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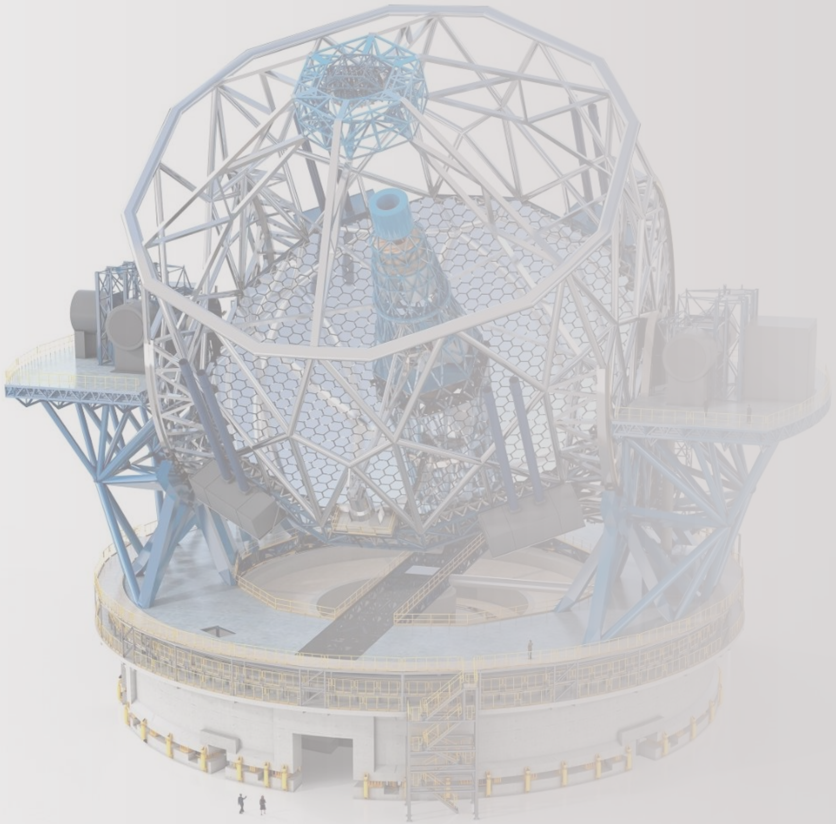

Vibration Mitigation in Adaptive Optics of Large Telescopes using Model Predictive Control

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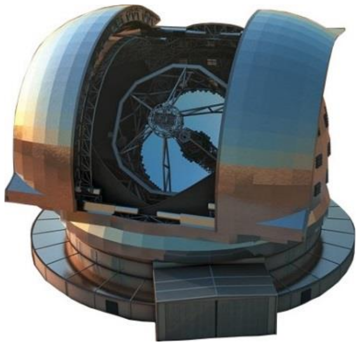
isys

Challenges of vibration mitigation in adaptive optics of extremely large telescopes

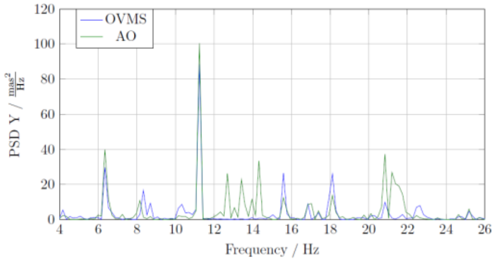
Limited telescope resolution by
- Atmospheric turbulences
- Structural vibrations
  - Dominant in tip-tilt modes (also defocus, …)
  - In interferometry OPD



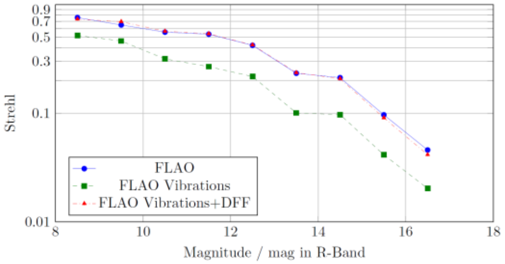
Images: ESO

Optical performance limited by dynamics of active components
- Tip-tilt mirror, large amplitudes, slow dynamics
- Deformable mirror, small amplitudes, high frequencies

Reduced bandwidth with faint NGS
- Slower sample rates for better SNR
- Poor performance for High frequency vibrations

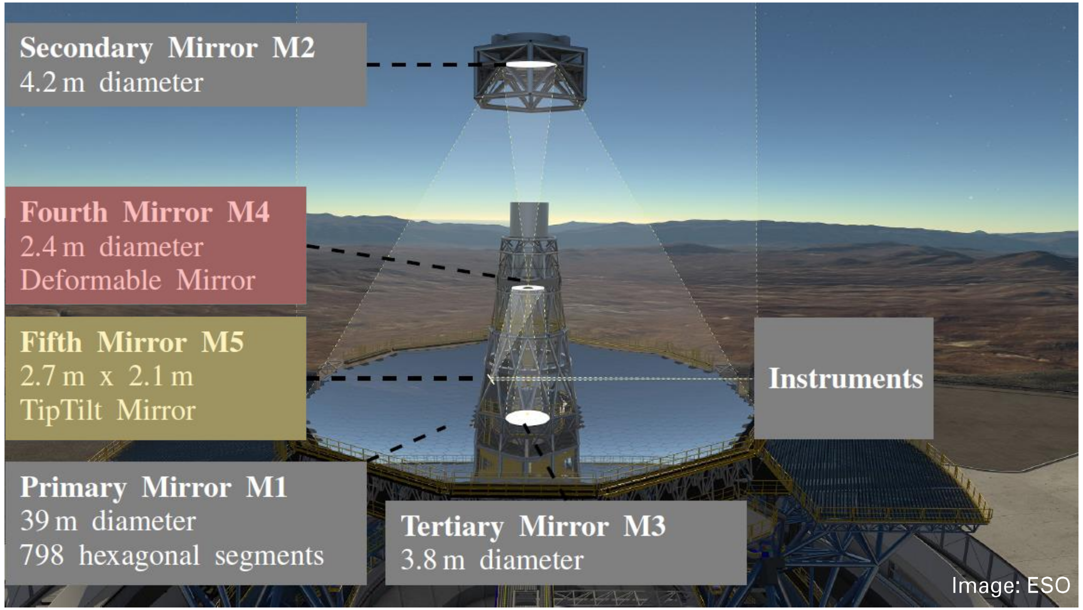


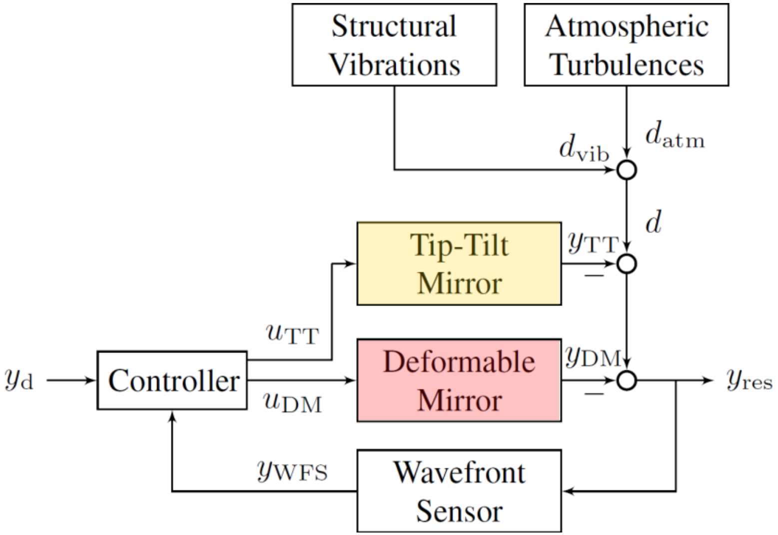
Vibrations at the LBT



Performance loss with an Integrator by the influence of vibrations

Achieving diffraction limited performance in the tip-tilt modes of the ELT



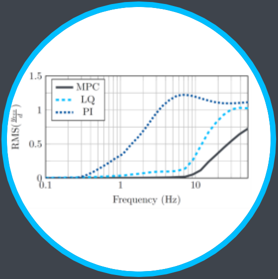


Goal: Designing a controller for the Tip-Tilt MISO system to achieve diffraction limited performance

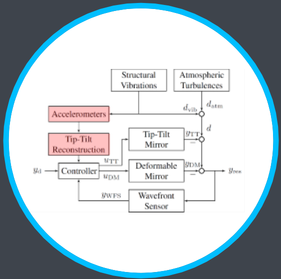
- ➢ Considering scenarios with strong atmospheric turbulences and structural vibrations
- ➢ Stroke limitations of the actuators (amplitude, slew rate)
- ➢ compensation mirror dynamics

➡ **Model Predictive Control**

Agenda

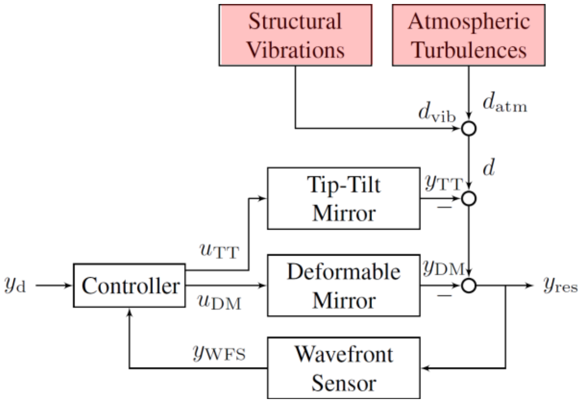


Model Predictive Control for Tip-Tilt Vibration Mitigation



Combining MPC with a Disturbance Feedforward Control for faint NGS

Modelling the disturbances of an adaptive optics system



Atmospheric turbulences

- Statistical spatial description by Kolmogorov
- Describing the temporal behavior by Taylor's "frozen flow" hypothesis

$$\dot{\phi}_{\mathrm{atm}}(x,y,t) = -\boldsymbol{v}\nabla\phi_{\mathrm{atm}}(x,y,t), \quad \phi_{\mathrm{atm}}(x,y,0) = g(x,y)$$

- Approximation of the temporal autocorrelation function by an AR2 model

Structural vibrations

- Each mirror of the optical path introduces vibrations due to the mounting
- Detection of cumulative vibrations by the wavefront sensors
- Approximation of the temporal autocorrelation function by an AR2 model
- Modelling Tip-Tilt telescope vibrations by an equivalent mechanical modal model

Discrete state space representation of a single natural frequency:

$$\underbrace{\begin{bmatrix} d_{\mathrm{vib},i}[k+1] \\ d_{\mathrm{vib},i}[k] \end{bmatrix}}_{x_{\mathrm{vib},i}[k+1]} = \begin{bmatrix} 2e^{-\omega_{1,i}\delta_i T_{\mathrm{s}}}\cos\left(\omega_{1,i}T_{\mathrm{s}}\sqrt{1-\delta_i^2}\right) & -e^{-2\omega_{1,i}\delta_{1,i}T_{\mathrm{s}}} \\ 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} d_{\mathrm{vib},i}[k] \\ d_{\mathrm{vib},i}[k-1] \end{bmatrix}}_{x_{\mathrm{vib},i}[k]} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_{\mathrm{vib}}[k+1]$$

$$d_{\mathrm{vib}}[k] = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_{\mathrm{vib}}} x_{\mathrm{vib}}[k],$$

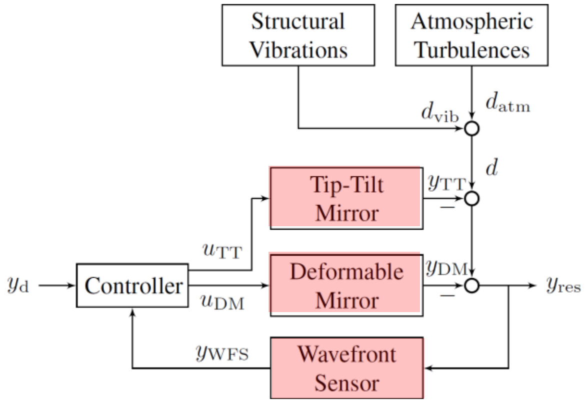
Models for sensing an compensating the tip-tilt residual wavefront error

WFS as a time delay system

- Receiving reconstructed WFS in Zernike modes
- Typically 2 samples time delay (exposure, reconstruction)

$$x_{\mathrm{T},i}[k+1] = \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}}_{A_{\mathrm{T},i}} \underbrace{\begin{bmatrix} y_{\mathrm{res},i}[k-1] \\ y_{\mathrm{res},i}[k-2] \end{bmatrix}}_{x_{\mathrm{T},i}[k]} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_{\mathrm{T},i}} y_{\mathrm{res},i}[k],$$

$$y_{\mathrm{WFS},i}[k] = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C_{\mathrm{T},i}} x_{\mathrm{T},i}[k]$$



Compensation mirror dynamics

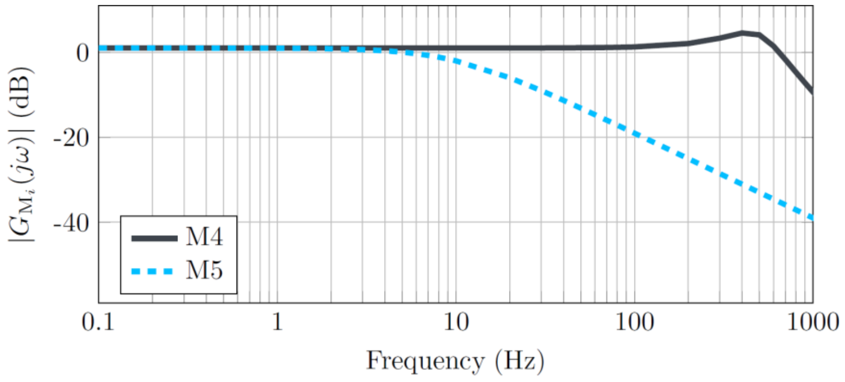
- M4 small amplitudes, large frequency range
- M5 large amplitudes, but small bandwidth

Mirror dynamics in tip-tilt modes

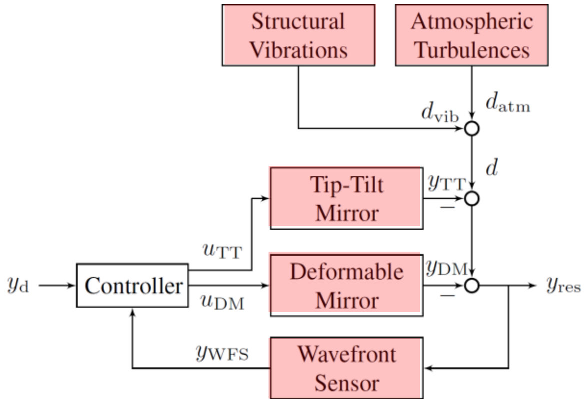
$$\begin{bmatrix} \dot{x}_{\mathrm{DM},i} \\ \dot{x}_{\mathrm{TT},i} \end{bmatrix} = \underbrace{\begin{bmatrix} A_{\mathrm{DM},i} & 0 \\ 0 & A_{\mathrm{TT},i} \end{bmatrix}}_{A_{\mathrm{CM},i}} \begin{bmatrix} x_{\mathrm{DM},i} \\ x_{\mathrm{TT},i} \end{bmatrix} + \underbrace{\begin{bmatrix} B_{\mathrm{DM},i} \\ B_{\mathrm{TT},i} \end{bmatrix}}_{B_{\mathrm{CM},i}} \begin{bmatrix} u_{\mathrm{DM},i} \\ u_{\mathrm{TT},i} \end{bmatrix}$$

$$y_{\mathrm{CM},i} = \underbrace{\begin{bmatrix} C_{\mathrm{DM},i} & C_{\mathrm{TT},i} \end{bmatrix}}_{C_{\mathrm{CM},i}} \begin{bmatrix} x_{\mathrm{DM},i} \\ x_{\mathrm{TT},i} \end{bmatrix}$$

Tip-tilt transfer functions



Creating an open-loop description of the AO system for the controller design



Measurement equation:

$$\underbrace{y_{\mathrm{WFS}}[k],i}_{y_i[k]} = \underbrace{\begin{bmatrix} C_{\mathrm{T},i} & 0 & 0 \end{bmatrix}}_{C_i} \underbrace{\begin{bmatrix} x_{\mathrm{T},i}[k] \\ x_{\mathrm{d},i}[k] \\ x_{\mathrm{CM},i}[k] \end{bmatrix}}_{x[k],i}$$

Corresponding dynamic model:

$$x_i[k+1] = A_i x_i[k] + B_i u_i[k] + V_i v_i[k],$$

$$A_i = \begin{bmatrix} A_{\mathrm{T},i} & B_{\mathrm{T},i} C_{\mathrm{d},i} & -B_{\mathrm{T},i} C_{\mathrm{CM},i} \\ 0 & A_{\mathrm{d},i} & 0 \\ 0 & 0 & A_{\mathrm{CM},i} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ B_{\mathrm{CM},i} \end{bmatrix}, V_i = \begin{bmatrix} 0 \\ V_{\mathrm{d},i} \\ 0 \end{bmatrix}$$

Design of a model predictive controller for an AO system

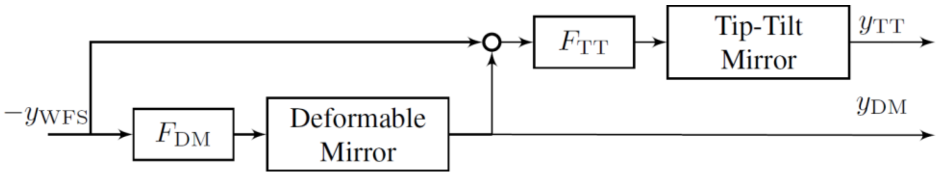
$$\begin{aligned}\min_{u} \quad & J_i = \sum_{j=0}^{N} \|y_i[k+j|k]\|_{R_\mathrm{y}} + \|u_i[k+j|k]\|_{R_\mathrm{u}} \\ \text{s.t.} \quad & x_i[k+j+1|k] = Ax_i[k+j|k] + Bu_i[k+j|k] \\ & y_i[k+j|k] = C_i x_i[k+j|k] \\ & |u_i[k+j|k]| \leq u_\mathrm{max}\end{aligned}$$

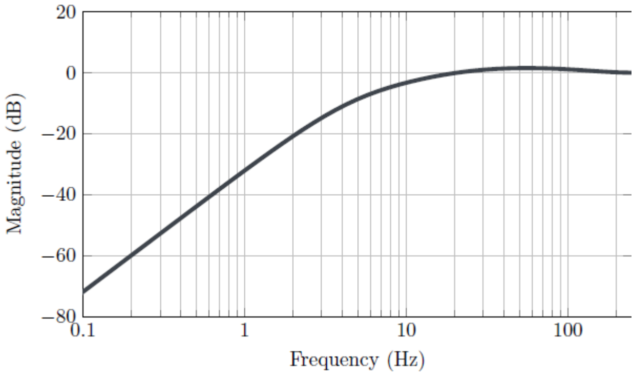
- State of the dynamic system is typically unknown, estimation of $x_i[k|k]$ by a Kalman filter
- Reformulation of the cost function as a quadratic program (QP) for the horizon N $\frac{1}{2}u^\mathrm{T}Hu + u^\mathrm{T}g$
- Solving the QP for **each time step** with e.g. qpOASES (2 ms)
- Choosing an applicable prediction horizon N (real time capability)

Comparison with the LQG control and the PI dual stage approach

PI control

- ➢ Current proposal for the controller of the ELT
- ➢ Dual-stage approach with PI controller





LQG Control

Cost function:

$$\min_{u} \quad J_i = \sum_{k=0}^{N_{\mathrm{LQ}}-1} x_i[k]^{\mathrm{T}} Q x_i[k] + u_i[k]^{\mathrm{T}} R u_i[k], \qquad Q = C^{\mathrm{T}} C, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{s.t.} \quad x_i[k+1] = A x_i[k] + B u_i[k]$$

Solving the optimal control problem:

$$u[k] = -K x[k]$$

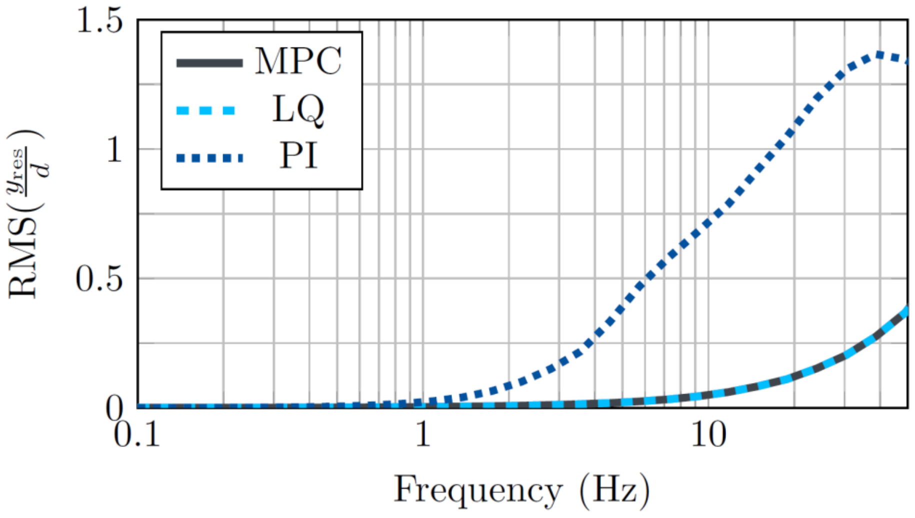
$$K[k] = \left(B^{\mathrm{T}} P[k] B + R\right)^{-1} B^{\mathrm{T}} P[k] A$$

$$P[k-1] = A^{\mathrm{T}} P[k] A + Q - K[k]^{\mathrm{T}} B^{\mathrm{T}} P[k] A$$

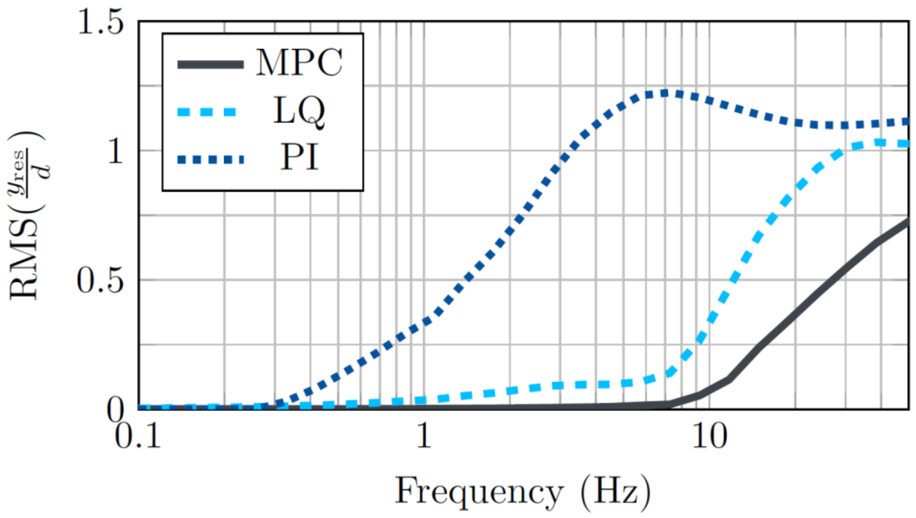
Influences on the residual tip-tilt for periodic disturbances and stroke limitations at the DM

- ➢ Sinusoidal disturbance with normalized amplitude of 1
- ➢ 0.1 amplitude Tip-Tilt input constraints at the deformable mirror

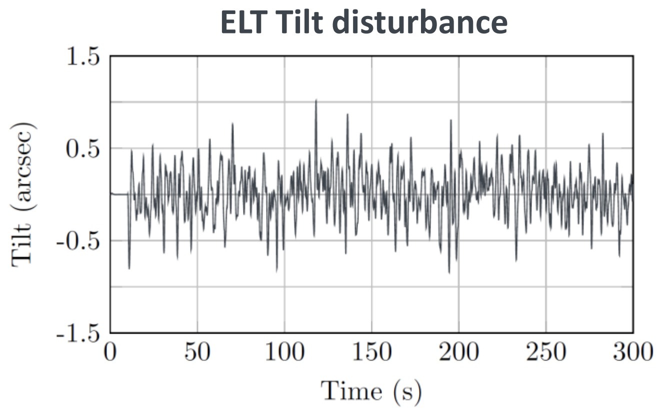
Without Input Constraints

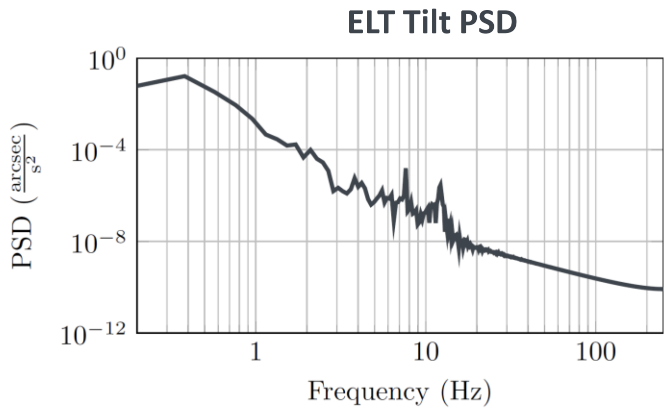


Input Constraints at the DM with 10% of the input disturbance

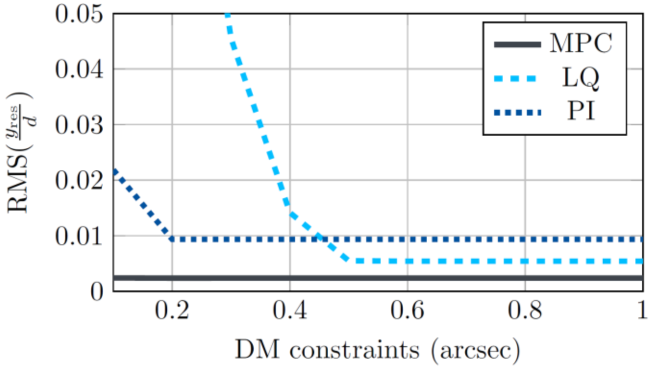


Evaluating the controller for a ELT tilt random signal



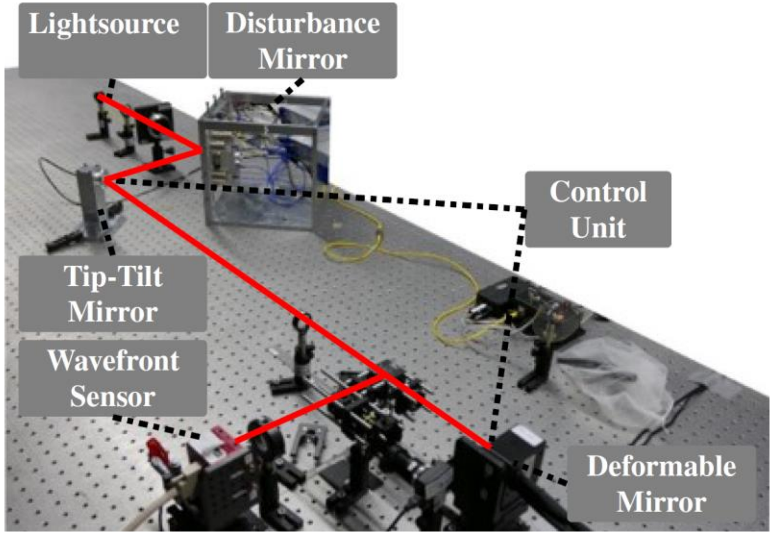


- LQ losses performance with input constraints and MPC yields best results



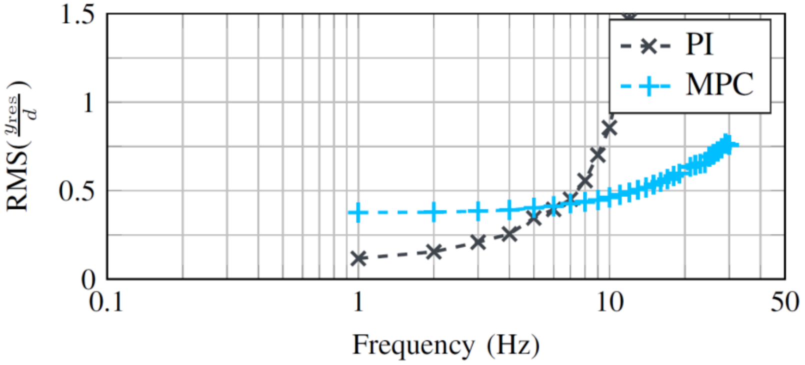
Investigations on the real-time capability of the MPC controller

- ➢ Scaled laboratory setup
- ➢ Injection of tip-tilt disturbances by a disturbance mirror
- ➢ Compensation with a tip-tilt and DM (ALPAO 52) mirror
  - ➢ ELT mirror dynamics considered by simulation
- ➢ High-Order modes compensated by a classical integral control
- ➢ AO control on a real-time computer (Sample Rate 2 ms)
- ➢ QP solved by qpOASES within 2ms



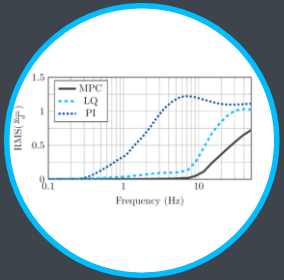
Future Work

- ➢ Improving the compensation performance
- ➢ Testing different QP solvers

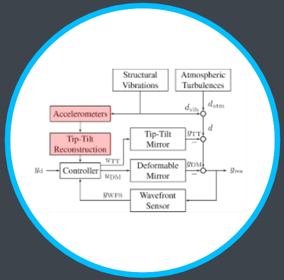


Alternative Approach for the ELT Tip-Tilt control

Agenda



Model Predictive Control for Tip-Tilt Vibration Mitigation



Combining MPC with a Disturbance Feedforward Control for faint NGS

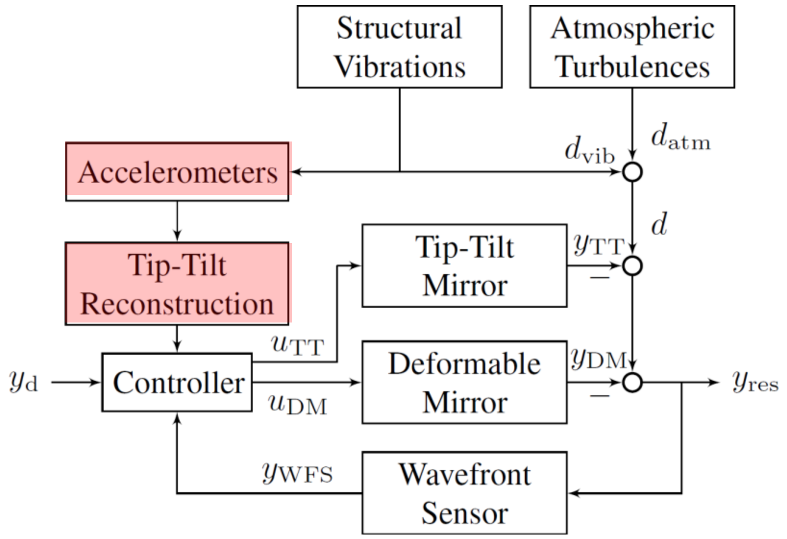
Disturbance Feedforward Control for the observations with faint NGSs

Disturbance Feedforward (DFF) Control

- Measuring vibrations with additional accelerometers
- Reconstruction of the optical aberrations (Tip,Tilt)
- Disturbance Feedforward at the compensation mirrors

➡ Independent of WFS exposure time

➡ Suppression of high frequency vibrations



Combining DFF control with a MPC approach

- Improving the vibration state estimation
- Optimal control for the compensation mirrors

➡ Increased Strehl for faint natural guide stars

Sensor fusion of WFS and accelerometers by a multi-rate observer

Combining the accelerometer and WFS vibration model

- Measuring of vibrations at each telescope mirror with accelerometers
- WFS measures cumulative vibrations

→

- Equivalent modal model for each telescope mirror
- Reconstructing tip-tilt modes of each mirror
- Calculating cumulative tip-tilt in the focal plane with a geometric model of the telescope

$$\underbrace{\begin{bmatrix} y_{\mathrm{WFS},i}[k] \\ y_{\mathrm{ACC},i}[k] \end{bmatrix}}_{y_i[k]} = \underbrace{\begin{bmatrix} C_{\mathrm{T},i} & 0 & 0 \\ 0 & C_{\mathrm{ACC},i} & 0 \end{bmatrix}}_{C_i} \underbrace{\begin{bmatrix} x_{\mathrm{T},i}[k] \\ x_{\mathrm{d},i}[k] \\ x_{\mathrm{CM},i}[k] \end{bmatrix}}_{x_i[k]}$$

Handling of different sample rates of WFS and accelerometers

- Accelerometer sample rate multiple of WFS rate
- Estimating the current system state by a Kalman Filter

$$\hat{x}[k|k-1] = A_i \hat{x}[k-1|k-1] + B_i u[k-1]$$
$$\hat{x}[k|k] = \hat{x}[k|k-1] + L_i[k]\left(y_i[k] - \hat{y}[k]\right)$$

Calculating the Kalman gain

→

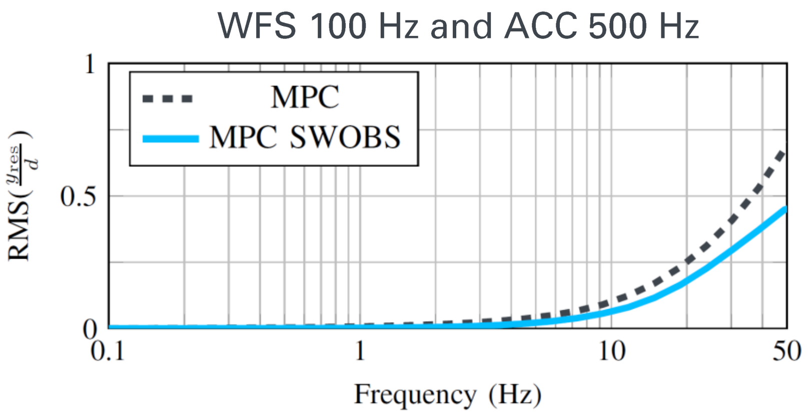
Adapting C to the incoming sensor signals

$$L_i[k] = P[k|k-1] C_i^{\mathrm{T}} S_i[k]^{-1}$$
$$P[k|k-1] = A_i P[k-1|k-1] A_i^{\mathrm{T}} + Q$$
$$S_i[k] = C_i P[k|k-1] C_i^{\mathrm{T}} + R$$
$$P[k|k] = P[k|k-1] - L_i[k] S_i[k] L_i[k]^{\mathrm{T}}$$

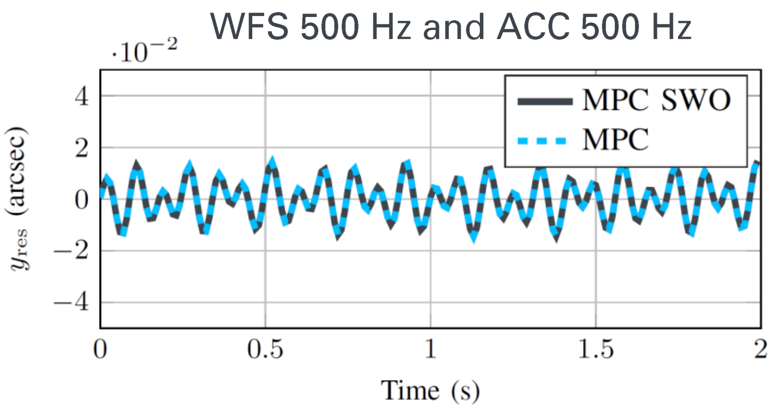
$$C_i = \begin{cases} \begin{bmatrix} C_{\mathrm{T},i} & 0 & 0 \\ 0 & C_{\mathrm{ACC},i} & 0 \end{bmatrix} & kT_{\mathrm{acc}} \bmod T_{\mathrm{WFS}} = 0 \\ \begin{bmatrix} 0 & C_{\mathrm{ACC},i} & 0 \end{bmatrix} & \text{else} \end{cases}$$

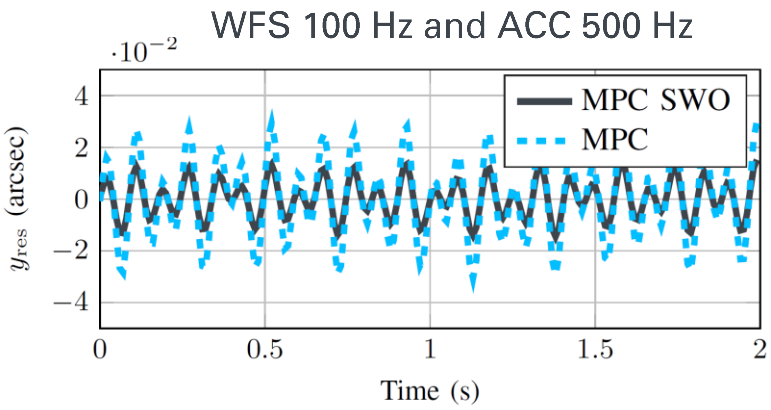
Results of a vibration mitigation based on a multi-rate Observer

Sinusoidal Excitation:



Random signal based on a ELT tip-tilt PSD:

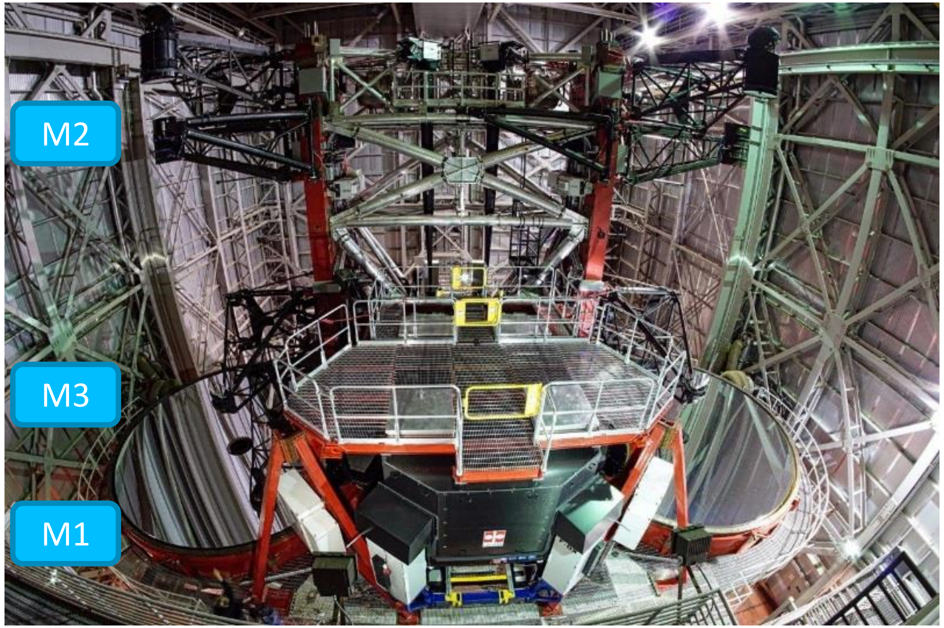




Investigations on the Disturbance Feedforward Control at the LBT

Current Status of Implementation

- Measuring Vibrations at each telescope mirror (already in use for OPD compensation)
- Reconstruction of the tip-tilt signal in the focal plane
- Transformation into DM Space
- Model-based latency compensation
- Sending Signals over telescope network to DM
- Combining WFS and accelerometer signals



➡ We're looking forward to test the implementation at the telescope!

Conclusion

- Investigations on the performance of the ELTs adaptive optics system
  - Designing a MPC controller for considering input constraints
  - Comparison with a LQ and PI controller
- Best results with a MPC by considering actuator constraints
- Improving the performance for faint NGS by using additional accelerometers within a multi-rate observer

Outlook

- Investigations on the speed of optimization algorithms
- Studying the stability and robustness of the MPC controller
- Implementing of a Disturbance Feedforward Control at the LBT

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Thank you!

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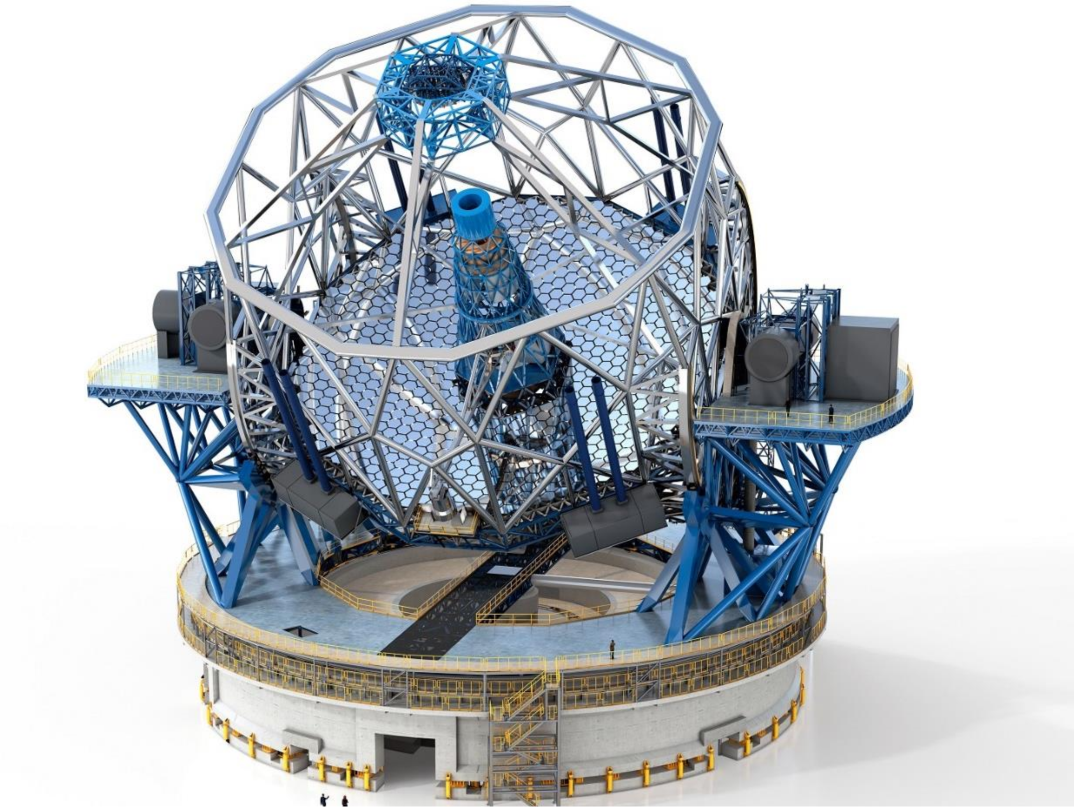

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