A few words about AO control performance evaluation

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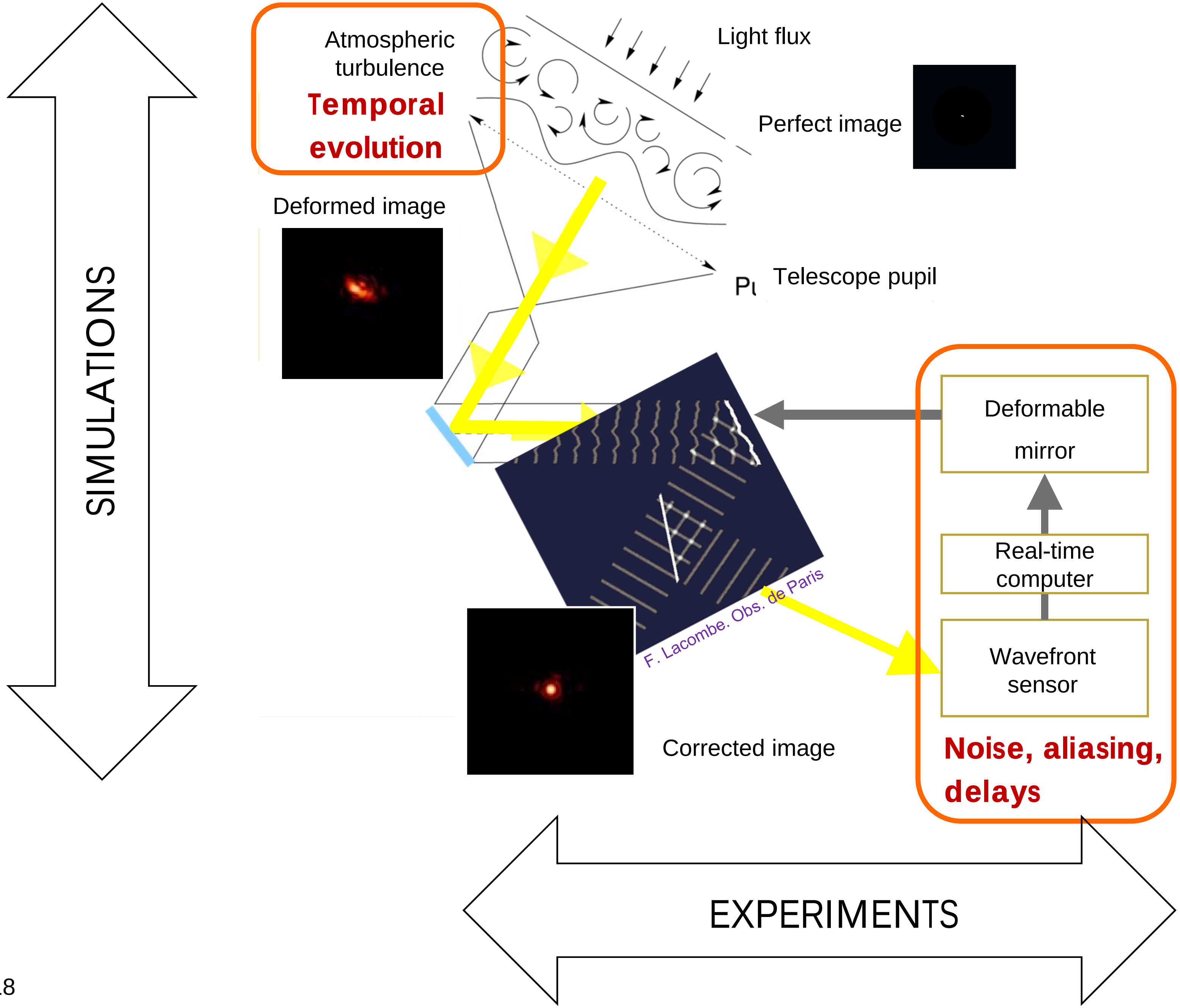
Adaptive optics principle

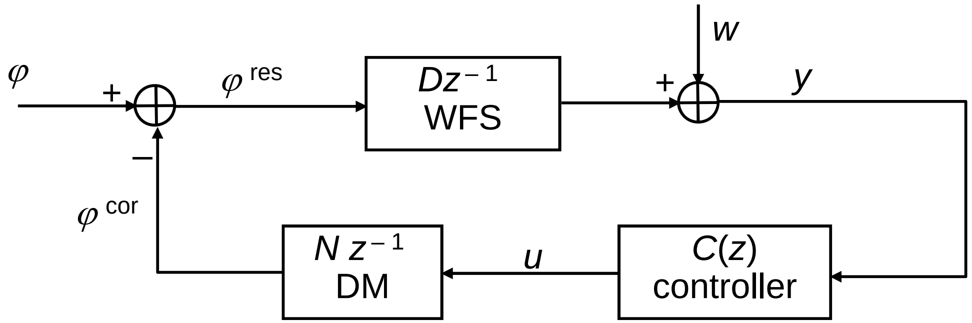


Outline

- Real system or replay from WFS data
  - equivalent block-diagram representations
- The scalar case: noise propagation and performance
- Performance evaluation in the multivariable case
- Why it fails, what we can do

Closed-loop AO control

- The control engineer point of view

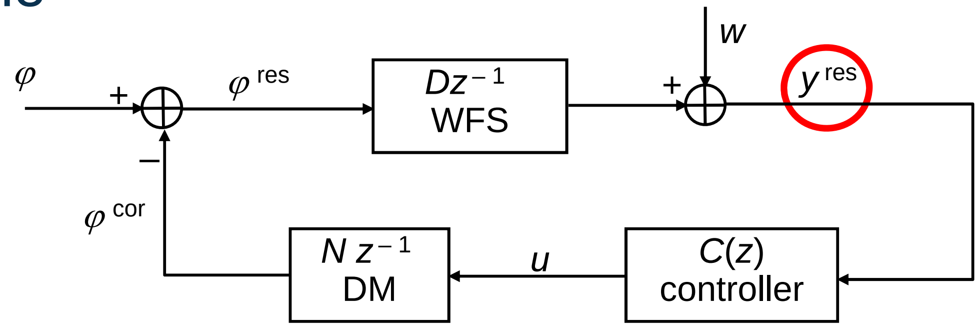


  - WFS: Wave Front Sensor (Shack-Hartmann, pyramid…)
  - DM: Deformable Mirror
  - Controller: *u* applied with a zero-order-hold

    ⇒ constant over one frame (WFS integration time)

AO control in replay

- On-sky scheme

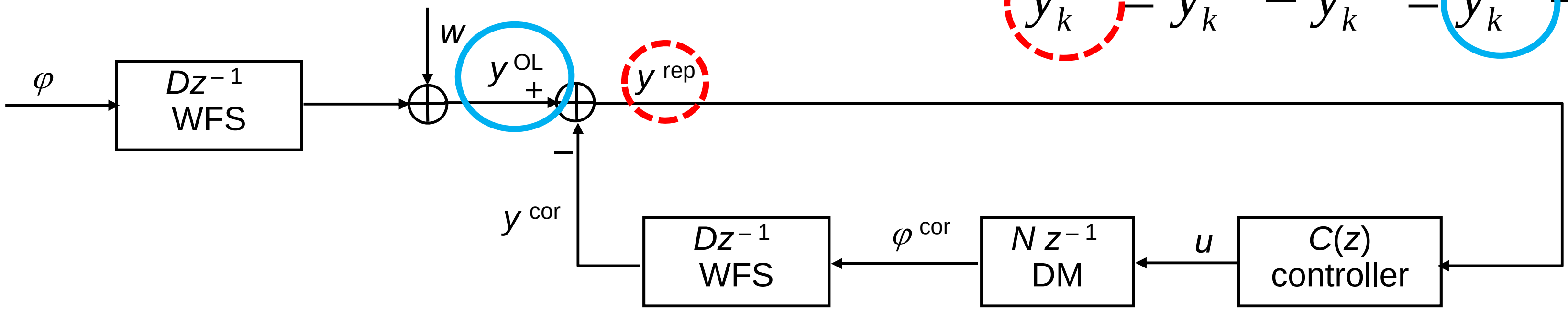


$$y_k^{\mathrm{res}} = D\varphi_{k-1}^{\mathrm{res}} + w_k = y_k^{\mathrm{OL}} - D\varphi_{k-1}^{\mathrm{cor}}$$

$$\|$$

$$y_k^{\mathrm{rep}} = y_k^{\mathrm{OL}} - y_k^{\mathrm{cor}} = y_k^{\mathrm{OL}} - D\varphi_{k-1}^{\mathrm{cor}}$$

- Evaluate control performance in replay



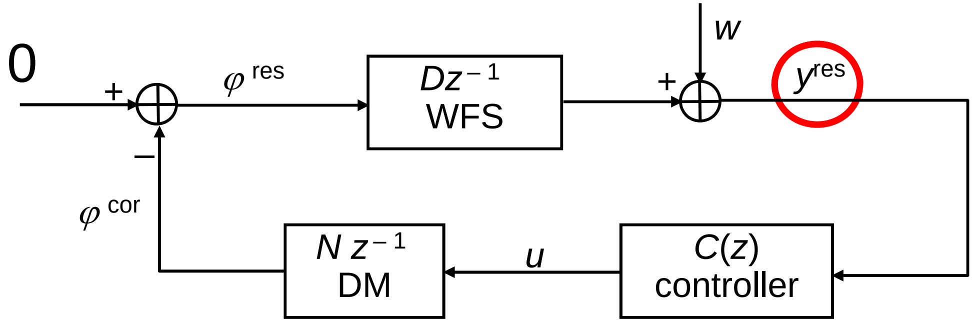
- For a scalar loop: $\sigma_{\mathrm{res}}^2 = \dfrac{\mathrm{Var}(y^{\mathrm{rep}}) - \sigma_w^2}{D^2}$

Noise propagation: the scalar case

- Noise propagation

$$\varphi_k^{\text{res}} = -\varphi_k^{\text{cor}} = -N u_{k-1}$$

$$\sigma^2_{\text{noise prop}} = N^2 \text{Var}(u)$$



- **Example: integrator**

$$u_k = u_{k-1} + g y_k^{\text{res}}$$
$$= u_{k-1} - gDN u_{k-2} + g w_k$$

$$X_{k+1} = A X_k + B w_k$$

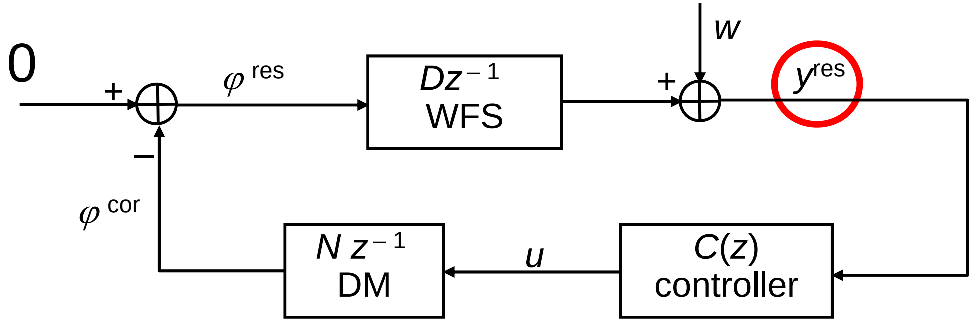
$$X_k = \begin{pmatrix} u_{k-1} \\ u_{k-2} \end{pmatrix}, \quad A = \begin{pmatrix} 1 & -gDN \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} g \\ 0 \end{pmatrix}$$

Solution to the Lyapunov equation $\text{Var}(X) = A\text{Var}(X)A^{\text{T}} + g^2 \sigma_w^2 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$$\sigma^2_{\text{noise prop}} = N^2 \text{Var}(X_{1,1})$$

Noise propagation in the multivariable case

- Noise propagation



  - Controller in state-space form

$$X_k^c = A^c X_{k-1}^c + B^c(-DNu_{k-2} + w_k)$$

  - Closed-loop formulation

$$X_k = \begin{pmatrix} X_k^c \\ u_{k-2} \end{pmatrix} \quad \Rightarrow \quad X_{k+1} = AX_k + Bw_k$$

$$u_k = PX_k$$

  - Variance evaluation (Lyapunov) $\mathrm{Var}(X) = A\mathrm{Var}(X)A^{\mathrm{T}} + B\Sigma_w B^{\mathrm{T}}$

Scalar case

$$\sigma^2_{\text{noise prop}} = N^2 \mathrm{Var}(X_{1,1})$$

Noise propagation variance

$$\sigma^2_{\text{noise prop}} = \mathrm{trace}(NP\,\mathrm{Var}(X)\,P^{\mathrm{T}}N^{\mathrm{T}})$$

Numerical application on a NAOS-like example

213 actuators
Noise: 0.2 rad^2
r_0 = 10 cm@ 0.5 µm
1 turbulent layer
Performance: rad^2 @ 1.65 µm

- Noise propagated variance
  - Lyapunov: $\sigma^2_{\text{noise prop}} = 0.097\ \text{rad}^2$
  - Simulations: $\sigma^2_{\text{noise prop}} = 0.097 \pm 0.001\ \text{rad}^2$

Noise propagation error is easy to estimate for any linear controller

Global performance evaluation in the multivariable case

- Residual phase variance
  - Scalar case

$$\sigma^2_{\mathrm{res}} = \frac{\mathrm{Var}(y^{\mathrm{rep}}) - \sigma^2_w}{D^2}$$

  - Reconstructed phase

$$\hat{\varphi} = R^{\mathrm{MAP}} y^{\mathrm{OL}} \qquad (y^{\mathrm{OL}} = D\varphi + w)$$

$$\hat{\varphi} = R^{\mathrm{MAP}} D\varphi + R^{\mathrm{MAP}} w$$

  - Closed-loop formulation

$$\hat{\varphi}^{\mathrm{res}} = \hat{\varphi} - \varphi^{\mathrm{cor}}$$

$$\hat{\sigma}^2_{\mathrm{res}} = \mathrm{Var}(\hat{\varphi}^{\mathrm{res}}) - R^{\mathrm{MAP}} \Sigma_w \left(R^{\mathrm{MAP}}\right)^{\mathrm{T}}$$

Performance evaluation on a NAOS-like example

- Residual phase variance with LQG regulator on 495 Zernike modes
  - Simulation of the real system: $\sigma^2_{res} = 0.54 \text{ rad}^2$
  - Phase reconstruction with 209 Zernike modes: $\hat{\sigma}^2_{res} = 0.48 \text{ rad}^2$
  - Phase reconstruction with 495 Zernike modes: $\hat{\sigma}^2_{res} = 0.29 \text{ rad}^2$

213 actuators (15x15)
Noise: 0.2 rad^2
r_0 = 10 cm@ 0.5 µm
1 turbulent layer
Performance:
rad^2 @ 1.65 µm

MAP phase estimation does not allow for error budget evaluation

Regulators performance comparison

- Residual phase variance with LQG regulator on 495 Zernike modes
  - Simulation of the real system: $\sigma^2_{res} = 0.54 \text{ rad}^2$
  - Phase reconstructed with 209 Zernike modes: $\hat{\sigma}^2_{res} = 0.48 \text{ rad}^2$

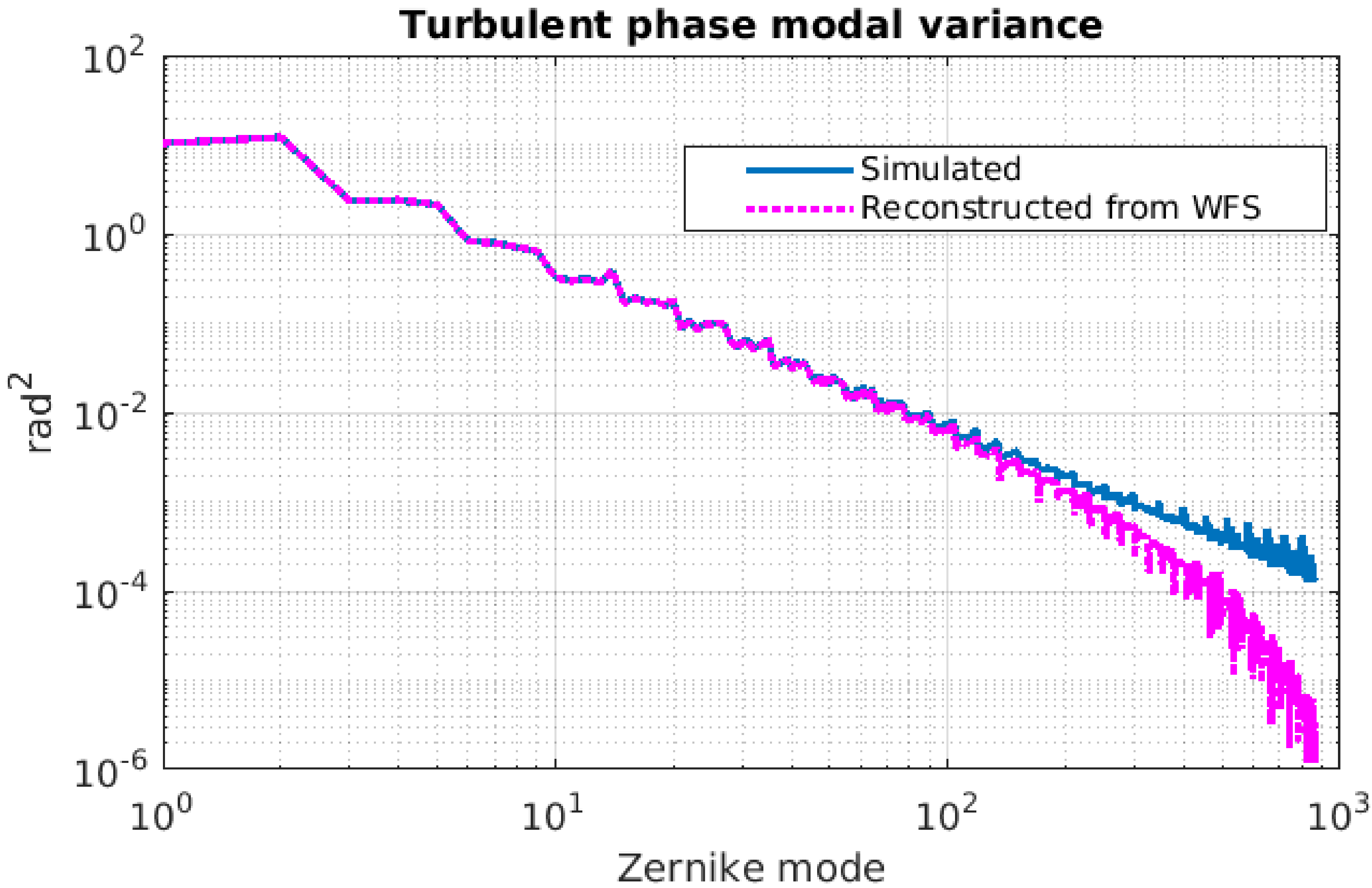
Variance increase of 37 %

- Residual phase variance with integral action regulator
  - Simulation of the real system: $\sigma^2_{res} = 0.74 \text{ rad}^2$
  - Phase reconstructed with 209 Zernike modes: $\hat{\sigma}^2_{res} = 0.56 \text{ rad}^2$

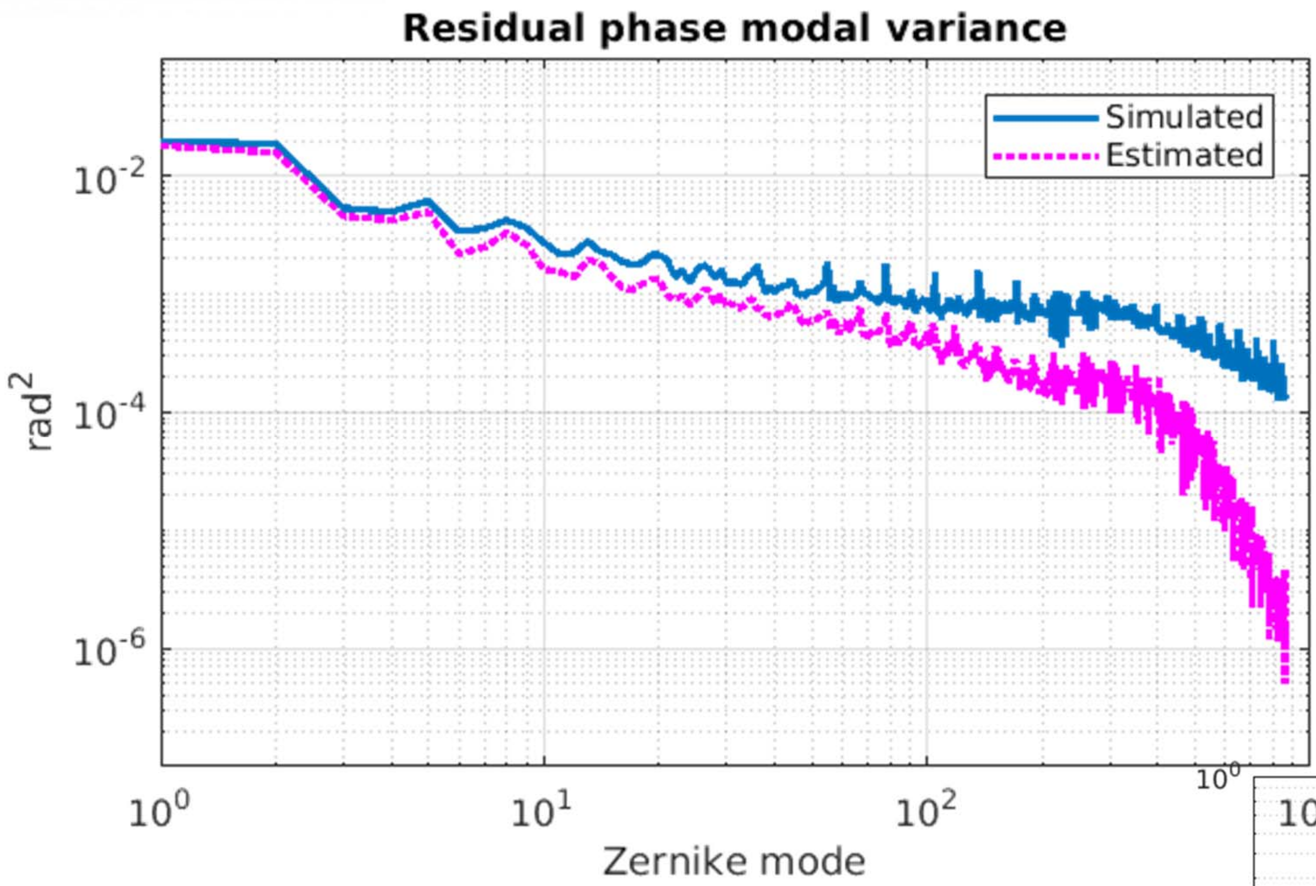
Variance increase of 17 %

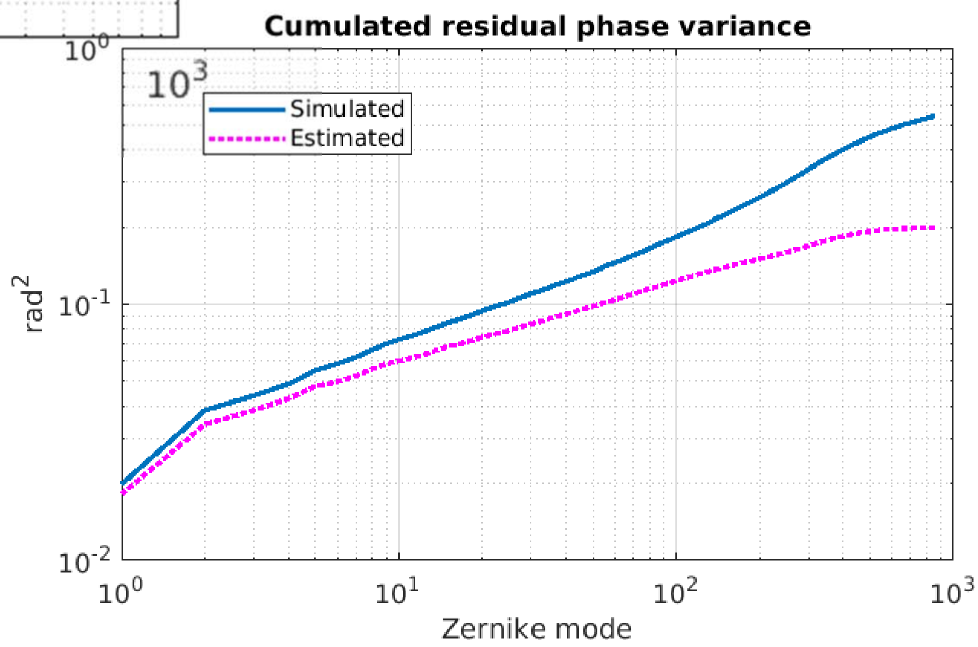
The gap in performance between controllers is not preserved

Turbulent phase reconstruction



Residual phase reconstruction





What we can do?

- Improve residual phase variance estimation
  - Spatio-temporal priors for high-order modes
  - Taylor+Kolmogorov

Performance criterion evaluation using Parseval

$$\sigma_{\text{res}}^2 = \text{trace}\frac{1}{\pi}\int_0^{\pi} T^{\text{rej}} S_{\varphi} T^{\text{rej}*} d\omega + \underbrace{\text{trace}\frac{1}{\pi}\int_0^{\pi} T^{\text{noise}} \Sigma_w T^{\text{noise}*} d\omega}_{\sigma_{\text{noise prop}}^2}$$

Power spectral density estimation with extrapolation

$$\hat{S}_{\varphi} = \left(\begin{array}{c|cc} S_{\hat{\varphi}} & \multicolumn{2}{c}{0} \\ \hline 0 & \hat{S}_{\overline{n}+1} & 0 \\ & 0 & \ddots \end{array}\right)$$

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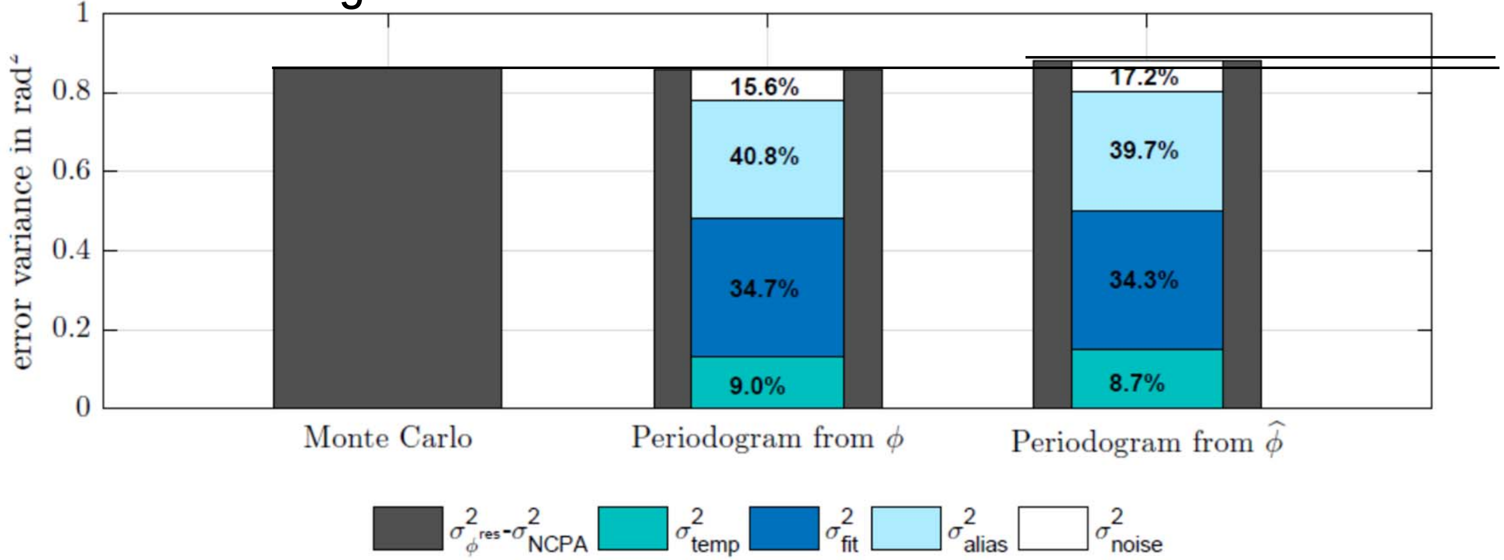
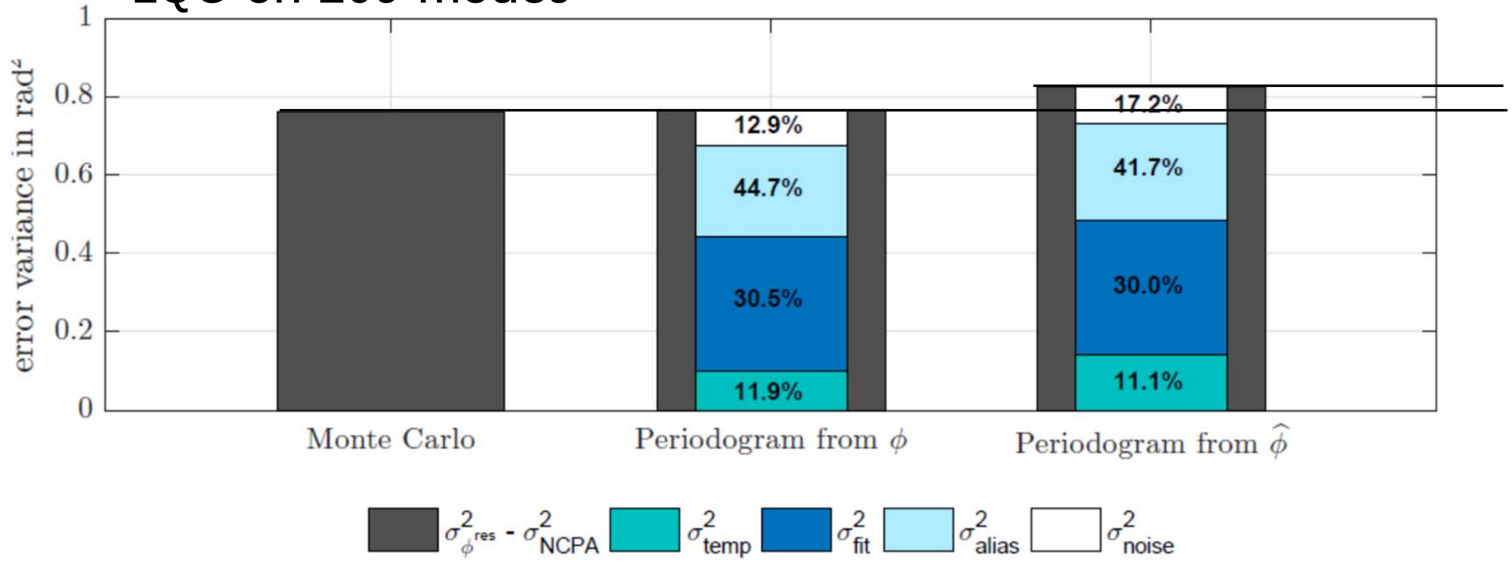
Performance evaluation from WFS data

Integrator



0.01 rad^2

LQG on 209 modes



0.05 rad^2

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Conclusion

- Noise propagation evaluation using state-space formulation is reliable
- Direct performance evaluation from WFS data fails
  - Residual phase variance is badly estimated from residual WFS measurements
- Spatial and temporal priors need to be used for phase PSD estimation
- Phase PSD estimation from WFS data needs priors on high-order modes
- Allows for "control dependent" error terms evaluation in error budget breakdown
- The method is able to cope with linear regulators in SCAO mode

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