analysis: We include nonlinearities of π

$$Z_{\mu\nu} [\hat{\pi}(x)] \partial^\mu \partial^\nu \delta\pi = 0$$



• $\beta > 1$: gradient instability

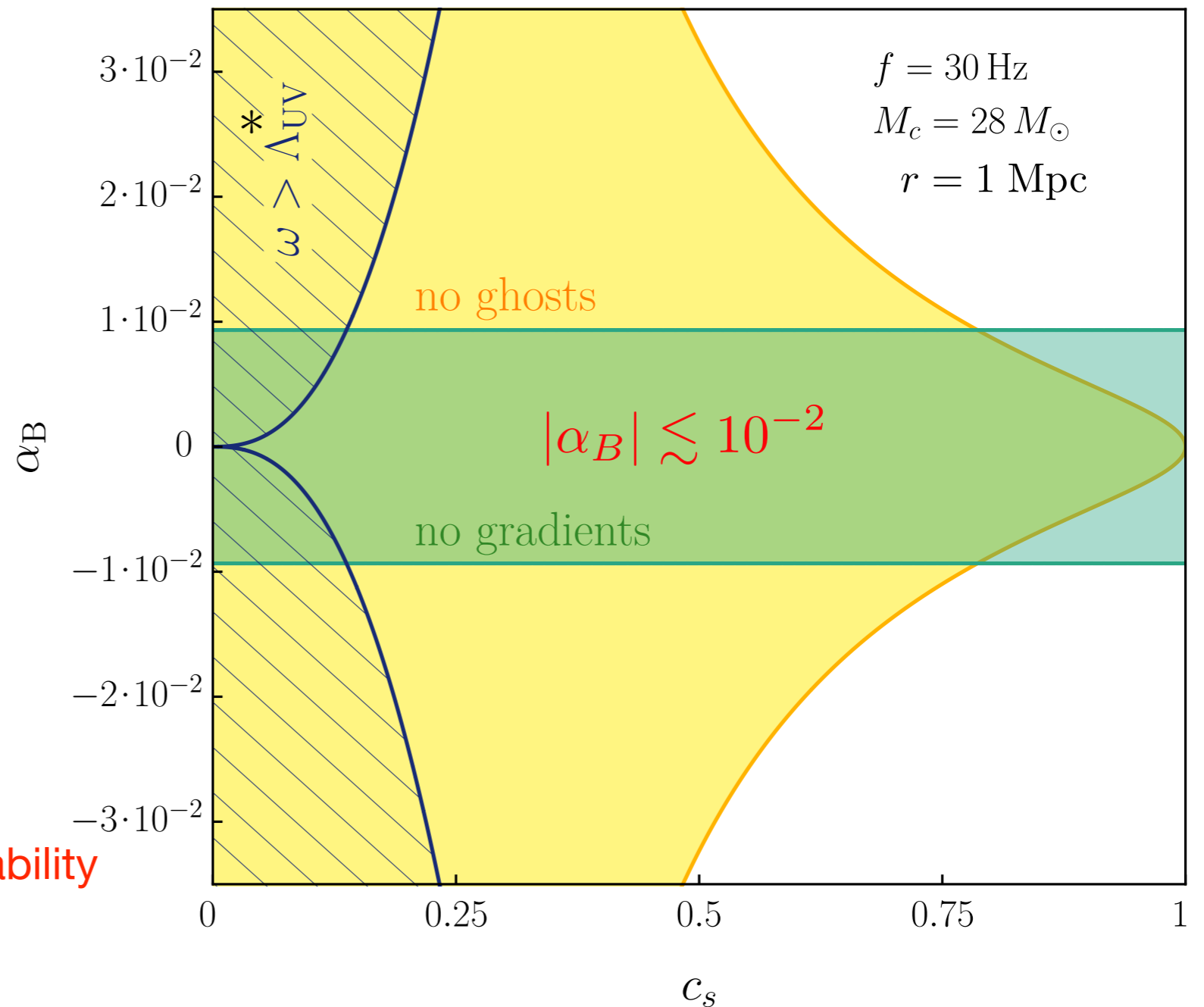
• $\beta^2 > (1 - c_s^2) c_s^{-4}$: ghost instability

We will also find ghost instabilities $Z_{00} < 0$ See also Babichev '20

Constraints for stellar-mass BHs

$$\beta = \frac{|\alpha_B|}{\alpha c_s^2} \frac{\omega}{H} h_0^+$$

$$h_0^+ \sim \frac{1}{\sqrt{2}} \cdot \frac{4}{r} (GM_c)^{5/3} (\pi f)^{2/3}$$



$\odot \beta > 1$: gradient instability

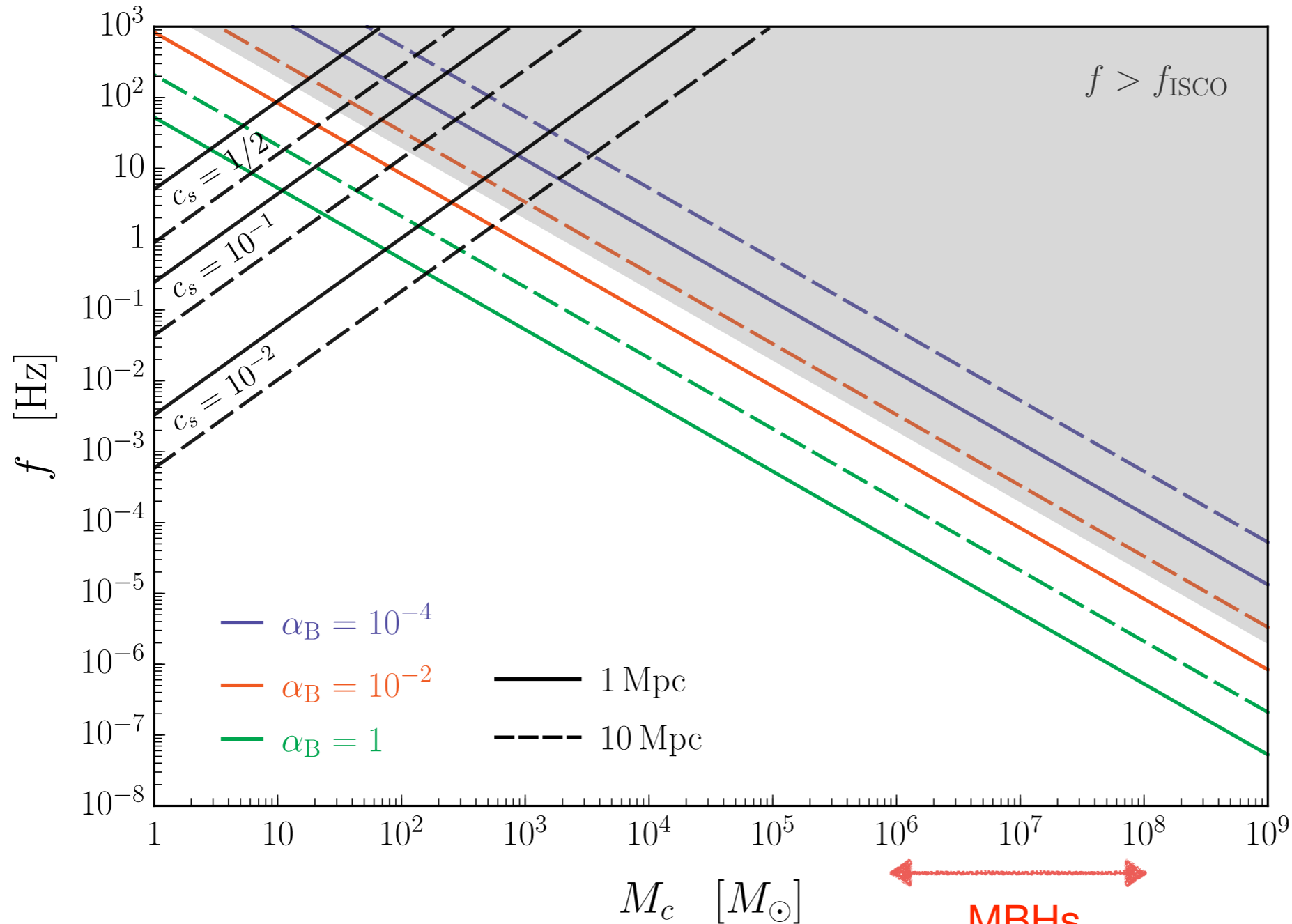
$\odot \beta^2 > (1 - c_s^2) c_s^{-4}$: ghost instability

$$\ast \Lambda_{UV} \sim \frac{\alpha^{1/2} c_s^{11/6}}{\alpha_B^{1/3}} \Lambda_3$$

Gradient instability, $\beta > 1$, for α_B

$$\beta = \frac{|\alpha_B|}{\alpha c_s^2} \frac{\omega}{H} h_0^+$$

$$h_0^+ \sim \frac{1}{\sqrt{2}} \cdot \frac{4}{r} (GM_c)^{5/3} (\pi f)^{2/3}$$



$$|\alpha_B| \lesssim 10^{-2}$$

Population of MBHs is
enough to globally
trigger the instability

Down to 10^{10} Km

The fate of UV instability

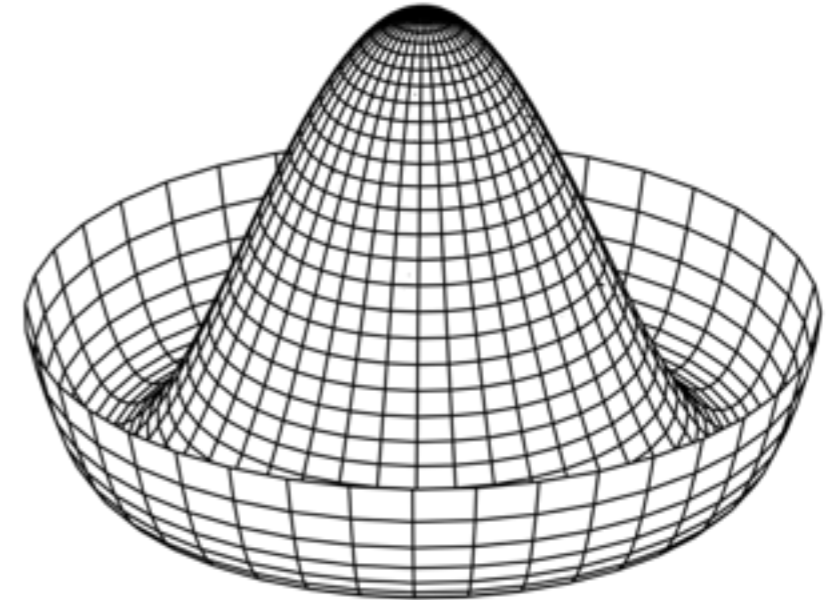
Most unstable modes are at the UV cutoff. Instability depends on the UV completion

$U(1)$ *ad hoc* example: complex scalar in the broken phase

See also Babichev, Ramazanov, Vikman '18

Integrating out radial direction $\Rightarrow \mathcal{L} \simeq -\frac{1}{4\lambda} X(\mu^2 - X)$

$$X = (\partial\phi)^2 \qquad \mu^2 = 4\lambda v^2$$



- **Gradient instabilities** for $\frac{\mu^2}{6} < \hat{X} < \frac{\mu^2}{2}$

In UV theory, instability is saturated at μ : stable for $k \gg \mu$

- **Ghost instabilities** for $\hat{X} > \frac{\mu^2}{2}$

EFT has no applicability: radial mode becomes massless

Time evolution of the system (Cauchy problem), and thus compatibility of the EFT with the data, depend on the UV completion

Theory after no decay and no instabilities

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + \cancel{G_3(\phi, X)\square\phi}$$

$$\square\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu} \phi_{;\mu} \phi_{;\nu}$$

$$\cancel{2G_{4,X}(\phi, X) \left[(\square\phi)^2 + (\phi_{;\mu\nu})^2 \right]}$$

$$\cancel{F_4(\phi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma} \phi_{;\mu} \phi_{;\mu'} \phi_{;\nu\nu'} \phi_{;\rho\rho'}}$$

• **Braiding:** $\alpha_B = \frac{\dot{\phi} X G_{3,X}}{H G_4}$

To trust the EFT: $|\alpha_B| \lesssim 10^{-2}$. Interestingly close to constraints from the large-scale structure

Summary and conclusion

Gravitational waves probe modified gravity as light probes material

In many cases very effectively, more than what large-scale structure can do

$$\begin{aligned}\mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \\ & - 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] \\ & - F_4(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'} \\ & - F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}\end{aligned}$$

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- Speed of GW: $|c_T - 1| \lesssim 10^{-15}$
- Resonant graviton decay $10^{-10} \lesssim |\alpha_H| \lesssim 10^{-20}$
- Perturbative decay and dispersion $|\alpha_H| \lesssim 10^{-10}$

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- Resonant graviton decay $10^{-10} \lesssim |\alpha_H| \lesssim 10^{-20}$
- Perturbative decay and dispersion $|\alpha_H| \lesssim 10^{-10}$
- Instabilities due to GW $|\alpha_B| \lesssim 10^{-2}$

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