(Semi-)analytical and non-linear approaches towards optical gain compensation for pyramid sensors

Victoria Hutterer & Iuliia Shatokhina

joint work with Ronny Ramlau and Andreas Obereder

Industrial Mathematics Institute, Johannes Kepler University (JKU), Linz, Austria.

Johann Radon Institute for Computational and Applied Mathematics (RICAM), Linz, Austria.

Paris, October 23, 2018

Pyramid topics

- The non-linearity issue of the pyramid sensor
- Methods for optical gain compensation and NCPA handling
- Non-linear wavefront reconstruction

Problem description

Pyramid sensor measuring process:

$$\mathbf{P}\Phi = s$$

(Φ ... incoming wavefront, s ... pyramid sensor measurements)

interaction-matrix-based:

$\tilde{\mathbf{P}}$ ... $2n_{sub} \times n_{act}$ - matrix

model-based:

$\mathbf{P}$ ... non-linear operator

Ideas for optical gain compensation/NCPA correction

usage of linear reconstructor and a frequency dependent gain

usage of non-linear reconstructor

linearize around a non-zero setpoint
(e.g., for correction in presence of large NCPAs)

Ideas for optical gain compensation

usage of linear reconstructor and a frequency dependent gain

- registration of measurements
- analyze the sensor response to fed frequencies in numerical simulations

[1] V. Korkiakoski et al., Improving the performance of a pyramid wavefront sensor with modal sensitivity compensation, Appl. Opt. 47, (2008).

[2] V. Korkiakoski et al., Applying sensitivity compensation for pyramid wavefront sensor in different conditions, Proc. SPIE 7015, (2008).

[3] V. Viotto et al., PWFSs on GMCAO: a different approach to the non-linearity issue, Proc. SPIE 9909, (2016).

[4] V. Deo et al., A modal approach to optical gain compensation for the pyramid wavefront sensor, Proc. SPIE 1070320, (2018).

[5] **see talk by V. Deo**

Which non-linear reconstructors exist?

- phase retrieval algorithm:
  - R. Clare et al., Phase retrieval from subdivision of the focal plane with a lenslet array, Appl. Opt. 43, (2004).
- Jacobian reconstruction method:
  - V. Korkiakoski et al., Comparison between a model-based and a conventional pyramid sensor reconstructor, Appl. Opt. 46, (2007).
- quasi-Newton method:
  - R. Frazin, Efficient, nonlinear phase estimation with the nonmodulated pyramid wavefront sensor, J. Opt. Soc. Am. A 35, (2018).
- learning approach: see talk by S. Haffert
- non-linear Landweber Iteration for Pyramid Sensors (LIPS)
- non-linear Kaczmarz-Landweber It. for Pyramid Sensors (KLIPS)

Nonlinear reconstruction - Landweber iteration

- non-linear technique for finding a minimizer of the quadratic functional

$$||\mathbf{P}\Phi - s||^2_{\mathcal{L}_2(\mathbb{R}^2)}$$

- using this iterative procedure and roof sensor approximation $\mathbf{R}$

$$\Phi_{k+1} = \Phi_k + \omega \mathbf{R}'(\Phi_k)^* (s - \mathbf{R}(\Phi_k)), \qquad k = 0, 1, 2, \ldots$$

V. H., R. Ramlau. *Non-linear wavefront reconstruction methods for pyramid sensors using Landweber and Landweber-Kaczmarz iteration*. Appl. Opt. 57(30), (2018).

Results for the non-modulated sensor at 2.2μm

	LE Strehl
zonal MVM	0.89
P-CuReD	0.87
non-linear KLIPS	0.85
linear KLIPS	0.84
modal MAP	0.62[1]

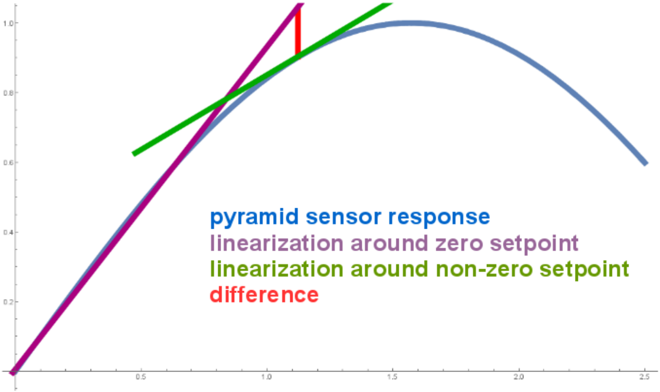
	METIS-like
telescope diameter	37m
central obstruction	30%
AO system	SCAO
sensor	74 $\times$ 74 PWFS
sensing band	K
evaluation band	K
modulation	0
atmosphere	median
Fried radius r_0@500nm	15.7 cm
photon flux	[50 – 10000] ph/px/frame
frame rate	1000 Hz
DM delay	1
mirror geometry	ELT M4
iterations	500
NCPAs	no

[1] M. Le Louarn et al., Latest AO simulation results for the E-ELT, poster AO4ELT5.

Characteristics of LIPS & KLIPS

- very sensitive to parameter choices
- experience (almost) no gain in using non-linear algorithms for modulated sensor
- Do non-linear algorithms outperform their corresponding linear estimators? – Yes, sometimes BUT ...
- First results for low flux cases are poor.
- Do the non-linear reconstructors give better results when the pyramid sensor is not in its linear regime?
  – No, it's confusing!

Linearization around a non-zero setpoint



Non-linear pyramid sensor model (transmission mask)

linearize around a non-zero setpoint
(e.g., for correction in presence of large NCPAs)

The **non-linear** pyramid sensor operator without modulation $\mathbf{P}_x : \mathcal{H}^{11/6}\left(\mathbb{R}^2\right) \rightarrow \mathcal{L}_2\left(\mathbb{R}^2\right)$ is given by

$$\begin{aligned}\mathbf{P}_x\Phi(x,y) \quad &:= \quad \mathcal{X}_\Omega(x,y)\frac{1}{\pi}\int\limits_{\Omega_y} \frac{\sin\left[\Phi(x',y)-\Phi(x,y)\right]}{x'-x}\, dx' \\ &+ \quad \frac{1}{\pi^3}\mathcal{X}_{\Omega_y}(x)\ p.v.\int\limits_{\Omega_y}\int\limits_{\Omega_x}\int\limits_{\Omega_x} \frac{\sin\left[\Phi(x',y')-\Phi(x,y'')\right]}{(x'-x)(y'-y)(y''-y)}\, dy''\ dy'\ dx'.\end{aligned}$$

Linearization around 0

The **linear** pyramid sensor operator without modulation
$\mathbf{P}_x^{lin} : \mathcal{H}^{11/6}\left(\mathbb{R}^2\right) \to \mathcal{L}_2\left(\mathbb{R}^2\right)$ **around 0** is given by

$$\begin{aligned}\mathbf{P}_x^{lin}\Phi(x,y) \quad &:= \quad \mathcal{X}_\Omega(x,y)\frac{1}{\pi}\int\limits_{\Omega_y}\frac{[\Phi(x',y)-\Phi(x,y)]}{x'-x}\,dx' \\ &+ \quad \frac{1}{\pi^3}\mathcal{X}_{\Omega_y}(x)\ p.v.\int\limits_{\Omega_y}\int\limits_{\Omega_x}\int\limits_{\Omega_x}\frac{[\Phi(x',y')-\Phi(x,y'')]}{(x'-x)(y'-y)(y''-y)}\,dy''\ dy'\ dx'.\end{aligned}$$

Linearization around non-zero setpoint

The **linear roof** sensor operator without modulation $\mathbf{R}_x^{lin} : \mathcal{H}^{11/6}\left(\mathbb{R}^2\right) \to \mathcal{L}_2\left(\mathbb{R}^2\right)$ **around the residual Φ_{res}** is given by

$$\mathbf{R}_x^{lin}\Phi(x,y) := \mathcal{X}_\Omega(x,y)\frac{1}{\pi}\int\limits_{\Omega_y} \frac{\boxed{\sin}\left[\Phi_{res}(x',y) - \Phi_{res}(x,y)\right]}{x'-x}\, dx'$$

$$-\frac{1}{\pi}\int\limits_{\Omega_y} \frac{\boxed{\cos}\left[\Phi_{res}(x',y) - \Phi_{res}(x,y)\right]}{x'-x}\left[\Phi_{res}\left(x',y\right) - \Phi_{res}\left(x,y\right)\right]\, dx'$$

$$+\frac{1}{\pi}\int\limits_{\Omega_y} \frac{\boxed{\cos}\left[\Phi_{res}(x',y) - \Phi_{res}(x,y)\right]}{x'-x}\left[\Phi\left(x',y\right) - \Phi\left(x,y\right)\right]\, dx'$$

Results: optical gain compensation

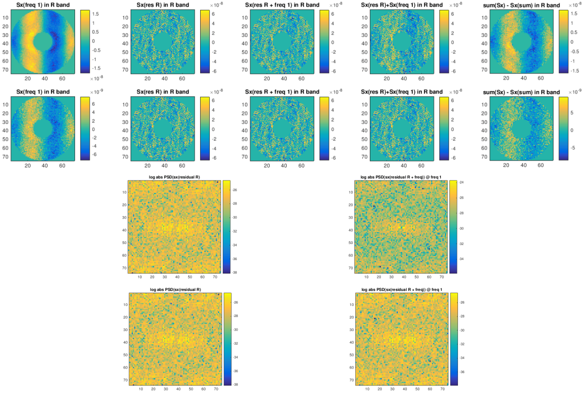
- results for $r_0 = 10$ cm, mod 4 λ/D, half-ELT setting, R-band sensing, no oversampling:

r0=10 cm, mod=4		H-band LE Strehl
environment	reconstructor	
octopus	Linear Landweber	0,408
	Non-linear Landweber	0,441
	modal MAP	0,456
compass	mvm (V. Deo)	0,470
octopus	cured	0,480
	P-cured	0,512
	OGC-P-cured	0,525
compass	OGC-mvm (V. Deo)	0,570

- to do: improve OG evaluation

Outlook: optical gain compensation

- analyze data in FD
- R-band, median atm, $rms_{res} \approx 100$nm, phase 100/30nm ampl.



- Conclusion: not all non-linearity can be corrected

Open question & outlook

- non-linear reconstructors
  - What is the influence of the **sensing wavelength, Fried parameter,...**?
  - Can we extend the regime in which the non-modulated PWFS usefully operates?
  - When do we **(want to) benefit** from a non-linear reconstructor?
  - What happens if **large NCPAs** are present?
    - Are linear reconstructors still usable?
    - Will the non-linear approaches outperform linear ones?
- develop OGC for linear model-based reconstructors (e.g., for P-CuReD)
- derive frequency dependent gain analytically

We need to get a deeper understanding of the non-linearity!

Thank you for your attention!

Reconstruction qualities of existing methods

Algorithm	**Quality in end-to-end simulations (OCTOPUS)** (LE Strehl ratios in the K-band)					
	SCAO	SCAO	SCAO	SCAO	XAO	XAO
Modulation (λ/D)	mod 0	mod 4	mod 0	mod 4	mod 0	mod 4
Photon flux (ph/pix/it)	10000	10000	10000	600	50	50
Frame rate (kHz)	1	0.5	1	0.5	3	3
Mirror geometry	M4	M4	M4	M4	Fried	Fried
Telescope spiders	✗	✗	✓	✓	✗	✗
Interaction matrix inversion: modal	≈ 0.62 [1]	0.888		0.859		0.96
Interaction matrix inversion: zonal		0.890	0.890	0.894		
Preprocessed CuReD (P-CuReD)	0.871	0.887	0.865	0.878	0.916	0.961
Conv. with Linearized Inverse Filter (CLIF)					0.88	0.94
Pyramid FTR (PFTR)					0.88	0.94
Finite Hilbert Transform Rec. (FHTR)	0.779	–		–	0.853	–
Singular Value Type Reconstructor (SVTR)	0.74	–		–	0.884	–
Conjugate Gradient for Normal Eq (CGNE).	0.842	0.860			0.901	
Steepest Descent (SD)	0.841	0.858				
Steepest Descent-Kaczmarz (SD-K)	0.841	0.858				
Linear Landweber iteration (LIPS)	0.840	0.860				
Linear Kaczmarz-Landweber iteration (KLIPS)	0.842	0.858			0.897	
Non-linear Landweber iteration (LIPS)	0.853	0.834				
Non-linear Kaczmarz-Landweber iteration (KLIPS)	0.853	0.826			0.903	

[1] M. Le Louarn et al., Latest AO simulation results for the E-ELT, poster AO4ELT5.