Rotating black hole in higher order theories

IJCLab UPS, CNRS

Rencontre des groupes de travail "Formes d'onde" et "Tests de la relativité générale et théories alternatives

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'$c_T = 1$' theories and their relation to Horndeski [part of EST: Crisostomi, Koyama or DHOST : Langloois, Noui]

Shift-symmetric scalar tensor theory $c_T = 1$ minimally coupled to matter : parametrized by K, A_3, G

$$\mathcal{L} = K(X) + G(X)R + A_3(X)\phi^\mu\phi_{\mu\nu}\phi^\nu\Box\phi + A_4(X)\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu + A_5(X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2\,,$$

- coupling functions depend only on $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.
- $K(X) = -\Lambda_{bare} + X + ..$ and the operators A_4, A_5 are fixed with respect to A_3, G
- $c_T = 1$ theories are mapped to Horndeski via,

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X)g_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$$

  for given functions C and D.
- One can start with a $c_T \neq 1$ Horndeski theory (solution) and map it to a $c_T = 1$ theory (solution) for a specific function D.
- Example

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Stealth solution of spherical symmetry [Babichev, CC, GEFarèse, Lehébel]

- Example Horndeski, $G_4 = \zeta + \beta X$

$$S = \int d^4x \sqrt{-g}\,[\zeta R - 2\Lambda_b - \eta X + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]\,,$$

- General spherically symmetric solution is known[Babichev, cc], $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, $\phi = \phi(t,r)$
- One such solution reads $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta}r^2$, $\phi = qt \pm \int dr\ \frac{q}{h}\sqrt{1-h}$ with $\Lambda_{\text{eff}} = -\zeta\eta/\beta$ and $q^2 = \frac{\zeta\eta + \Lambda_b\beta}{\beta\eta}$.
- Go to ($c_T = 1$ theory) via a disformal transformation:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}X}\phi_\mu\phi_\nu\,.$$

- The disformed metric is still a black hole, $\tilde{X}$ constant
- Solution stable in a Λ_b-dependent window.

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- $X = g^{\mu\nu}\phi_\mu\phi_\nu = -\frac{q^2}{h} + q^2\frac{f(1-h)}{h^2} = -q^2$ is constant ([Kobayashi and Tanahashi])
- Change of coordinates shows that ϕ is regular either at the EH or the CH
- Go to ($c_T = 1$ theory) via a disformal transformation:

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Going beyond spherical symmetry

- How can we implement rotation?
- We need to know how a rotating black hole behaves in general- in the ringdown phase- and further on for black hole binaries.
- For numerics we need to have candidate, approximate solutions in order to relax them to full fledged numerical solutions
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- Now we start with a $c_T = 1$ theory.

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Going beyond spherical symmetry-the role of geodesics

[cc, Crisostomi, Gregory, Stergioulas]

Consider an Einstein metric, $R_{\mu\nu} = \Lambda g_{\mu\nu}$ and $X = X_0$ constant.
When are such metric and scalar solutions to the field equations of a $c_T = 1$ theory?

When :

- $A_3(X_0) = 0$ (rotation)
- $\Lambda = -K/(2G)|_{X_0}$, $(K_X + 4\Lambda G_X)|_{X_0} = 0$ (self-tuning conditions)

- Any theory with A_3 having a zero at some value $X = X_0$ is enough to guarantee a solution.
- What does $X = \nabla_\mu\phi\nabla^\mu\phi$ constant really mean?
- Take $Y_a = \partial_a\phi$ then the derivative of $X = Y_a Y_b g^{ab} = X_0$ is simply $a^b = Y^a\nabla^b Y_a = 0$
- Acceleration is zero hence ϕ is related to a geodesic congruence in the given spacetime.
- the scalar field ϕ is the Hamilton-Jacobi generating function S where $\frac{\partial S}{\partial \lambda} = g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu}$

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The example of Carter's solution (de Sitter-Kerr)

- Rotating black hole Einstein metric

$$ds^2 = -\frac{\Delta_r}{\Xi^2\rho^2}\left[dt - a\sin^2\theta d\varphi\right]^2 + \rho^2\left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\right)$$
$$+ \frac{\Delta_\theta \sin^2\theta}{\Xi^2\rho^2}\left[a\,dt - \left(r^2 + a^2\right)d\varphi\right]^2 ,$$
$$\Delta_r = \left(1 - \frac{r^2}{\ell^2}\right)\left(r^2 + a^2\right) - 2Mr\,, \quad \Xi = 1 + \frac{a^2}{\ell^2}\,,$$
$$\Delta_\theta = 1 + \frac{a^2}{\ell^2}\cos^2\theta\,, \qquad \rho^2 = r^2 + a^2\cos^2\theta\,,$$

- Black hole parameters are $a, M, \Lambda = 3/l^2$ which describe a black hole with an inner, outer event and cosmological horizon for $\Lambda > 0$.
- To evaluate the HJ generating function we need to know the inverse metric and solve a first order differential equation.
- Is there such a scalar which is well defined in the black hole spacetime?

The example of Carter's solution (de Sitter-Kerr)

- The Hamilton Jacobi potential reads [Carter],

$$\mathcal{S} = -E\,t + L_z\varphi + S(r,\theta)\,,$$

since ∂_t and ∂_ϕ are Killing vectors and is separable $S(r,\theta) = S_r(r) + S_\theta(\theta)$!

$$S_r = \pm \int \frac{\sqrt{R}}{\Delta_r}dr\,, \qquad S_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta}d\theta\,,$$

$$\begin{aligned} R &= \Xi^2\left[E\left(r^2+a^2\right) - a\,L_z\right]^2 \\ &- \Delta_r\left[\mathcal{Q} + \Xi^2\left(a\,E - L_z\right)^2 + m^2r^2\right]\,, \qquad (1) \\ \Theta &= -\Xi^2\sin^2\theta\left(a\,E - \frac{L_z}{\sin^2\theta}\right)^2 \\ &+ \Delta_\theta\left[\mathcal{Q} + \Xi^2\left(a\,E - L_z\right)^2 - m^2a^2\cos^2\theta\right]\,. \qquad (2) \end{aligned}$$

- Note we have $E, m, L_z, \mathcal{Q}$ parametrising the Energy at infinity, rest mass, angular momentum and Carter's separation constant.
- We want to identify $\phi = \mathcal{S}$
- ϕ (unlike $\mathcal{S}$) needs to be well defined in all the permitted domain of the coordinates. For a start we need that Θ and R are positive functions.
- Regularity : $L_z = 0$ and $\mathcal{Q} + \Xi^2a^2E^2 = m^2a^2$,

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Rotating black hole $\Lambda = 0$

- We have,

$$\phi(t,r,\theta) = -E\,t + \phi_r + \phi_\theta\,,$$

where,

$$\phi_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr\,, \qquad \phi_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta} d\theta\,,$$

$$\Theta = a^2 m^2 \sin^2\theta \left(\Delta_\theta - \eta^2\right), R = m^2(r^2+a^2)\left(\eta^2(r^2+a^2) - \Delta_r\right)$$

where we define $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$

- Take $\Lambda = 0$, ie Kerr, we have $\eta = 1$
- The scalar ϕ then has no θ dependance. Coincides with known solution if $a = 0$ ($E = m = q$).
- Solution is regular at the event horizon for one of the branches by going to advanced EF coordinates.
- $v = t + \int dr\, \frac{r^2+a^2}{\Delta_r}, \qquad \bar{\varphi} = \varphi + a \int \frac{dr}{\Delta_r}$
- Solution then is Kerr with a non trivial scalar field

Rotating black hole $\Lambda = 0$

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Rotating black hole $\Lambda > 0$

- Scalar reads,

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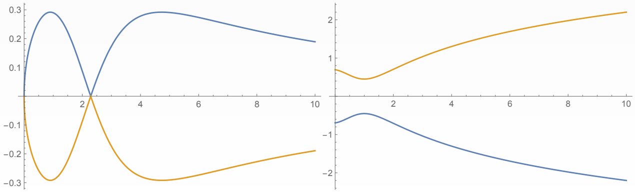
  where $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$.
- η_c is the limiting value of $R > 0$. ie., it is such that R has a double zero at $r_{EH} < r_0 < r_{CH}$
- we have $\eta_c < 1$ and as Λ increases η_c decreases
- We have two branches of solutions. Going to EF coords we see that one chart is regular at the EH while the latter at the CH but none at both.
- $v = t \pm \int dr\, \frac{r^2+a^2}{\Delta_r}, \qquad \bar{\varphi} = \varphi \pm a \int \frac{dr}{\Delta_r}$



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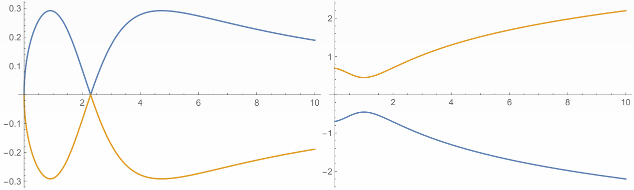
$$\phi(t,r,\theta) = -E\,t + \phi_r + \phi_\theta\,,$$
$$\phi_r = \pm\int \frac{\sqrt{R}}{\Delta_r}dr\,, \qquad \phi_\theta = \pm\int \frac{\sqrt{\Theta}}{\Delta_\theta}d\theta\,,$$
$$\Theta = a^2m^2\sin^2\theta\left(\Delta_\theta - \eta^2\right)\,, R = m^2(r^2+a^2)\left(\eta^2(r^2+a^2) - \Delta_r\right)$$
where $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$.
- η_c is the limiting value of $R > 0$. ie., it is such that R has a double zero at $r_{EH} < r_0 < r_{CH}$
- we have $\eta_c < 1$ and as Λ increases η_c decreases
- We have two branches of solutions. Going to EF coords we see that one chart is regular at the EH while the latter at the CH but none at both.
- $v = t \pm \int dr\, \frac{r^2+a^2}{\Delta_r}, \qquad \bar{\varphi} = \varphi \pm a\int \frac{dr}{\Delta_r}$



Regular Rotating black hole with $\Lambda \neq 0$

- Scalar reads,

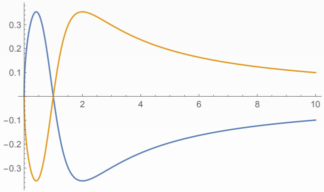
$$\phi(t,r,\theta) = -E\,t + \phi_r + \phi_\theta\,,$$

$$\phi_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr\,, \qquad \phi_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta} d\theta\,,$$

$$\Theta = a^2 m^2 \sin^2\theta \left(\Delta_\theta - \eta^2\right)\,, R = m^2(r^2+a^2)\left(\eta^2(r^2+a^2) - \Delta_r\right)$$

  where $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$.

- Fixing $\eta = \eta_c$ the two branches join with C_2 regularity at $r = r_0$.
- Then using both branches ie., $\phi_r = H[r-r_0]\int_{r_0}^{r} \frac{\sqrt{R}}{\Delta_r} - H[r_0 - r]\int_{r_0}^{r} \frac{\sqrt{R}}{\Delta_r}$, we have a regular scalar field everywhere



Conclusions

- We have obtained a rotating black hole with hair which is everywhere regular.
- Unlike GR for $\Lambda < 0$ there is no regular rotating solution!
- Rotating black hole construction is akin to $c_T = 1$ theories
- Spin 2 perturbations yield separable Teukolsky equation with source [CC, Crisostomi, Langlois, Noui] but scalar kinetic matrix is degenerate ([Babichev, et al], [De Rham, Zhang])
- Can obtain any GR vacuum solution with well defined hair in such theories
- One can use this stealth solution to construct numerically other non Kerr solutions by relaxation techniques