Relativistic effects in macroscopically delocalized quantum superpositions

Albert Roura

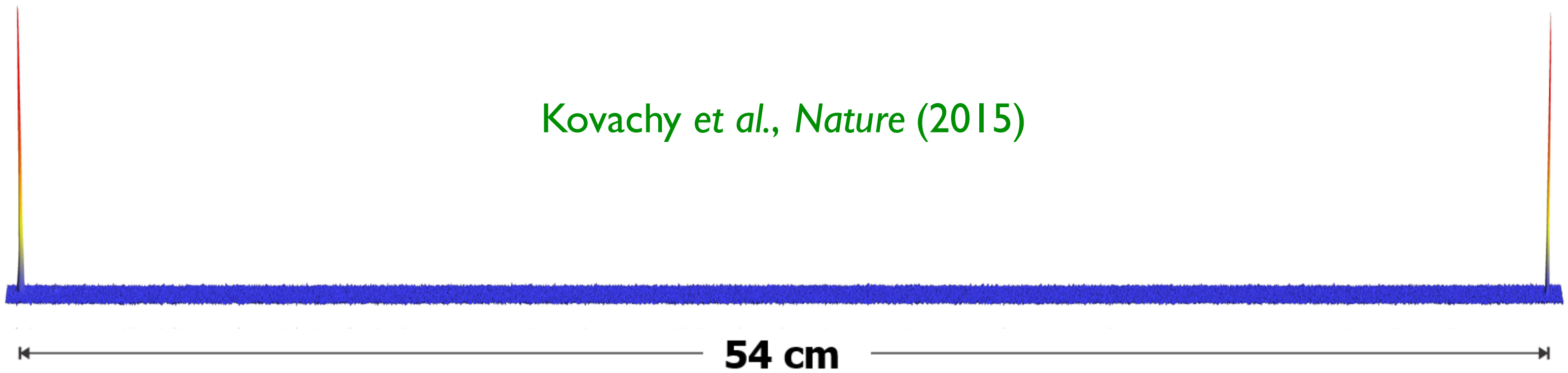
based on arXiv:1810.06744

Paris, 28 June 2019

Relativistic effects in macroscopically delocalized quantum superpositions

- *Macroscopically* delocalized quantum superpositions:

  coherent superposition of atomic wave packets



- Differences in *dynamics* of superposition components

  ⟶ entirely Newtonian

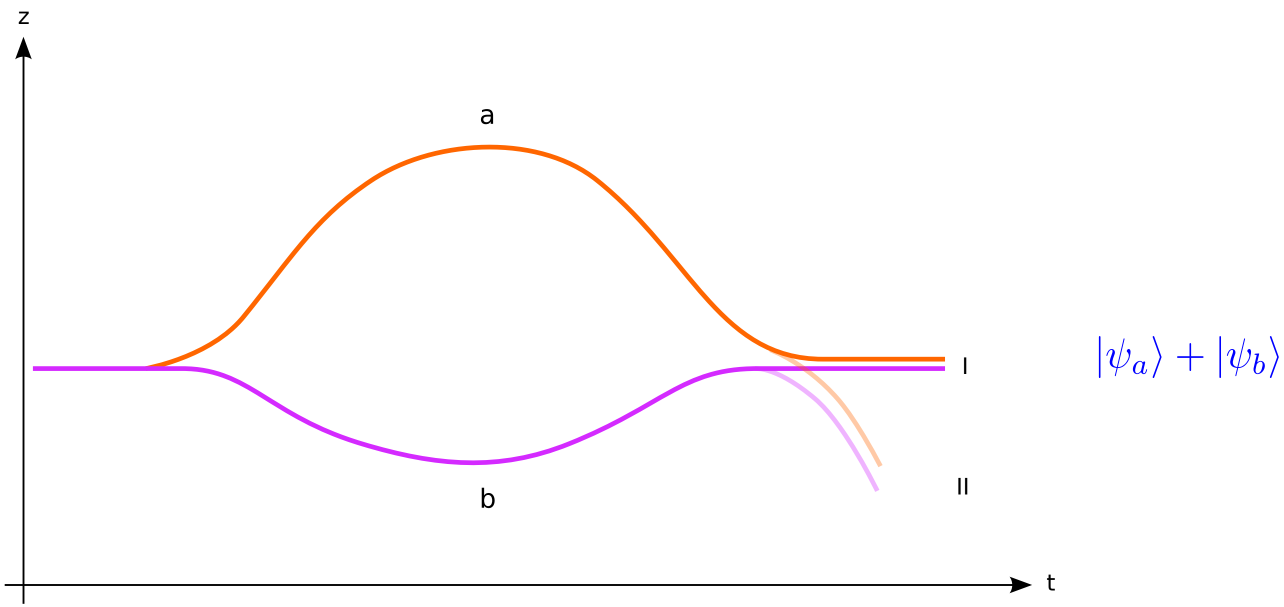
- Same relativistic effects on superposition components

  (e.g. atomic clocks)

★ <u>Goal</u> (QM + GR): experiment with general relativistic effects acting *non-trivially* on the quantum superposition

Proper time as *which-way* information

- Quantum superposition of clocks (*COM* + *internal state*) experiencing different proper times



→ *reduced* *visibility* of interference signal

Zych *et al.*, *Nat. Comm.* (2011)

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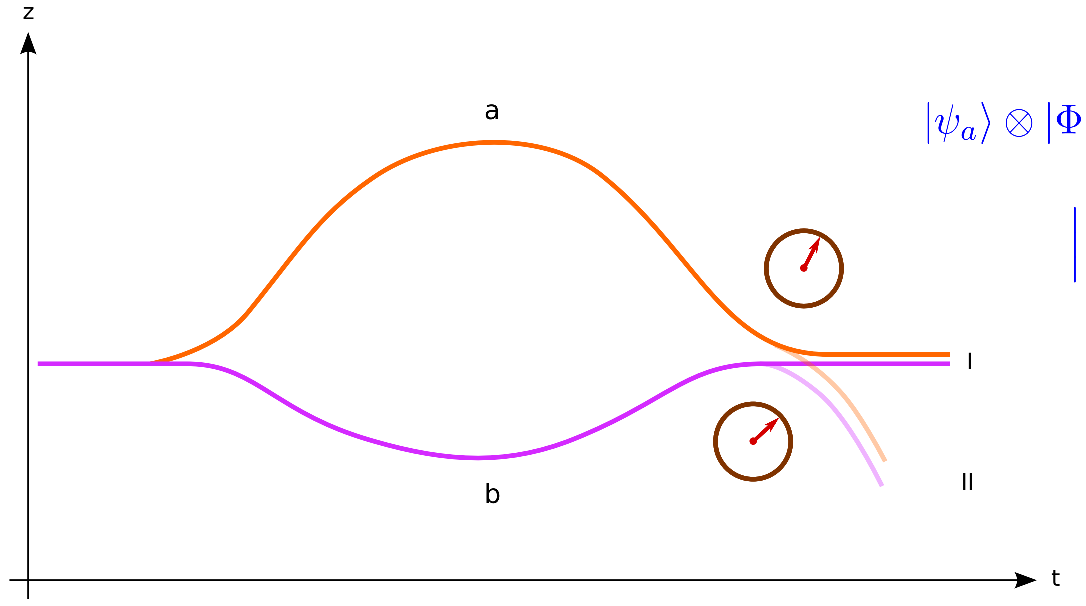


$$|\psi_a\rangle \otimes |\Phi(\tau_a)\rangle + |\psi_b\rangle \otimes |\Phi(\tau_b)\rangle$$

$$\left|\langle\Phi(\tau_b)|\Phi(\tau_a)\rangle\right| < 1$$

→ *reduced visibility* of interference signal

Zych *et al.*, *Nat. Comm.* (2011)

Outline

1. Relativistic effects in macroscopically delocalized quantum superpositions

2. Key elements of *quantum-clock interferometry*

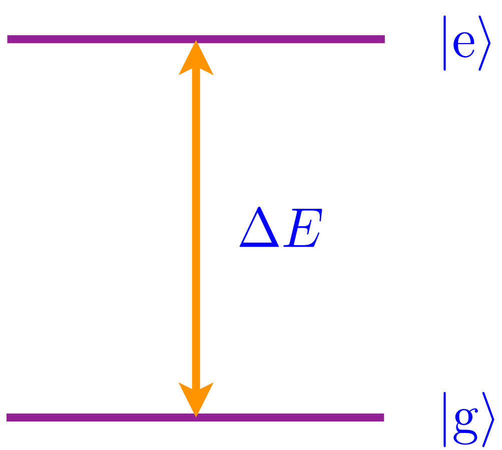
3. Major challenges in quantum-clock interferometry

4. *Doubly differential* scheme for *gravitational-redshift* measurements

5. Feasibility and extensions

Key elements
of quantum-clock interferometry

Quantum-clock model



- *Initialization* pulse:

$$|\mathrm{g}\rangle \;\rightarrow\; \big|\Phi(0)\big\rangle = \frac{1}{\sqrt{2}}\Big(|\mathrm{g}\rangle + i\,e^{i\varphi}|\mathrm{e}\rangle\Big)$$

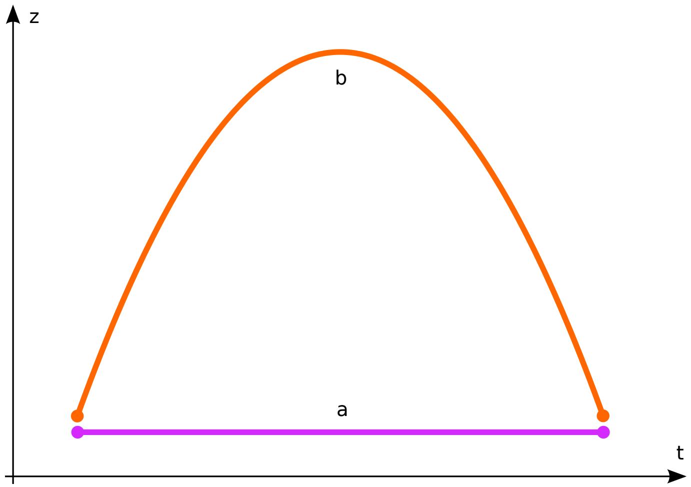
- Evolution:

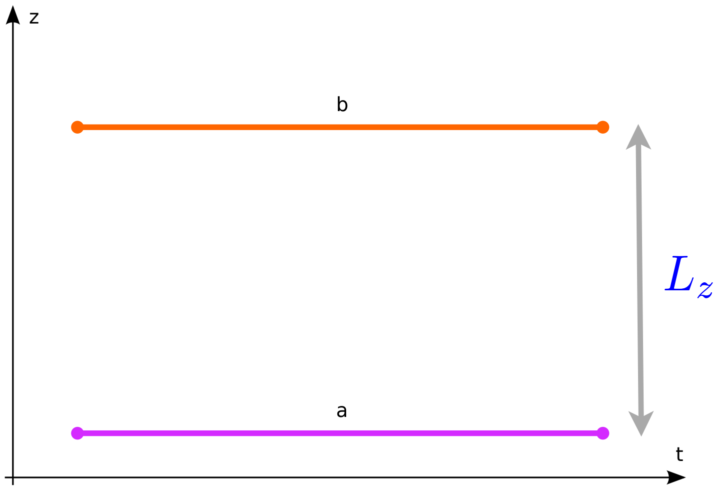
$$\big|\Phi(\tau)\big\rangle \propto \frac{1}{\sqrt{2}}\Big(|\mathrm{g}\rangle + i\,e^{i\varphi}e^{-i\Delta E\,\tau/\hbar}|\mathrm{e}\rangle\Big)$$

- Quantum overlap:

$$\Big|\langle\Phi(\tau_b)|\Phi(\tau_a)\rangle\Big| = \cos\left(\frac{\Delta E}{2\hbar}\,(\tau_b-\tau_a)\right)$$

- Comparison of independent clocks (after *read-out* pulse):





$$\Delta\tau_b - \Delta\tau_a \approx \left(g\, L_z/c^2\right) \Delta t$$

for optical *atomic clocks*

$\Delta E \sim 1\,\mathrm{eV}$ $\quad L_z \sim 1\,\mathrm{cm}$

- Instead of independent clocks we pursue a quantum superposition at *different heights*.

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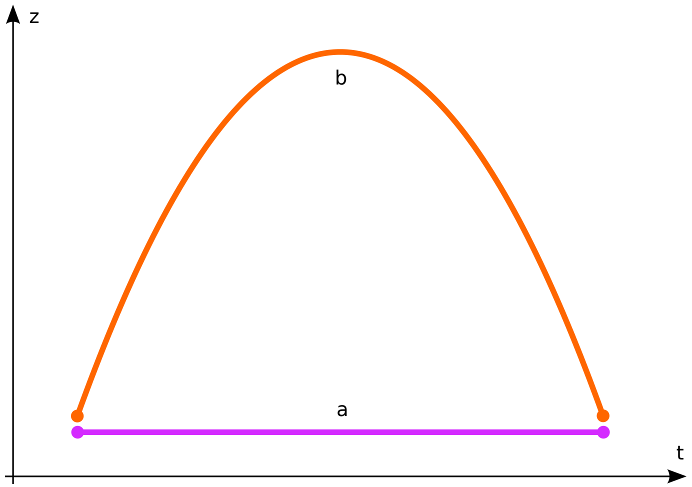

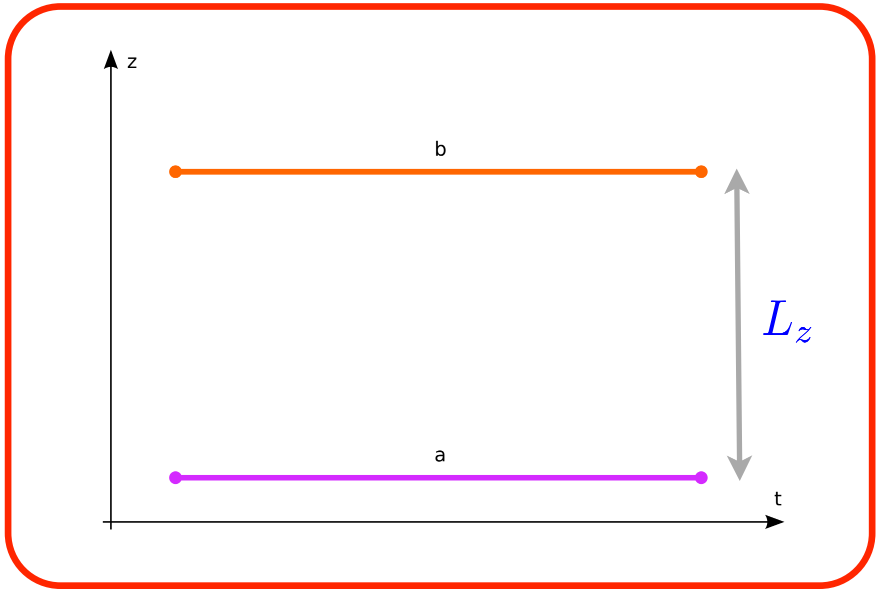

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- Theoretical description of the clock

  - *two-level* atom (*internal* state):

$$\hat{H} = \hat{H}_1 \otimes |\mathrm{g}\rangle\langle\mathrm{g}| + \hat{H}_2 \otimes |\mathrm{e}\rangle\langle\mathrm{e}|$$

$$m_1 = m_{\mathrm{g}}$$

$$m_2 = m_{\mathrm{g}} + \Delta m$$

$$\Delta m = \Delta E / c^2$$

  - classical action for *COM* motion:

$$S_n\left[x^\mu(\lambda)\right] = -m_n c^2 \int d\tau = -m_n c \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \qquad (\, n = 1, 2\,)$$

← free fall

$$S_n\left[x^\mu(\lambda)\right] = -m_n c^2 \int d\tau - \int d\tau\, V_n(x^\mu)$$

← including external forces

Atom interferometry in curved spacetime (including relativistic effects)

- Wave-packet evolution in terms of
  - *central trajectory* (satisfies *classical e.o.m.*) $X^\mu(\lambda)$
  - *centered* wave packet $|\psi_c^{(n)}(\tau_c)\rangle$
- Fermi-Walker frame associated with the *central trajectory*
  - valid for *freely falling* wave packet (*geodesic*)
  - but also with *external forces* / *guiding* potential (*accel.* trajectory)
  - approximately *non-relativistic* dynamics for centered wave packet

$$\Delta p/m \ll c \qquad \Delta x \ll \ell$$

curvature radius

- Metric in *Fermi-Walker* coordinates: $X^\mu(\tau_c) = \big(c\,\tau_c\,,\mathbf{0}\big)$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = g_{00}\,c^2 d\tau_c^2 \,+\, 2\,g_{0i}\,c\,d\tau_c\,dx^i \,+\, g_{ij}\,dx^i dx^j$$

$$\begin{aligned} g_{00} &= -\big(1+\delta_{ij}\,a^i(\tau_c)\,x^j/c^2\big)^2 - R_{0i0j}(\tau_c,\mathbf{0})\,x^i x^j + O\big(|\mathbf{x}|^3\big) \\ g_{0i} &= -\frac{2}{3}R_{0jik}(\tau_c,\mathbf{0})\,x^j x^k + O\big(|\mathbf{x}|^3\big) \\ g_{ij} &= \delta_{ij} - \frac{1}{3}R_{ikjl}(\tau_c,\mathbf{0})\,x^k x^l + O\big(|\mathbf{x}|^3\big) \end{aligned}$$

- Expanding the action for the *centered wave packet:*

$$S_n\big[\mathbf{x}(t)\big] \approx \int d\tau_c \left[-m_n c^2 - V_n(\tau_c,\mathbf{0}) + \frac{m_n}{2}\,\mathbf{v}^2 - \frac{1}{2}\,\mathbf{x}^{\mathrm{T}}\Big(\mathcal{V}^{(n)}(\tau_c) - m_n\Gamma(\tau_c)\Big)\,\mathbf{x} - V^{(n)}_{\mathrm{anh.}}(\tau_c,\mathbf{x})\right]$$

- Hamiltonian: $$\hat{H}_n = m_n c^2 + V_n(\tau_c, \mathbf{0}) + \hat{H}_c^{(n)}$$

$$\mathcal{V}_{ij}^{(n)}(\tau_c) = \partial_i \partial_j V_n(\tau_c, \mathbf{x})\big|_{\mathbf{x}=\mathbf{0}}$$

$$\hat{H}_c^{(n)} = \frac{1}{2m_n}\hat{\mathbf{p}}^2 + \frac{1}{2}\hat{\mathbf{x}}^{\mathrm{T}}\Big(\mathcal{V}^{(n)}(\tau_c) - m_n\Gamma(\tau_c)\Big)\hat{\mathbf{x}}$$

- Wave-packet evolution: $$\big|\psi^{(n)}(\tau_c)\big\rangle = e^{i\mathcal{S}_n/\hbar}\big|\psi_c^{(n)}(\tau_c)\big\rangle$$

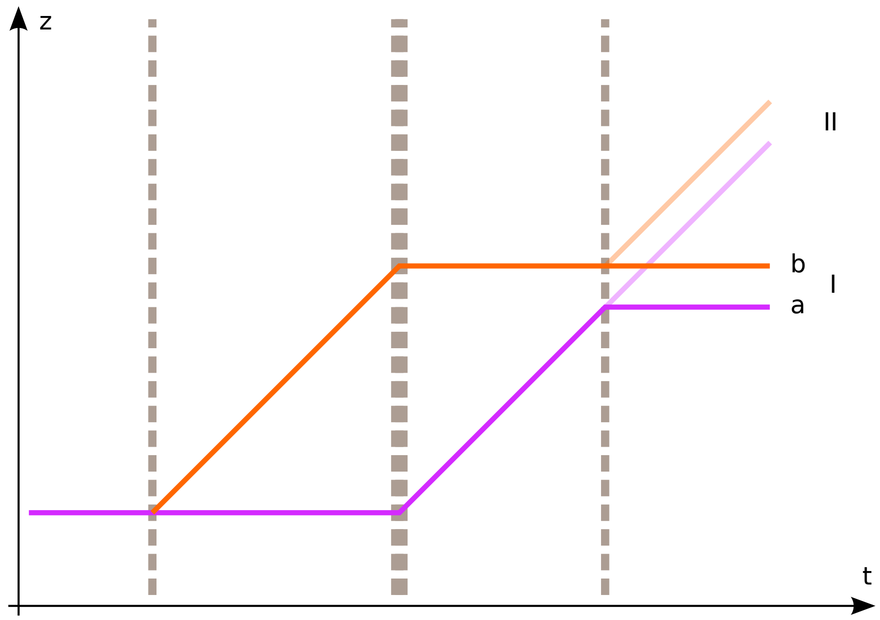
  - *propagation phase*

$$\mathcal{S}_n = -\int_{\tau_1}^{\tau_2} d\tau_c\,\left(m_n c^2 + V_n(\tau_c, \mathbf{0})\right)$$

  - *centered wave packet*

$$i\hbar\frac{d}{d\tau_c}\big|\psi_c^{(n)}(\tau_c)\big\rangle = \hat{H}_c\big|\psi_c^{(n)}(\tau_c)\big\rangle$$

- Full interferometer (including *laser kicks*):



propagation + *laser* phases

$$|\psi_{\mathrm{I}}\rangle = \frac{1}{2}\left(e^{i\phi_a} + e^{i\phi_b}\right)|\psi_{\mathrm{c}}\rangle$$

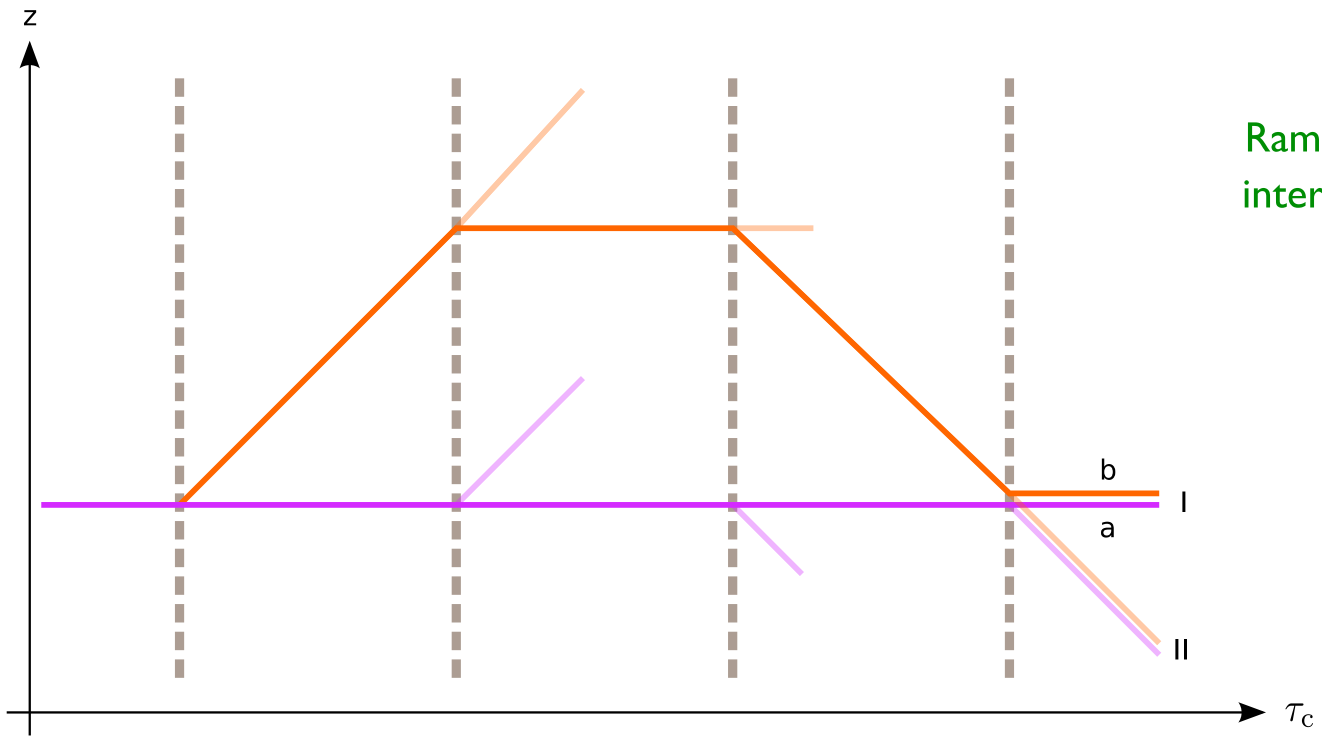
- Detection *probability* at the *exit port(s)*: $\langle\psi_{\mathrm{I}}|\psi_{\mathrm{I}}\rangle = \frac{1}{2}(1 + \cos\delta\phi)$

- *Phase shift:* $\delta\phi = \phi_b - \phi_a + \delta\phi_{\mathrm{sep}}$

Major challenges in quantum-clock interferometry

Insensitivity to gravitational redshift (in a *uniform field*)

- Consider a freely falling frame:



- Proper-time difference between the two interferometer branches → independent of g

(small dependence due to pulse timing suppressed by $(v_{\rm rec}/c) \sim 10^{-10}$)

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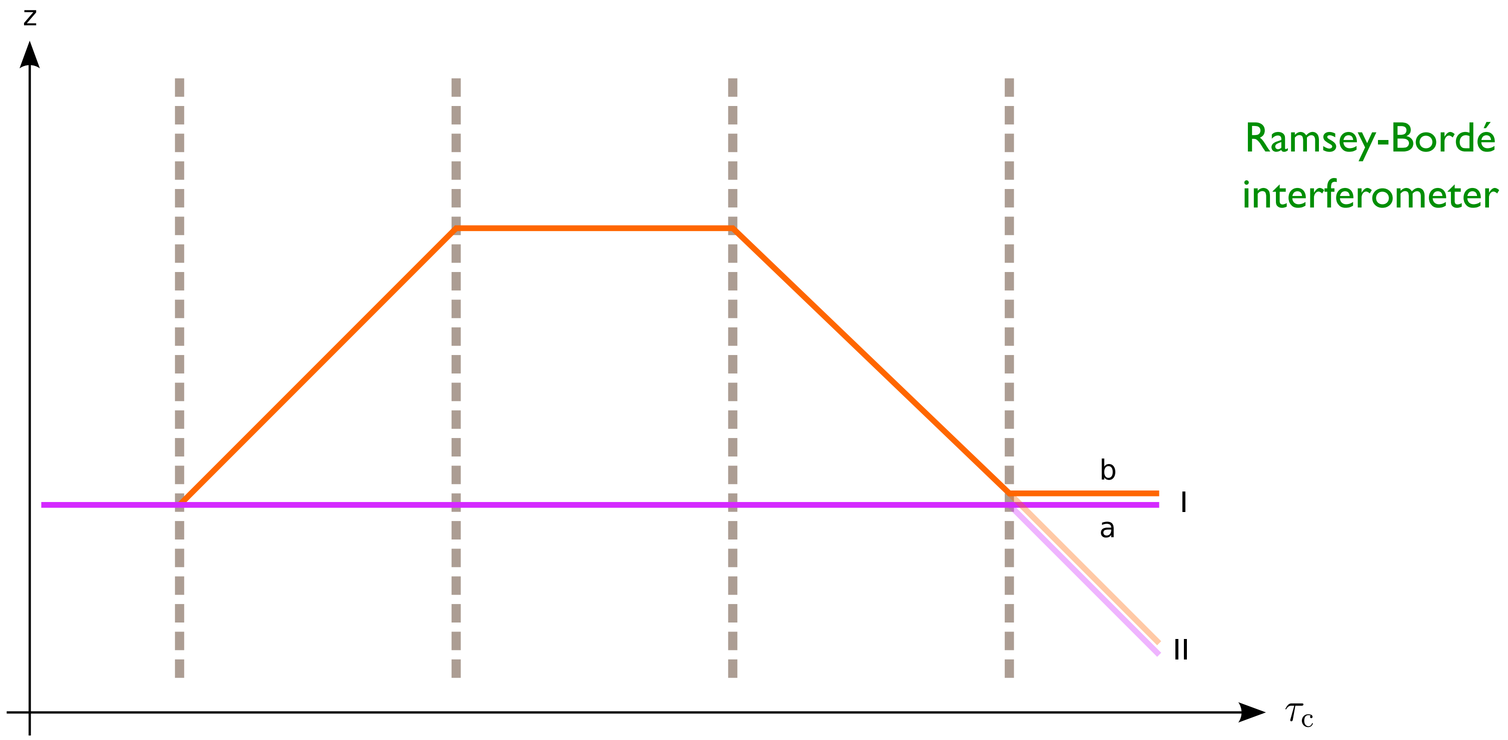

- Proper-time difference between the two interferometer branches ⟶ independent of g

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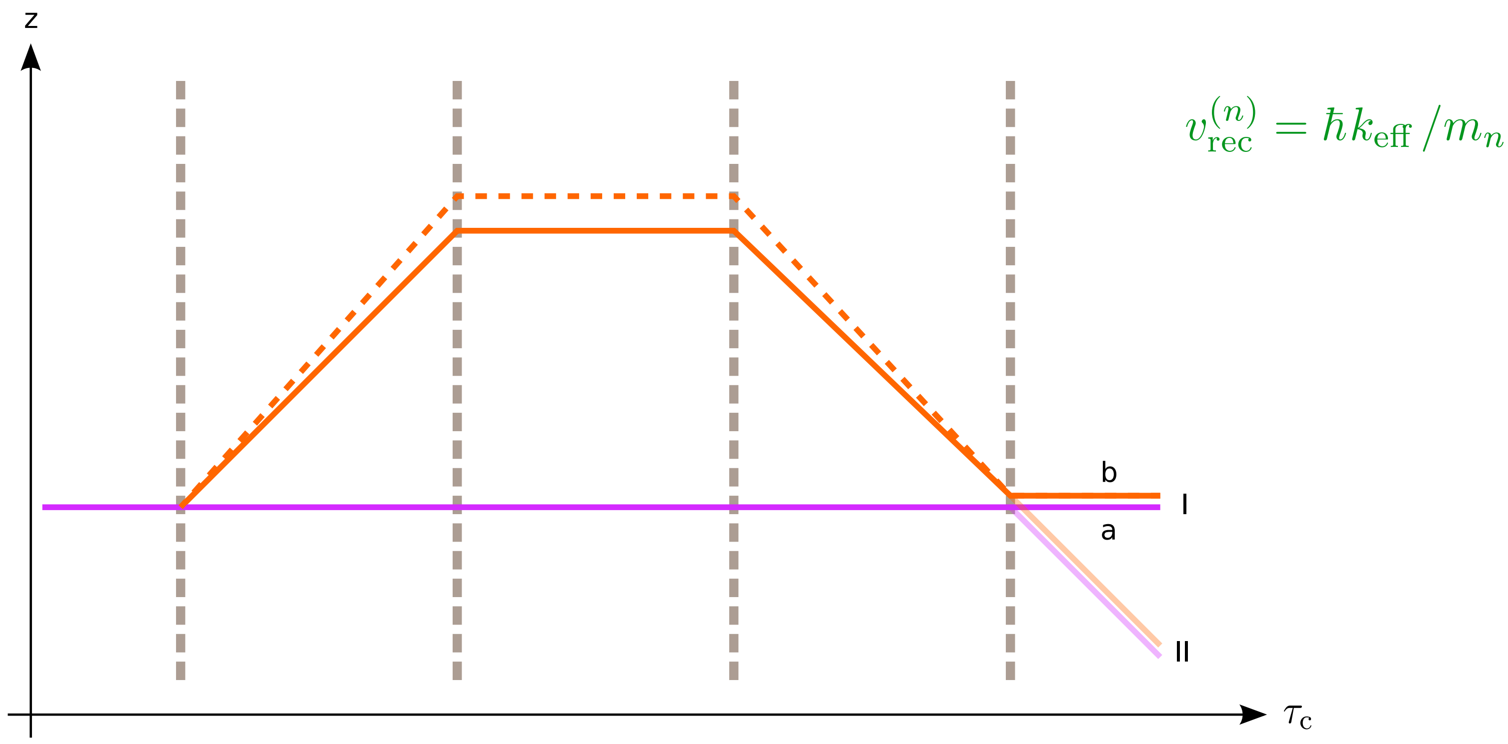
Differential recoil

- Different *recoil velocities* → different *central trajectories*



- Implied changes of proper-time difference are comparable to signal of interest.

Small visibility changes

- Reduced interference visibility due to deceasing quantum overlap of *clock states*:

$$\langle \Psi_{\rm I} | \Psi_{\rm I} \rangle = \frac{1}{2} + \frac{1}{2} \left| \langle \Phi(\tau_b) | \Phi(\tau_a) \rangle \right| \cos \delta\phi$$

$$\left| \langle \Phi(\tau_b) | \Phi(\tau_a) \rangle \right| = \cos\left(\frac{\Delta E}{2\hbar} (\tau_b - \tau_a) \right)$$

- Small effect for feasible parameter range:

$$\left| \langle \Phi(\tau_b) | \Phi(\tau_a) \rangle \right| = \cos\left(\frac{\omega_0}{2} \frac{g\,\Delta z}{c^2} \Delta t \right) \approx 1 - \left(10^{-3}\right)^2 / 2$$

$$\Delta E/\hbar = \omega_0 \approx 2\pi \times 4 \times 10^2\,\mathrm{THz}$$

$$\Delta z = 1\,\mathrm{cm}$$

$$\Delta t = 1\,\mathrm{s}$$

- Extremely difficult to measure such small *changes of visibility*, which are masked by *other effects* leading also to *loss of visibility*.

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$$\Delta E/\hbar = \omega_0 \approx 2\pi\times 4\times 10^2\,\mathrm{THz}$$

$$\Delta z = 1\,\mathrm{m}$$

$$\Delta t = 1\,\mathrm{s}$$

- Extremely difficult to measure such small *changes of visibility*, which are masked by *other effects* leading also to *loss of visibility*.

Doubly differential scheme for gravitational-redshift measurement

Differential phase-shift measurement

- Detection probability at first exit port (*independent* of *internal state*):

$$\begin{aligned}\langle \Psi_\mathrm{I} | \Psi_\mathrm{I} \rangle &= \frac{1}{2}\Big(\langle \Psi_\mathrm{I}^{(1)} | \Psi_\mathrm{I}^{(1)} \rangle + \langle \Psi_\mathrm{I}^{(2)} | \Psi_\mathrm{I}^{(2)} \rangle \Big) \\ &= \frac{1}{4}\Big(1 + \cos\delta\phi^{(1)} \; + \; 1 + \cos\delta\phi^{(2)} \Big) \\ &= \frac{1}{2} + \underbrace{\frac{1}{2}\cos\left(\frac{\delta\phi^{(2)} - \delta\phi^{(1)}}{2}\right)}_{\text{visibility}} \cos\left(\frac{\delta\phi^{(1)} + \delta\phi^{(2)}}{2}\right)\end{aligned}$$

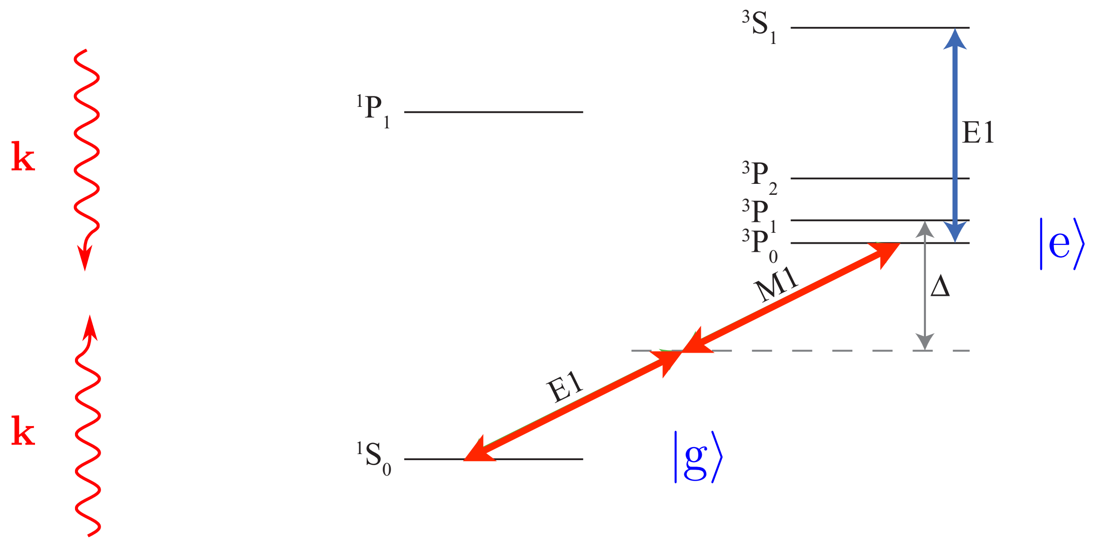
- Phase-shift difference directly related to visibility reduction.

- Precise differential phase-shift measurement involving *state-selective detection* is much more viable.

  (*immune* to spurious loss of contrast + *common-mode rejection* of phase noise)

Two-photon pulse for clock initialization

- Level structure for group-II-type atoms (e.g. *Sr*, *Yb*) employed in optical atomic clocks:



Alden *et al.*, *Phys. Rev. A* (2014)

- Two-photon process *resonantly* connecting the two clock states.

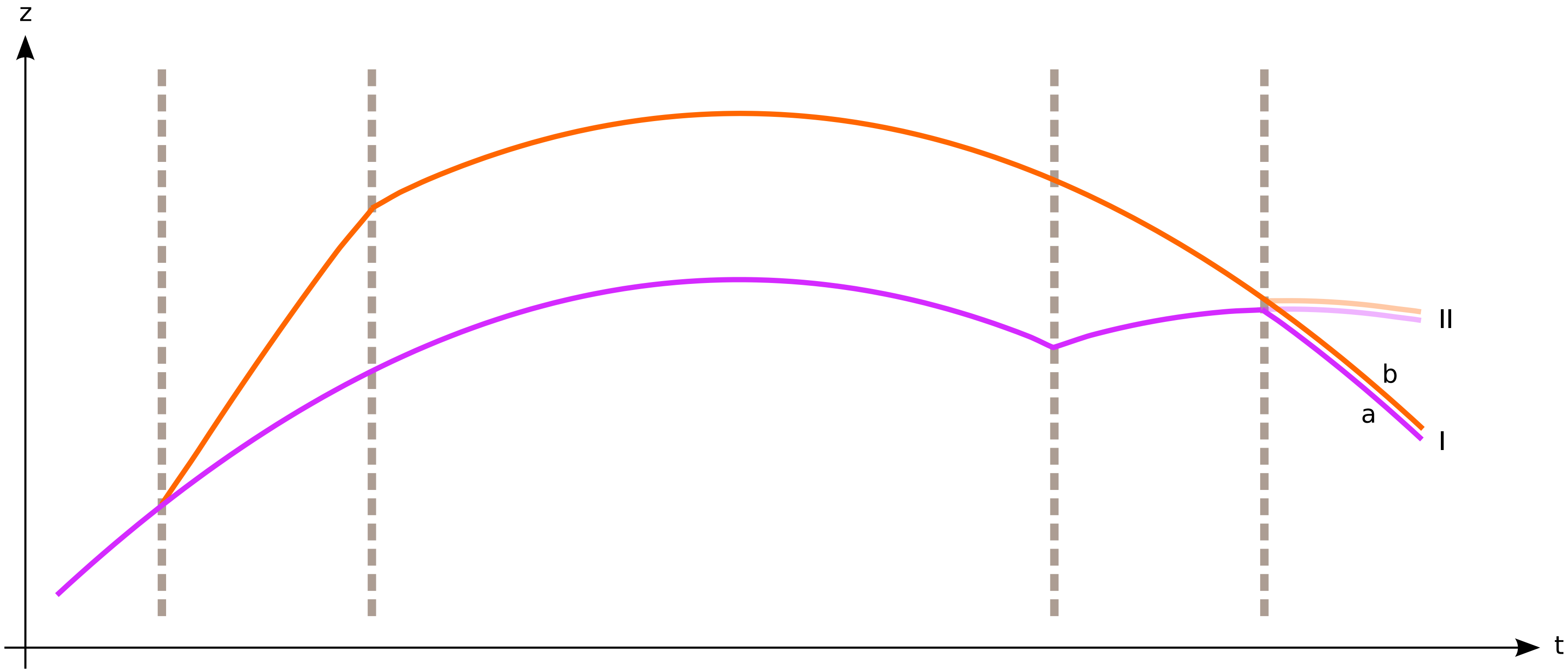
- Equal-frequency *counter-propagating* laser beams in lab frame:

  constant *effective phase* ⟶ *simultaneity hypersurfaces* in lab frame

$$e^{i\omega t}e^{i\mathbf{k}\cdot\mathbf{x}} \times e^{i\omega t}e^{-i\mathbf{k}\cdot\mathbf{x}} = e^{i\,2\omega t}$$

Laboratory frame

- Compare *differential phase-shift* measurements for different initialization times:



$$\left(\delta\phi^{(2)}(t_{\rm i}') - \delta\phi^{(1)}(t_{\rm i}')\right) - \left(\delta\phi^{(2)}(t_{\rm i}) - \delta\phi^{(1)}(t_{\rm i})\right) = \frac{\Delta E}{2\hbar}\,(\Delta\tau_b - \Delta\tau_a) = \Delta m\, g\, \Delta z\,(t_{\rm i}' - t_{\rm i})/\hbar$$

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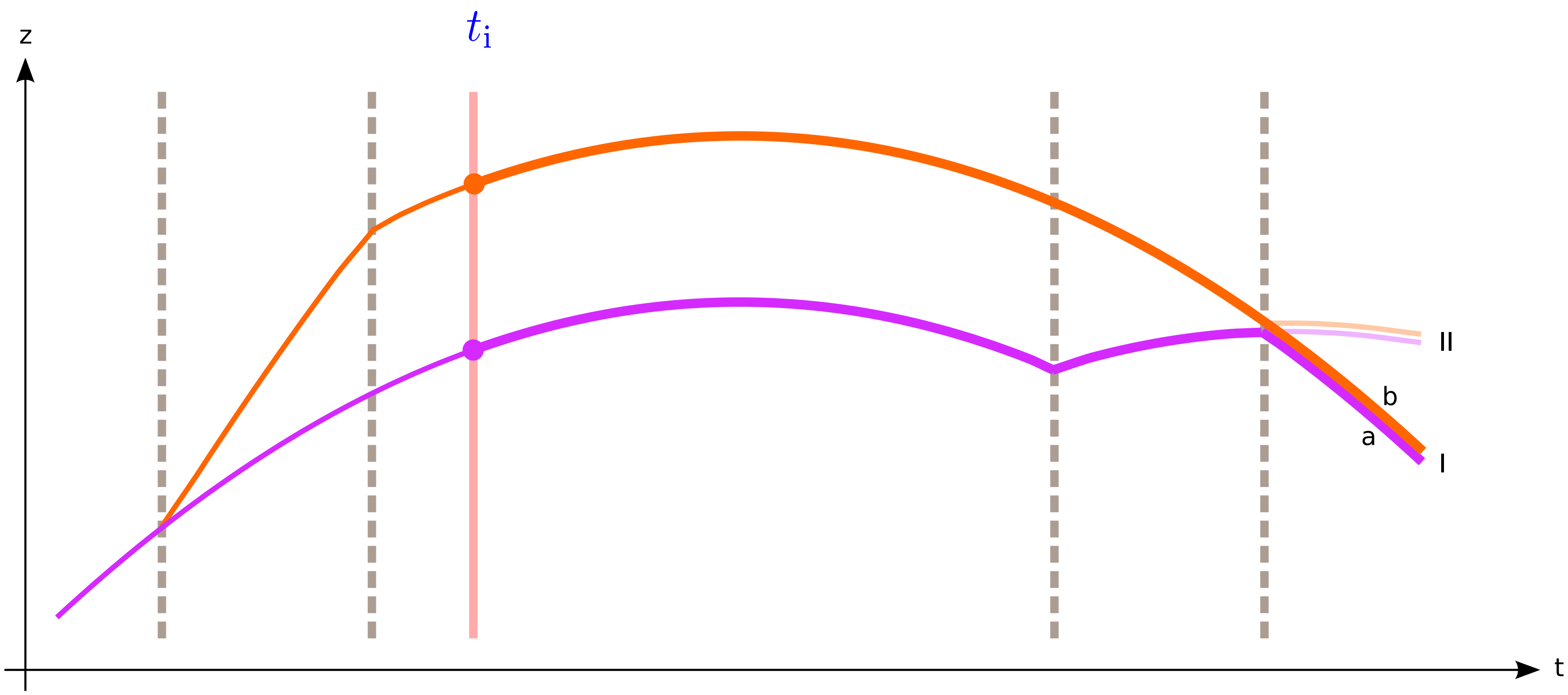

$$\left(\delta\phi^{(2)}(t'_{\rm i}) - \delta\phi^{(1)}(t'_{\rm i})\right) - \left(\delta\phi^{(2)}(t_{\rm i}) - \delta\phi^{(1)}(t_{\rm i})\right) = \frac{\Delta E}{2\hbar}\,(\Delta\tau_b - \Delta\tau_a) = \Delta m\, g\, \Delta z\,(t'_{\rm i} - t_{\rm i})/\hbar$$

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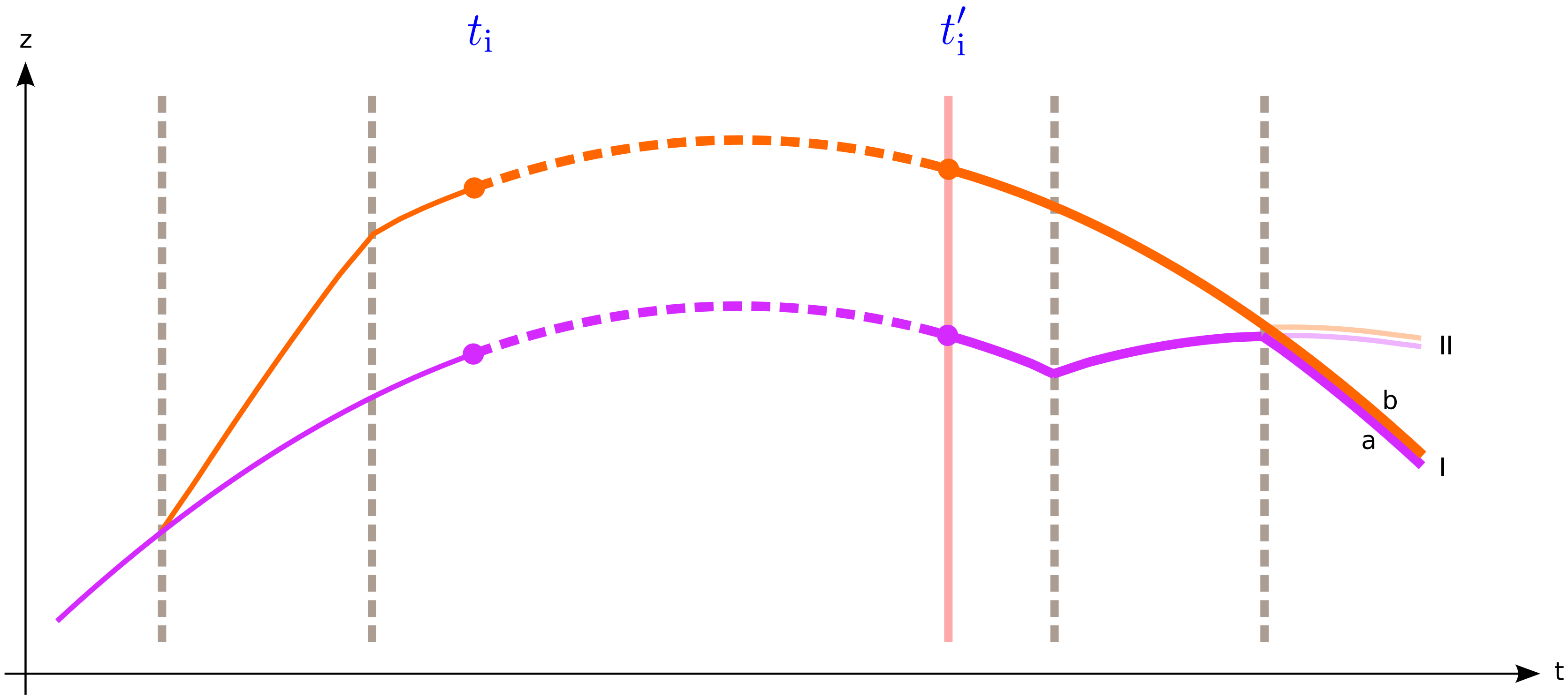

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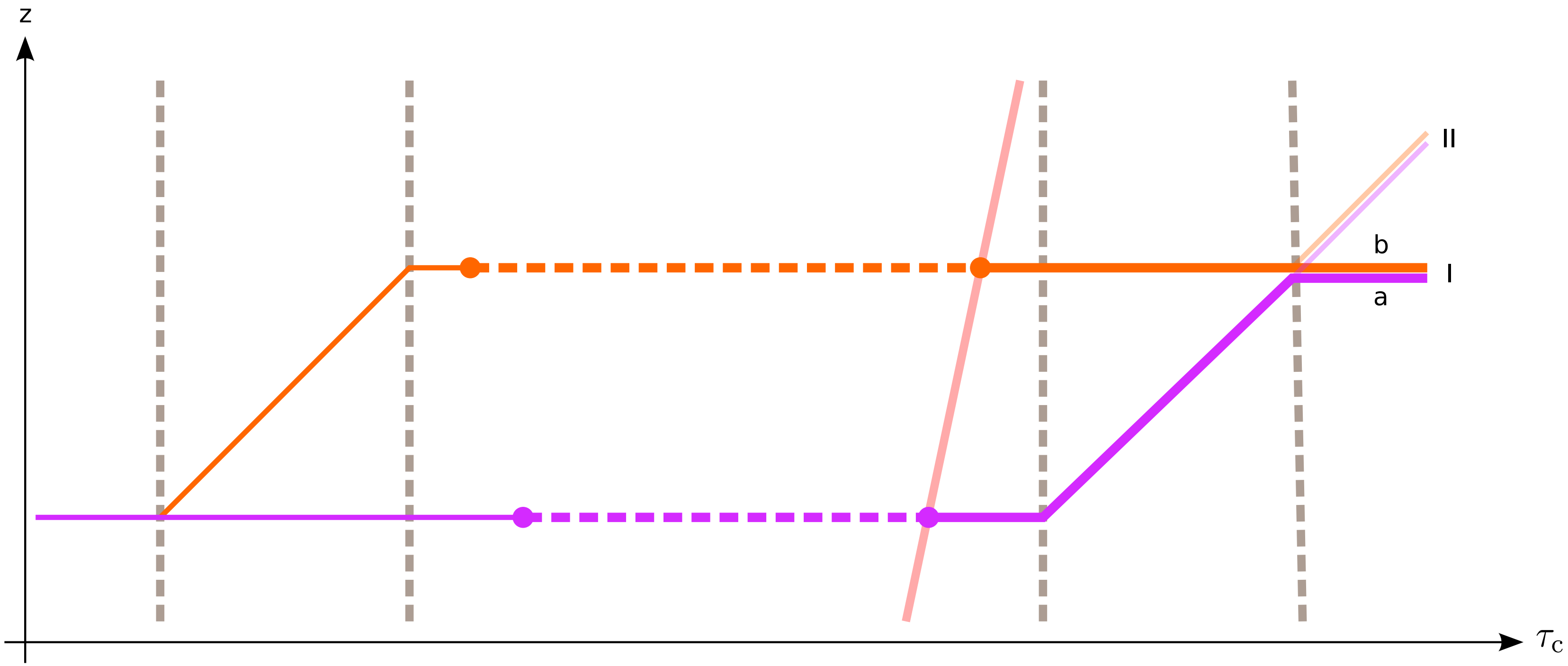
Freely falling frame

- Relativity of simultaneity: $\Delta\tau_{\rm c} \approx -v(t)\,\Delta z/c^2 = g\,(t - t_{\rm ap})\,\Delta z/c^2$



$$\left(\delta\phi^{(2)}(t_{\rm i}') - \delta\phi^{(1)}(t_{\rm i}')\right) - \left(\delta\phi^{(2)}(t_{\rm i}) - \delta\phi^{(1)}(t_{\rm i})\right) = \frac{\Delta E}{2\hbar}\,(\Delta\tau_b - \Delta\tau_a) = \Delta m\, g\, \Delta z\, (t_{\rm i}' - t_{\rm i})/\hbar$$

Challenges addressed

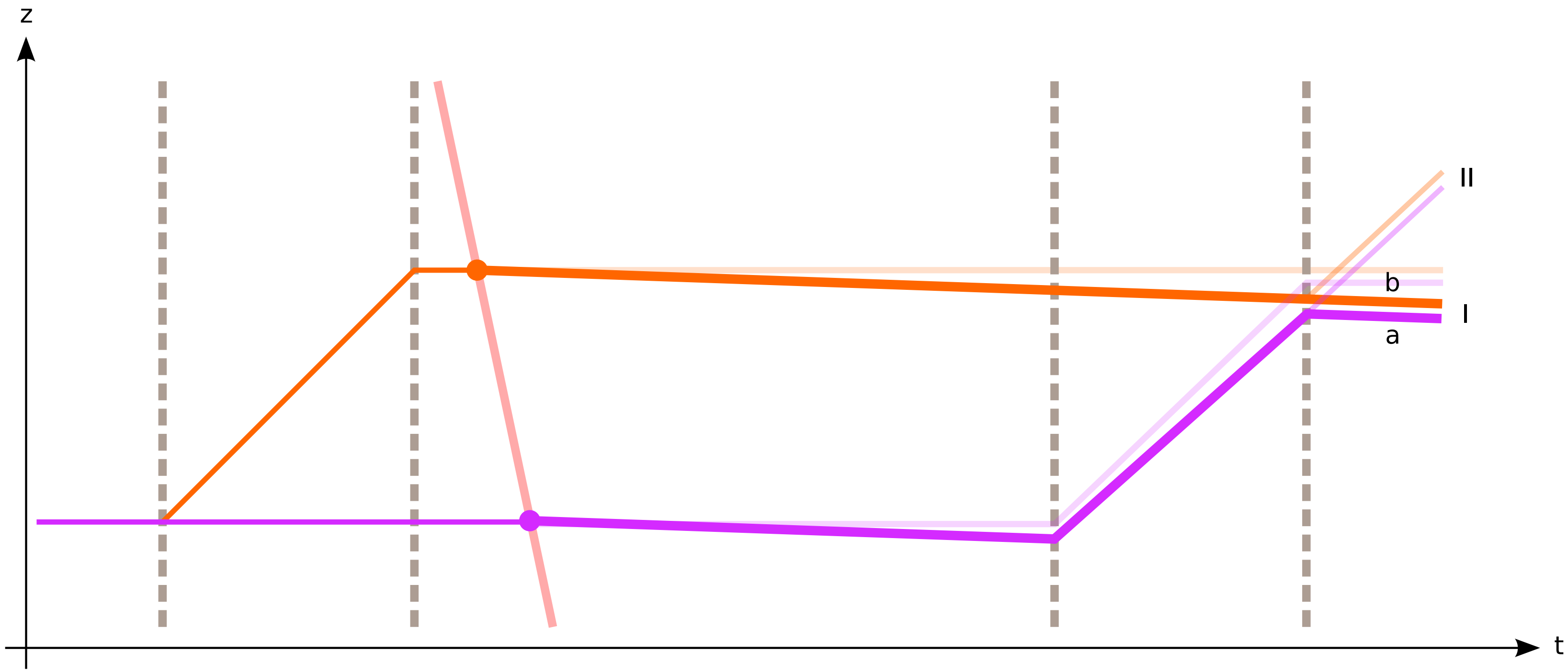
- Differential phase-shift measurement → *precise measurement*, *common-mode rejection* (of *noise* & *systematics*)

- Comparing measurements with different initialization times
  → sensitive to *gravitational redshift* + further *immunity*

- Almost no recoil from *initialization pulse*,
  small *residual recoil* with no impact on gravitational redshift measurement,

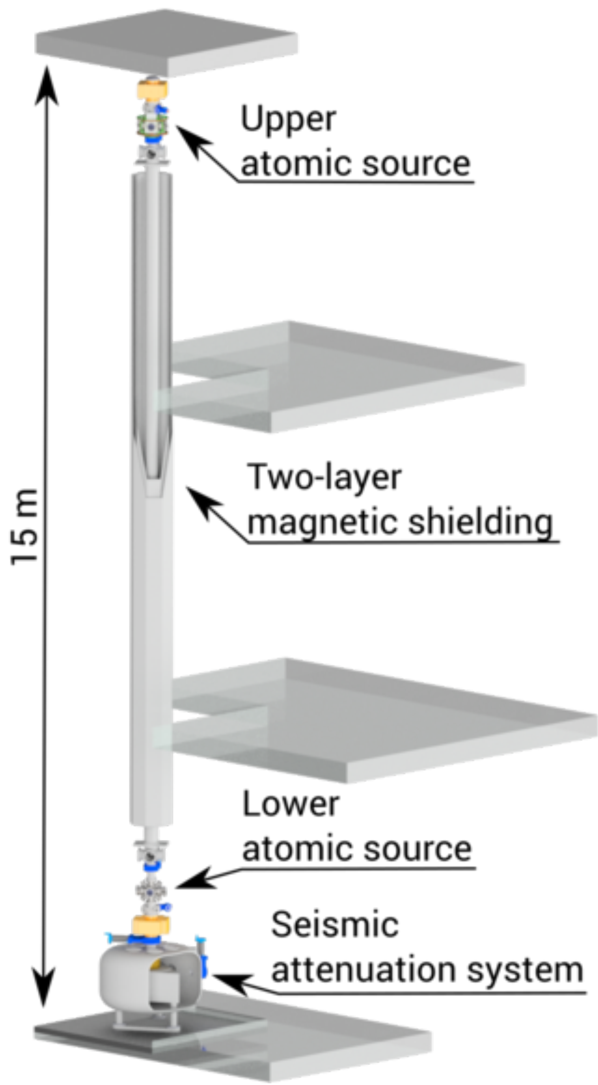
  effect of *differential recoil* from second pair of Bragg pulses cancels out in *doubly differential* measurement.

- Residual recoil with no influence on the *phase-shift* for the *excited* state:



Feasibility and extensions

Feasible implementation



HITec (Hannover)

- 10-m atomic fountains operating with *Sr*, *Yb* in *Stanford* & *Hannover* respectively.
- More than $2\,\mathrm{s}$ of free evolution time.
- *Doubly differential phase shift* of $1\,\mathrm{mrad}$ for

$$\Delta E/\hbar = \omega_0 \approx 2\pi \times 4 \times 10^2\,\mathrm{THz}$$

$$\Delta z = 1\,\mathrm{cm}$$

$$\Delta t_{\mathrm{i}} = 1\,\mathrm{s}$$

- Resolvable in a *single shot* for atomic clouds with $N = 10^6$ atoms (*shot-noise* limited)
- More compact set-ups possible with *guided* or *hybrid* interferometry (less mature).

Conclusion

- Measurement of relativistic effects in *macroscopically* delocalized quantum superpositions with *quantum-clock interferometry*.

- Important *challenges* in quantum-clock interferometry and its application to gravitational-redshift measurement.

- Promising doubly differential scheme that overcomes them.

- Feasible implementation in facilities soon to become operational.

  Applicable also to more compact set-ups based on *guided* or *hybrid* interferometry.

- If one considers a consistent framework for parameterizing violations of Einstein's *equivalence principle*, (e.g. *dilaton models*)

  both for comparison of *independent clocks* and for the above *quantum-clock interferometry* scheme one obtains

$$\frac{\Delta\bar{\tau}_b - \Delta\bar{\tau}_a}{\Delta\bar{\tau}_a} \approx \left(1 + \alpha_{\text{e-g}}\right)\Big(U(\mathbf{x}_b) - U(\mathbf{x}_a)\Big)/c^2 \qquad \alpha_{\text{e-g}} = \frac{m_1}{\Delta m}\left(\beta_2 - \beta_1\right)$$

→ test of *universality of gravitational redshift* with delocalized quantum superpositions

Related work

- Collaboration on the experimental realization of the proposed scheme with *Leibniz University Hannover*:

Sina Loriani   Dennis Schlippert   Ernst Rasel

- Related theoretical work at *Ulm University*:

PHYSICAL REVIEW A **99**, 013627 (2019)

Proper time in atom interferometers: Diffractive versus specular mirrors

Enno Giese,[1,*] Alexander Friedrich,[1] Fabio Di Pumpo,[1] Albert Roura,[1] Wolfgang P. Schleich,[1,2]
Daniel M. Greenberger,[3] and Ernst M. Rasel[4]

Stephan Kleinert   Christian Ufrecht

- Discussion of special relativistic effects:

Interference of Clocks: A Quantum Twin Paradox

Sina Loriani[1†], Alexander Friedrich[2†*], Christian Ufrecht[2], Fabio Di Pumpo[2], Stephan Kleinert [2], Sven Abend[1], Naceur Gaaloul[1], Christian Meiners[1], Christian Schubert[1], Dorothee Tell[1], Étienne Wodey[1], Magdalena Zych[3], Wolfgang Ertmer[1], Albert Roura[2], Dennis Schlippert[1], Wolfgang P. Schleich[2,4], Ernst M. Rasel[1] and Enno Giese[2]

arXiv:1905.09102

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Thank you for your attention.

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aufgrund eines Beschlusses
des Deutschen Bundestages

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